PRINTABLE VERSION

Quiz 14

You scored 0 out of 100

Question 1

You did not answer the question.

Find the interval of convergence.

$$\sum (k+1)x^{k+4}$$

a) $(-1, 4)$

b) $(-4, 4)$

c) $(-1, 1)$

d) $[-1, 1]$

e) $[-4, 4]$

let $a_k=(k+1)x^{k+4}$.

$\left|\frac{a_{k+1}}{a_k}\right| = \left|\frac{(k+2)x^{k+5}}{(k+1)x^{k+4}}\right| = \left|\frac{k+2}{k+1}x\right| \to |x|<1$. as $k\to\infty$

$\Rightarrow |x|<1 \Rightarrow -1<x<1$

$x\in(-1,1)$.

Then check $x=1$. $\Rightarrow \Sigma(k+1)$ diverges (BDT) $\Rightarrow x\in(-1,1)$

$x=-1 \Rightarrow \Sigma(k+1)(-1)^{k+4}$ diverges. (BDT)

Question 2

You did not answer the question.

Find the interval of convergence.

$$\sum \frac{x^k}{(2k+2)!}$$

a) $[-2, 2]$

b) $(-2, 2)$

c) $(-\infty, \infty)$

d) $[-1, 1)$

e) $(0, 2)$

let $a_k=\frac{x^k}{(2k+2)!}$

$\left|\frac{a_{k+1}}{a_k}\right| = \left|\frac{x^{k+1}}{[2(k+1)+2]!}\cdot\frac{(2k+2)!}{x^k}\right| = \left|\frac{1}{(2k+4)(2k+3)}x\right| \to 0$ as $k\to\infty$

$(2k+2+2)! = (2k+4)!$

$= (2k+4)(2k+3)(2k+2)!$

which is alway "<1" $\Rightarrow x\in(-\infty,\infty)$

Question 3

You did not answer the question.

Find the interval of convergence.

$$\sum \frac{2^k x^k}{(k+3)^2}$$

a) $\left[-\frac{1}{2}, \frac{1}{2}\right]$

b) $[-3, 3]$

c) $[-2, 2]$

d) $(-1, 1)$

e) $\left(-\frac{1}{2}, \frac{1}{2}\right)$

let $a_k=\frac{2^k x^k}{(k+3)^2}$

$\left|\frac{a_{k+1}}{a_k}\right| = \left|\frac{2^{k+1}x^{k+1}}{(k+4)^2}\cdot\frac{(k+3)^2}{2^k x^k}\right|$

$= \left|\frac{(k+3)^2}{(k+4)^2}\cdot 2x\right| \to |2x|<1 \Rightarrow |x|<\frac{1}{2}$. as $k\to\infty$

$\Rightarrow -\frac{1}{2}<x<\frac{1}{2}$

$\Rightarrow x\in\left(-\frac{1}{2},\frac{1}{2}\right)$

check $x=\frac{1}{2}$, $\Sigma\frac{2^k\cdot(\frac{1}{2})^k}{(k+3)^2} = \Sigma\frac{1}{(k+3)^2}$ converges (p-series)

$x=-\frac{1}{2}$, $\Sigma\frac{2^k(-\frac{1}{2})^k}{(k+3)^2} = \Sigma\frac{(-1)^k}{(k+3)^2}$ converges (Alternating)

$\Rightarrow x\in\left[-\frac{1}{2},\frac{1}{2}\right]$

Question 4

You did not answer the question.

Find the interval of convergence.

$$\sum \frac{x^k}{(k+3)3^k}$$

a) $[-4, 4]$

b) $(-4, 4)$

c) $[-1, 1]$

d) $(-3, 3)$

e) $[-3, 3)$

let $a_k=\frac{x^k}{(k+3)3^k}$, then $\left|\frac{a_{k+1}}{a_k}\right| = \left|\frac{x^{k+1}}{(k+4)3^{k+1}}\cdot\frac{(k+3)3^k}{x}\right|$

$= \left|\frac{k+3}{k+4}\cdot\frac{x}{3}\right| \to \left|\frac{x}{3}\right|<1 \Rightarrow -3<x<3$

check $x=3$, $\Sigma\frac{3^k}{(k+3)3^k} = \Sigma\frac{1}{k+3}$ diverges (p-series)

check $x=-3$ $\Sigma\frac{(-3)^k}{(k+3)3^k} = \Sigma\frac{(-1)^k}{(k+3)}$ converge (Alternating)

$\Rightarrow x\in[-3,3)$

Question 5

You did not answer the question.

