Tayby polynomial P_n at $x=0$ for $f(x)$

$$P_n = f(0) + \frac{f'(0)}{1!}x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \cdots + \frac{f^{(n)}(0)}{n!}x^n$$

PRINTABLE VERSION

Quiz 13

You scored 0 out of 100

Question 1

You did not answer the question.

Find the Taylor polynomial $P_4(x)$ centered at $x=0$ for the given function.

$f(x) = x - 3\cos(x)$ $\Rightarrow f(0) = -3$

$f'(x) = 1 + 3\sin(x) \Rightarrow f'(0) = 1$

$f''(x) = 3\cos(x) \Rightarrow f''(0) = 3$

$f'''(x) = -3\sin(x) \Rightarrow f'''(0) = 0$

$f^{(4)}(x) = -3\cos(x) \Rightarrow f^{(4)}(0) = -3$

$P_4(x) = -3 + x + \frac{3}{2!}x^2 + \frac{0}{3!}x^3 - \frac{3}{4!}x^4$

$= -3 + x + \frac{3}{2}x^2 - \frac{1}{8}x^4$

a) $-3 + x + \frac{3}{2}x^2 - \frac{1}{4}x^4$

b) $-1 + 3x + x^2 + \frac{1}{4}x^4$

c) $\boxed{-3 + x + \frac{3}{2}x^2 - \frac{1}{8}x^4}$

d) $3 - x - \frac{3}{2}x^2 - \frac{1}{8}x^4$

e) $-3 - x + \frac{3}{4}x^2 - \frac{1}{4}x^4$

Question 2

You did not answer the question.

Find the Taylor polynomial $P_4(x)$ centered at $x=0$ for the given function.

$f(x) = 8\sqrt{1+x} = 8(1+x)^{\frac{1}{2}} \Rightarrow f(0) = 8$

$f'(x) = 4(1+x)^{-\frac{1}{2}} \Rightarrow f'(0) = 4$

$f''(x) = -2(1+x)^{-\frac{3}{2}} \Rightarrow f''(0) = -2$

$f'''(x) = 3(1+x)^{-\frac{5}{2}} \Rightarrow f'''(0) = 3$

$f^{(4)}(x) = -\frac{15}{2}(1+x)^{-\frac{7}{2}} \Rightarrow f^{(4)}(0) = -\frac{15}{2}$

a) $-8 - 4x - x^2 - \frac{1}{2}x^3 - \frac{5}{16}x^4$

b) $8 + 4x + \frac{1}{2}x^2 + \frac{1}{4}x^3 + \frac{7}{16}x^4$

c) $\boxed{8 + 4x - x^2 + \frac{1}{2}x^3 - \frac{5}{16}x^4}$

$P_4(x) = 8 + \frac{4}{1!}x + \frac{-2}{2!}x^2 + \frac{3}{3!}x^3 - \frac{15}{2}\cdot\frac{1}{4!}x^4$

$= 8 + 4x - x^2 + \frac{x^3}{2} - \frac{15}{48}x^4$

d) $8 + 4x + x^2 + \frac{1}{2}x^3 + \frac{5}{16}x^4$

e) $8 + 2x - \frac{1}{2}x^2 + \frac{1}{4}x^3 - \frac{5}{32}x^4$

$(*)$ $\cos(x) = 1 - \frac{1}{2!}x^2 + \frac{1}{4!}x^4 - \frac{1}{6!}x^6 + \frac{1}{8!}x^8 -$

$\sin(x) = x - \frac{1}{3!}x^3 + \frac{1}{5!}x^5 - \frac{1}{7!}x^7 + \cdots$

Question 3

You did not answer the question.

Find the Taylor polynomial $P_5(x)$ centered at $x=0$ for the given function.

$f(x) = 3x\cos(4x^2)$

$\boxed{\text{Method 1}}$ Product rule.

$f'(x) = 3\cos(4x^2) + 3x\cdot 16x(-\sin(4x^2))$

$f''(x)$ ……

or $\boxed{\text{Method 2}}$ by $(*)$.

