NAME: Sol

PS ID:

MATH 1432 - QUIZ 10

August 12, 2014

Show your work to get proper credit.

(1)[4 Pts] Use the basic divergence test to check if the series converges or diverges:

(a)

$$\sum_{k=1}^{\infty} \frac{4}{28+3^{-k}}$$

let $a_k = \frac{4}{28+3^{-k}} = \frac{4}{28+\frac{1}{3^k}}$. $\lim_{k\to\infty} a_k = \frac{4}{28} \neq 0$ By B.D.T. $\left(\frac{1}{3^k} \to 0 \text{ as } k\to\infty\right)$

This series diverges

(b)

$$\sum_{k=1}^{\infty} \frac{5\ln(2+k)}{3(2+k)}$$

let $a_k = \frac{5\ln(2+k)}{3(2+k)}$, $\lim_{k\to\infty} a_k = \lim_{k\to\infty} \frac{5\ln(2+k)}{3(2+k)} \underset{(\frac{\infty}{\infty})}{\overset{(L')}{=}} \lim_{k\to\infty} \frac{\frac{5}{2+k}}{3} = 0$

BDT fails.

Try integral test. check $\int_1^{\infty} \frac{5\ln(2+x)}{3(2+x)}dx \overset{\text{Improper}}{=} \lim_{b\to\infty} \int_0^b \frac{5}{3}\ln(2+x)\cdot\frac{dx}{(2+x)} = \lim_{b\to\infty}\left[\frac{5}{3}(\ln(2+x))^2\right]_0^b$

$\Rightarrow$ diverges

(2)[4 Pts] Use the geometric series test to check if the series converges or diverges. If it converges, what does it converge to?

(a)

$$\sum_{n=1}^{\infty} \left(\frac{5}{3}\right)^{n-1} = \sum_{n=1}^{\infty} \frac{3}{5}\left(\frac{5}{3}\right)^n$$

since $\left(\frac{5}{3}\right) > 1 \Rightarrow$ diverges by geometric series.

(b)

$$\sum_{n=1}^{\infty} \frac{5}{3^n} = \sum_{n=1}^{\infty} 5\cdot\left(\frac{1}{3}\right)^n = \frac{\frac{5}{3}}{1-\frac{1}{3}} = \frac{5}{3}\cdot\frac{3}{2} = \frac{5}{2}$$ converges

$\boxed{a = \frac{5}{3}\ (\text{as } n=1),\ r = \frac{1}{3} < 1}$ $\sum_{n=0}^{\infty} ar^n = \frac{a}{1-r}$

(3)[2 Pts] Use the integral test to check if the series converges or diverges:

$$\sum_{k=1}^{\infty} \frac{5\ln(2+k)}{3(2+k)}$$

See (1) part (b)

$$\int_1^{\infty} \frac{5\ln(2+x)}{3(2+x)}dx = \lim_{b\to\infty}\left[\frac{5}{3}(\ln(2+x))^2\right]_0^b$$

$$= \lim_{b\to\infty}\left[\underbrace{\frac{5}{3}(\ln(2+b))^2}_{\text{diverges}} - \underbrace{\frac{5}{3}(\ln 2)^2}_{\text{Fixed}}\right] \to \text{diverges}$$