Find the interval of convergence.

$$\sum \frac{x^k}{(k+5)^2 3^k}$$

a) $[-5, 5]$

b) $(-3, 3)$

let $a_k=\frac{x^k}{(k+5)^2 3^k} \to \left|\frac{a_{k+1}}{a_k}\right| = \left|\frac{x^{k+1}}{(k+6)^2 3^{k+1}}\cdot\frac{(k+5)^2 3^k}{x^k}\right|$

$= \left|\frac{(k+5)^2}{(k+6)^2}\cdot\frac{x}{3}\right| \to \left|\frac{x}{3}\right|<1$

$\Rightarrow -3<x<3$

check $x=3$, $\Sigma\frac{3^k}{(k+5)^2 3^k} = \Sigma\frac{1}{(k+5)^2}$ converge (p-series) $\Rightarrow x\in[-3,3]$

check $x=-3$, $\Sigma\frac{(-3)^k}{(k+5)^2 3^k} = \Sigma\frac{(-1)^k}{(k+5)^2}$ converges (Alternating)

c) [-1, 1]

d) [-3, 3]

e) (-5, 5)

Q6, let $a_k = \frac{(k-5)x^{k+4}}{k-4}$.

$\left|\frac{a_{k+1}}{a_k}\right| = \left|\frac{[(k+1)-5]\,x^{k+5}}{(k+1)-4}\cdot\frac{k-4}{(k-5)x^{k+4}}\right|$

$= \left|\frac{(k-4)(k-4)}{(k-3)(k-5)}\cdot x\right| \to |x|<1 \Rightarrow -1<x<1$

check $x=-1$, $\sum \frac{(k-5)(-1)^{k+4}}{k-4}$ diverges (BDT)

check $x=1$, $\sum \frac{(k-5)}{(k-4)}1^{k+4}$ diverges (BDT)

$\Rightarrow x\in(-1,1)$

Question 6

You did not answer the question.

Find the interval of convergence.

$$\sum \frac{(k-5)x^{k+4}}{k-4}$$

a) [-4, 4)

b) [-1, 1)

c) [-1, 1]

d) (-4, 4]

e) (-1, 1)

Q7. let $a_k = \frac{4k^2x^{k+1}}{e^{k+1}}$. $\left|\frac{a_{k+1}}{a_k}\right| = \left|\frac{4(k+1)^2x^{k+2}}{e^{k+2}}\cdot\frac{e^{k+1}}{4k^2x^{k+1}}\right|$

$= \left|\frac{(k+1)^2}{k^2}\frac{1}{e}\cdot x\right| \to \left|\frac{x}{e}\right|<1 \Rightarrow -e<x<e$

Check $x=e$, $\sum \frac{4k^2e^{k+1}}{e^{k+1}} = \sum 4k^2$ diverges (BDT)

$x=-e$ $\sum 4k^2\frac{(-e)^{k+1}}{e^{k+1}} = \sum (-1)^{k+1}4k^2$ diverges (BDT)

$\Rightarrow x\in(-e,e)$

Question 7

You did not answer the question.

Find the interval of convergence.

$$\sum \frac{4k^2x^{k+1}}{e^{k+1}}$$

a) [-1, 1]

b) (-e, e)

c) [-e, e]

d) (-1, 1)

e) (-4, 4)

Question 8

You did not answer the question.

Q8. let $a_k = \frac{(-1)^k(x-8)^k}{k^k}$.

$\sqrt[k]{|a_k|} = \sqrt[k]{\frac{(x-8)^k}{k^k}} = \frac{x-8}{k} \to 0$ which is always "< 1"

$\Rightarrow x\in(-\infty,\infty)$

Find the interval of convergence.

$$\sum \frac{(-1)^k(x-8)^k}{k^k}$$

a) (-8, 8)

b) $(-\infty, \infty)$

c) [-8, 8]

d) [-1, 1]

e) (-1, 1)

Q9. let $a_k = (k+2)!\,x^{k+3}$

$\left|\frac{a_{k+1}}{a_k}\right| = \left|\frac{(k+3)!\,x^{k+4}}{(k+2)!\,x^{k+3}}\right| = |k+3||x|<1$

$\Leftrightarrow |x|=0$ (only one point)

Question 9

You did not answer the question.