$\cos(4x^2) = 1 - \frac{1}{2!}[4x^2]^2 + \frac{1}{4!}[4x^2]^4$

$= 1 - \frac{16}{2}x^4 + \cancel{\frac{16\cdot 16}{24}x^8} \Rightarrow$ We don't need this term for finding P_5

$\Rightarrow P_5 = 3x\left(1 - \frac{16}{2}x^4\right) = 3x - 24x^5$

a) $-3x - 12x^5$

b) $3x + 6x^5$

c) $-x - 2x^5$

d) $x + 24x^5$

e) $\boxed{3x - 24x^5}$

Question 4

You did not answer the question.

Determine the nth Taylor polynomial P_n centered at $x=0$ for the given function.

$f(x) = e^{-7x}$

$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots + \frac{x^n}{n!} + \cdots$

$= \sum_{n=0}^{\infty} \frac{x^n}{n!}$

$\Rightarrow e^{-7x} = \sum_{n=0}^{\infty} \frac{(-7x)^n}{n!}$

$= \sum_{n=0}^{\infty} \frac{(-1)^n \cdot 7^n}{n!}x^n$

a) $\sum_{k=1}^{n} \frac{(-1)^{k+1}7^k x^k}{k!}$

b) $\sum_{k=1}^{n} \frac{7^k x^k}{k!}$

c) $\sum_{k=0}^{n} \frac{7^{k+1} x^k}{k!}$

d) $\boxed{\sum_{k=0}^{n} \frac{(-1)^k 7^k x^k}{k!}}$

e) $\sum_{k=0}^{n} \frac{(-1)^k x^k}{k!}$

Question 5

You did not answer the question.

Determine the nth Taylor polynomial P_n centered at $x = 0$ for the given function.

$f(x) = \sinh(9x) = \frac{e^{9x} - e^{-9x}}{2}$

a) $\sum_{k=0}^{\frac{1}{2}(n-1)} \frac{(-1)^k 9^{2k-1} x^{2k-1}}{(2k-1)!}$

b) $\sum_{k=0}^{\frac{1}{2}n} \frac{9^{2k-1} x^{2k-1}}{(2k-1)!}$

c) $\sum_{k=1}^{n} \frac{(-1)^k 9^{2k} x^{2k}}{(2k)!}$

d) $\sum_{k=0}^{\frac{1}{2}(n-1)} \frac{9^{2k+1} x^{2k+1}}{(2k+1)!}$

e) $\sum_{k=0}^{n} \frac{x^{2k+1}}{(2k+1)!}$

Since $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + \frac{x^6}{6!} + \cdots$

Then $e^{9x} = 1 + (9x) + \frac{(9x)^2}{2!} + \frac{(9x)^3}{3!} + \frac{(9x)^4}{4!} + \frac{(9x)^5}{5!} + \frac{(9x)^6}{6!} + \cdots$

and $e^{-9x} = 1 - 9x + \frac{(9x)^2}{2!} - \frac{(9x)^3}{3!} + \frac{(9x)^4}{4!} - \frac{(9x)^5}{5!} + \frac{(9x)^6}{6!} + \cdots$

$\Rightarrow e^{9x} - e^{-9x} = 18x + 2\frac{(9x)^3}{3!} + 2\frac{(9x)^5}{5!} + \cdots$

$\Rightarrow \frac{e^{9x} - e^{-9x}}{2} = 9x + \frac{(9x)^3}{3!} + \frac{(9x)^5}{5!} + \cdots$

$= \sum_{k=0}^{\infty} \frac{(9x)^{2k+1}}{(2k+1)!} \Rightarrow P_n = \sum_{k=0}^{\frac{n-1}{2}} \frac{(9x)^{2k+1}}{(2k+1)!}$

$(2k+1 \leq n) \Rightarrow k \leq \frac{n-1}{2}$

Question 6

You did not answer the question.

