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MATH 1432 - QUIZ 3

Show your work to get proper credit. Formula: $P(t) = P_0 \cdot e^{rt}$

P_0: initial date
r: rate
t: time

(1)[5 Pts] A population P of insects increases at a rate proportional to the current population. Suppose there are 500 insects initially and 1,000 insects 7 days later. $t=7$

P_0 $\quad P(7)$

3 (a) Find an expression for the number $P(t)$ of insects at any time t.

2 (b) How many insects will there be in 14 days? In 49 days?

(a) $P(t) = P_0 e^{rt}$ $\Rightarrow$ $\boxed{P(t) = 500 e^{t \cdot \frac{1}{7}\ln 2}}$

$1000 = P(7) = 500 \cdot e^{r \cdot 7}$

$\div 500$

$\Rightarrow 2 = \frac{1000}{500} = \frac{500}{500} \cdot e^{r7}$

Take "ln"

$\Rightarrow \ln 2 = \ln e^{r7} = r7$

$\Rightarrow r = \frac{1}{7}\ln 2$

(b) $t = 14$ (days)

We have

$P(14) = \boxed{500 e^{14 \cdot \frac{\ln 2}{7}}}$

$= 500 \cdot e^{2\ln 2} = 500 e^{\ln 2^2} = 500 \cdot 2^2$

$= 2000$

$t = 49$.

We have

$P(49) = \boxed{500 e^{49 \cdot \frac{\ln 2}{7}}}$

$= 500 e^{7\ln 2}$

$= 500 e^{\ln 2^7} = 500 \cdot 2^7$

$= 64000$

(2)[2 Pts] Differentiate the function $f(x) = \tan^{-1}\sqrt{4x}$.

$f'(x) = \frac{1}{1+(\sqrt{4x})^2} \cdot \frac{1}{2}\frac{1}{\sqrt{4x}} \cdot 4$

$= \frac{2}{\sqrt{4x}[1+(\sqrt{4x})^2]} = \boxed{\frac{2}{\sqrt{4x}(1+4x)}}$ or $\frac{1}{\sqrt{x}(1+4x)}$

(3)[3 Pts] Evaluate the indefinite integral:

$$\int \frac{1}{x\sqrt{1-(\ln x)^2}}\,dx.$$

let $u = \ln x$, $du = \frac{dx}{x}$

$= \int \frac{1}{\sqrt{1-u^2}}\,du = \arcsin(u) + c$.

$= \boxed{\arcsin(\ln x) + c}$