Find the interval of convergence.

$$\sum (k+2)!\,x^{k+3}$$

a) (-2, 2)

b) {0}

c) {1}

d) [-1, 1]

e) (-1, 1)

Q10, let $a_k = \frac{(-1)^k6^kx^k}{8^{k+1}}$

$\left|\frac{a_{k+1}}{a_k}\right| = \left|\frac{(-1)^{k+1}6^{k+1}x^{k+1}}{8^{k+2}}\cdot\frac{8^{k+1}}{(-1)^k6^kx^k}\right|$

$= \left|\frac{6}{8}\cdot x\right|<1 \Rightarrow |x|<\frac{4}{3} \Rightarrow -\frac{4}{3}<x<\frac{4}{3}$

Check $x=\frac{4}{3}$, $\sum\frac{(-1)^k6^k(\frac{4}{3})^k}{8^{k+1}} = \sum\frac{(-1)^k(2)^k}{8\cdot(2)^k} = \sum\frac{(-1)^k}{8}$ Diverges (BDT)

Question 10

You did not answer the question.

Find the interval of convergence.

$$\sum \frac{(-1)^k6^kx^k}{8^{k+1}}$$

$x=-\frac{4}{3}$, $\sum\frac{(-1)^k6^k(-\frac{4}{3})^k}{8^{k+1}} = \sum\frac{(-1)^{2k}}{8}$ Diverges (BDT)

a) $\left[-\frac{1}{6}, \frac{1}{6}\right]$

b) (-8, 8)

$\Rightarrow x\in\left(-\frac{4}{3},\frac{4}{3}\right)$

c) $\left(-\frac{3}{4}, \frac{3}{4}\right)$

d) $\left[-\frac{4}{3}, \frac{4}{3}\right]$

e) $\left(-\frac{4}{3}, \frac{4}{3}\right)$

Q11. let $a_k = \frac{(-1)^k k!(x-2)^k}{(k+1)^3}$

$$\left|\frac{a_{k+1}}{a_k}\right| = \left|\frac{(-1)^{k+1}(k+1)!\,(x-2)^{k+1}}{(k+2)^3} \cdot \frac{(k+1)^3}{(-1)^k k!\,(x-2)^k}\right|$$

$$= \left|\frac{k+1}{1} \frac{(k+1)^3}{(k+2)^3} \cdot (x-2)\right| \to |k+1|\,|(x-2)| < 1.$$

$\Leftrightarrow$ $x = 2$. (Only one point)

Question 11

You did not answer the question.

Find the interval of convergence.

$$\sum \frac{(-1)^k k!\,(x-2)^k}{(k+1)^3}$$

a) $\{0\}$

b) $\{2\}$

c) $(-2, 2)$

d) $(-1, 1)$

e) $\{1\}$

Q12. $a_k = \frac{(-1)^k k^2 (x+2)^k}{(k+3)!}$

$$\left|\frac{a_{k+1}}{a_k}\right| = \left|\frac{(-1)^{k+1}(k+1)^2(x+2)^{k+1}}{(k+4)!} \cdot \frac{(k+3)!}{(-1)^k k^2 (x+2)^k}\right|$$

$$= \left|\frac{1}{k+4} \cdot \frac{(k+1)^2}{k^2} \cdot (x+2)\right| \to 0 \text{ (always } < 1)$$

$\Rightarrow x \in (-\infty, \infty)$

Question 12

You did not answer the question.

Find the interval of convergence

$$\sum \frac{(-1)^k k^2 (x+2)^k}{(k+3)!}$$

a) $(-1, 1)$

b) $\{0\}$

c) $[-1, 1]$

d) $\{1\}$

e) $(-\infty, \infty)$

Q13. let $a_k = \frac{k^3 (x-10)^k}{e^k}$

$$\left|\frac{a_{k+1}}{a_k}\right| = \left|\frac{(k+1)^3 (x-10)^{k+1}}{e^{k+1}} \cdot \frac{e^k}{k^3 (x-10)^k}\right| = \left|\frac{(k+1)^3}{k^3} \cdot \frac{1}{e} (x-10)\right|$$

Question 13

You did not answer the question.

$\to \left|\frac{1}{e}(x-10)\right| < 1$

Find the interval of convergence.

$$\sum \frac{k^3 (x-10)^k}{e^k}$$

$\Rightarrow -e < x - 10 < e$

$\Rightarrow 10 - e < x < e + 10$

a) $[-1, 1]$

check $x = 10 - e$, $\sum \frac{k^3(-e)^k}{e^k} = \sum k^3 (-1)^k$ diverges (BDT).

b) $(-e + 10, e + 10)$

c) $[-e + 10, e + 10]$

check $x = 10 + e$. $\sum \frac{k^3 e^k}{e^k} = \sum k^3$ diverges (BDT)

d) $(-e, e)$

e) $(-1, 1)$

$\Rightarrow x \in (10 - e, e + 10)$

Question 14

You did not answer the question.

Find the interval of convergence.

$$\sum \frac{(-1)^k (k+4) x^k}{2^k}$$

let $a_k = \frac{(-1)^k (k+4) x^k}{2^k}$ $\quad \left|\frac{a_{k+1}}{a_k}\right| = \left|\frac{(-1)^{k+1}(k+5)x^{k+1}}{2^{k+1}} \cdot \frac{2^k}{(-1)^k (k+4) x^k}\right|$

a) $(-6, 2)$

$= \left|\frac{k+5}{k+4} \cdot \frac{x}{2}\right| \to \left|\frac{x}{2}\right| < 1 \Rightarrow -2 < x < 2$

b) $(-4, 0)$

c) $(-2, 2)$

check $x = 2$, $\sum \frac{(-1)^k (k+4) 2^k}{2^k} = \sum (-1)^k (k+4)$ diverges (BDT)

d) $(-2, 6)$

e) $(-4, 4)$

check $x = -2$, $\sum \frac{(-1)^k (k+4)(-2)^k}{2^k} = \sum (k+4)$ diverges (BDT)

$\Rightarrow x \in (-2, 2)$

Question 15

You did not answer the question.

Expand in powers of x.

$$\frac{1}{(1-x)^7} = \left(\frac{1}{1-x}\right)^7 = \left(\sum x^n\right)^7.$$

$= (1 + x + x^2 + \cdots)^7$

$= 1 + 7x + (7+6+5+4+3+2+1)x^2 + \cdots$

$= 1 + 7x + \frac{7 \cdot 8}{2} x^2 + \cdots$

a) $1+7x+\frac{7}{2}(8)x^2+\cdots+\frac{(n+6)!\,x^n}{n!\,(6)!}+\cdots$

b) $1+14x+\frac{7}{4}(8)x^2+\cdots+\frac{(n+6)!\,x^n}{(n-1)!\,(7)!}+\cdots$

c) $1+7x+\frac{7}{2}(6)x^2+\cdots+\frac{(n-5)!\,x^n}{n!\,(6)!}+\cdots$

d) $1+7x+\frac{7}{4}(8)x^2+\cdots+\frac{(n+6)!\,x^n}{n!\,(7)!}+\cdots$

e) $1+14x+7(8)x^2+\cdots+\frac{(n+6)!\,x^n}{(n-1)!\,(6)!}+\cdots$

$$(*)\ \tan(x) = x+\frac{x^3}{3}+\frac{2}{15}x^5+\frac{17}{315}x^7+\cdots$$

$$Q16.\ [\ln(1-9x)]' = \frac{-9}{1-9x} = \sum_{n=0}^{\infty} -9(9x)^n$$

(compare with $\frac{a}{1-r}=\sum ar^n$)

$$\Downarrow$$

① $\ln(1-9x) = \int \sum_{n=0}^{\infty} -9(9x)^n\,dx$

$$= -9\sum_{n=0}^{\infty}\int 9^n x^n\,dx$$

$$= -9\sum_{n=0}^{\infty}\frac{9^n}{n+1}x^{n+1}+C$$

② Find C, let $x=0$

$$\Rightarrow 0=\ln 1 = -9\sum_{n=0}^{\infty}\frac{9^n}{n+1}\cdot 0^{n+1}+C$$

$$= 0+C \Rightarrow C=0$$

③ $\ln(1-9x) = \sum_{n=0}^{\infty}\frac{-9^{n+1}x^{n+1}}{n+1}$

Question 16

You did not answer the question.

Expand in powers of x.

$$\ln(1-9x)$$

a) $-9x-\frac{1}{3}9^{(2)}x^2-\frac{1}{4}9^{(3)}x^3-\cdots-\frac{9^{n+1}x^{n+1}}{n+2}-\cdots$

b) $9x+\frac{9}{2}x^2+\frac{1}{3}9^{(2)}x^3+\cdots+\frac{9^n x^n}{n+1}+\cdots$

c) $-9x-\frac{1}{2}9^{(2)}x^2-\frac{1}{3}9^{(3)}x^3-\cdots-\frac{9^{n+1}x^{n+1}}{n+1}-\cdots$

d) $-9x-9^{(2)}x^2-\frac{1}{2}9^{(3)}x^3-\cdots-\frac{9^{n+1}x^{n+1}}{n}-\cdots$

e) $9x+\frac{1}{2}9^{(2)}x^2+\frac{1}{3}9^{(3)}x^3+\cdots+\frac{9^{n+1}x^{n+1}}{n+1}+\cdots$

$$Q17.\ \int 4\sec^2(4x)\,dx$$

$$= \tan(4x)+C$$

$$= 4x+\frac{(4x)^3}{3}+\frac{2}{15}(4x)^5+\frac{17}{315}(4x)^7+\cdots$$

$$\Rightarrow 4\sec^2(4x)$$

$$= [\tan(4x)]'$$

Question 17

You did not answer the question.

Expand in powers of x.

$$4\sec^2(4x) = 4+4^3x^2+\frac{2\cdot 4^5}{3}x^4+\frac{17\cdot 4^7}{45}x^6$$

a) $-4-4^{(2)}x^3-\frac{2}{3}4^{(4)}x^5-\frac{17}{45}4^{(6)}x^7-\cdots$

b) $4+4^{(2)}x^2+\frac{2}{3}4^{(4)}x^4+\frac{17}{45}4^{(6)}x^6+\cdots$

c) $-4-4^{(3)}x^3-\frac{2}{3}4^{(5)}x^5-\frac{17}{45}4^{(7)}x^7-\cdots$

d) $4+4^{(3)}x^2+\frac{2}{3}4^{(5)}x^4+\frac{17}{45}4^{(7)}x^6+\cdots$

e) $4+4^{(3)}x^3+\frac{2}{3}4^{(5)}x^5+\frac{17}{45}4^{(7)}x^7+\cdots$

$$Q18.\ [\ln(\cos(4x))]' = \frac{-\sin(4x)}{\cos(4x)}\cdot 4 = -4\tan(4x)$$

$$= -4\cdot 4x-4\frac{(4x)^3}{3}-4\cdot\frac{2}{15}(4x)^5-4\frac{17}{315}(4x)^7+\cdots$$

$$\ln(\cos(4x)) = \int -4\tan(4x)\,dx$$

Question 18

You did not answer the question.

Expand in powers of x.

$$\ln(\cos(4x)) = -\frac{1}{2}4^2x-\frac{4^4}{12}x^4-\frac{4^6}{45}x^6-\frac{17}{2520}4^8x^8+\cdots$$

a) $-\frac{1}{2}4^{(2)}x^2-\frac{1}{12}4^{(3)}x^4-\frac{1}{45}4^{(4)}x^6-\frac{17}{2520}4^{(5)}x^8-\cdots$

b) $\frac{1}{2}4^{(2)}x^2+\frac{1}{12}4^{(4)}x^4+\frac{1}{45}4^{(6)}x^6+\frac{17}{2520}4^{(8)}x^8+\cdots$

c) $\frac{1}{3}4^{(2)}x^2+\frac{2}{15}4^{(4)}x^4+\frac{17}{315}4^{(6)}x^6+\frac{17}{2520}4^{(8)}x^8+\cdots$

d) $-\frac{1}{2}4^{(2)}x^2-\frac{1}{12}4^{(4)}x^4-\frac{1}{45}4^{(6)}x^6-\frac{17}{2520}4^{(8)}x^8-\cdots$

e) $-\frac{1}{3}4^{(2)}x^2-\frac{2}{15}4^{(3)}x^4-\frac{17}{315}4^{(4)}x^6-\frac{17}{2520}4^{(5)}x^8-\cdots$

Question 19

You did not answer the question.

Expand in powers of x.

$$\frac{6x}{1-x^2} = 6x\cdot\frac{1}{1-x^2} = 6x\sum_{n=0}^{\infty}(x^2)^n$$

(compare with $\frac{a}{1-r}=\sum_{n=0}^{\infty}ar^n$)

$$= 6x\sum_{n=0}^{\infty}x^{2n} = \sum_{n=0}^{\infty}6x^{2n+1}.$$

a) $\sum_{k=0}^{\infty}(-6x^{2k})$

b) $\sum_{k=0}^{\infty}(-6x^{2k+1})$

c) $\sum_{k=0}^{\infty}6^k x^{4k+2}$

d) $\sum_{k=0}^{\infty}6x^{2k-1}$

e) $\boxed{\sum_{k=0}^{\infty}6x^{2k+1}}$

Q20. ⊛ $\ln(1+x) = 0 + x + \frac{(-1)}{2}x^2 + \frac{1}{3}x^3 + \frac{1}{4}x^4 + \frac{x^5}{5} - \frac{\cdots}{6} \cdots$

$= \sum_{k=1}^{\infty} (-1)^{k+1}\frac{x^k}{k}$

$$\left(\begin{aligned} f(x) &= \ln(1+x) && \Rightarrow f(0) = 0 \\ f'(x) &= (1+x)^{-1} && \Rightarrow f'(0) = 1 \\ f''(x) &= (-1)(x+1)^{-2} && \Rightarrow f''(0) = (-1) \\ f'''(x) &= (-1)(-2)(x+1)^{-3} && \Rightarrow f'''(0) = (-1)(-2) \\ &\vdots \\ f^{(n)}(x) &= (-1)(-2)\cdots(-(n-1))(x+1)^{-n} && \Rightarrow f^{(n)}(0) = (-1)^{n-1}(n-1)! \\ &= \frac{(-1)^{n-1}(n-1)!}{(x+1)^n} \end{aligned}\right)$$

Question 20

You did not answer the question.

Expand in powers of x.

$5x\ln(1+x^6)$ $= 5x \cdot \sum_{k=1}^{\infty} (-1)^{k+1}\frac{(x^6)^k}{k} = 5x\sum_{k=1}^{\infty}(-1)^{k+1}\frac{x^{6k}}{k}$

$= \sum_{k=1}^{\infty}\frac{(-1)^{k+1}5x^{6k+1}}{k}$

a) $\sum_{k=1}^{\infty}\frac{5(-1)^k x^{6k-1}}{k}$

b) $\sum_{k=1}^{\infty}\frac{(-1)^{k+1}5^k x^{6k}}{k}$

c) $\sum_{k=1}^{\infty}\frac{(-1)^{k+1}5^k x^{6k+1}}{k}$

d) $\boxed{\sum_{k=1}^{\infty}\frac{5(-1)^{k+1}x^{6k+1}}{k}}$

e) $\sum_{k=1}^{\infty}\frac{5(-1)^k x^{6k+1}}{k}$

* $\boxed{\text{Review}}$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots + \frac{x^n}{n!} + \cdots = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} + \cdots + (-1)^{n+1}\frac{x^n}{n} + \cdots = \sum_{n=0}^{\infty} (-1)^{n+1}\frac{x^n}{n}.$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots + (-1)^n \frac{x^{2n}}{(2n)!} + \cdots = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$$

$$\sin x = \frac{x}{1} - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots + (-1)^n \frac{x^{2n+1}}{(2n+1)!} + \cdots = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$$

$$\tan x = x + \frac{x^3}{3} + \frac{2}{15}x^5 + \frac{17}{315}x^7 + \cdots$$