Determine the nth Taylor polynomial P_n centered at $x = 0$ for the given function.

$f(x) = e^{9x}$

a) $\sum_{k=0}^{n} \frac{(-1)^k x^k}{k!}$

b) $\sum_{k=0}^{n} \frac{9^{k+1} x^{k+1}}{(k+1)!}$

c) $\sum_{k=0}^{n} \frac{9^k x^k}{k!}$

d) $\sum_{k=0}^{n} \frac{(-1)^k 9^k x^k}{k!}$

e) $\sum_{k=0}^{n} \frac{x^k}{k!}$

Since P_n of e^x at $x=0$ is

$1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots + \frac{x^n}{n!} = \sum_{k=0}^{n} \frac{x^k}{k!}$

Then P_n of e^{9x} at $x=0$ is

$\sum_{k=0}^{n} \frac{(9x)^k}{k!} = \sum_{k=0}^{n} \frac{9^k x^k}{k!}$

Question 7

Q7

You did not answer the question.

Use the values in the table below and the formula for Taylor polynomials to give the 4th degree Taylor polynomial for f centered at $x = 0$.

$f(0)$	$f'(0)$	$f''(0)$	$f'''(0)$	$f^{(4)}(0)$
-5	-2	-2	4	-5

4^{th} degree $\Rightarrow P_4$

$P_4 = f(0) + \frac{f'(0)}{1!}x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \frac{f^{(4)}(0)}{4!}x^4$

$= -5 - \frac{2}{1!}x - \frac{2}{2!}x^2 + \frac{4}{3!}x^3 - \frac{5}{4!}x^4$

$= -5 - 2x - x^2 + \frac{2}{3}x^3 - \frac{5}{24}x^4$

a) $-5 - 2x - 2x^2 + 2x^3 - \frac{5}{6}x^4$

b) *not enough information*

c) $-5 - 2x - x^2 + \frac{2}{3}x^3 - \frac{5}{24}x^4$

d) $-5 - 2x - x^2 + \frac{4}{3}x^3 - \frac{5}{4}x^4$

e) $-5 - 2x - 2x^2 + 4x^3 - 5x^4$

Question 8

You did not answer the question.

Determine the nth Taylor polynomial P_n centered at $x = 0$ for the given function.

$f(x) = \cos(2x)$

Since P_n of $\cos x$ at $x=0$ is

$1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots + \text{—} = \sum_{k=0}^{\frac{n}{2}} \frac{x^{2k}}{(2k)!} \cdot (-1)^k$

Then

$\cos 2x = \sum_{k=0}^{\frac{n}{2}} \frac{(-1)^k (2x)^{2k}}{(2k)!}$

$= \sum_{k=0}^{\frac{n}{2}} \frac{(-1)^k 2^{2k} \cdot x^{2k}}{(2k)!}$

a) $\sum_{k=0}^{n} \frac{2^{2k} x^{2k}}{(2k)!}$

b) $\sum_{k=0}^{\frac{1}{2}n} \frac{(-1)^k 2^{2k} x^{2k}}{(2k)!}$

c) $\sum_{k=0}^{n} \frac{(-1)^k 2^{2k-1} x^{2k-1}}{(2k-1)!}$

d) $\sum_{k=0}^{\frac{1}{2}n} \frac{(-1)^k 2^{2k+1} x^{2k+1}}{(2k-1)!}$

e) $\sum_{k=0}^{\frac{1}{2}n} \frac{(-1)^{2k} 2^{2k} x^{2k}}{(2k)!}$

Question 9

You did not answer the question.

Let P_n be the nth Taylor Polynomial of the function $f(x)$ centered at $x = 0$. Assume that f is a function such that $|f^{(n)}(x)| \leq 1$ for all n and x (the sine and cosine functions have this property.) Estimate the error if $P_6(\frac{1}{3})$ is used to approximate $f(\frac{1}{3})$.

Q9. Use the Lagrange formula

$$\boxed{R_n = \frac{f^{n+1}(c)}{(n+1)!} x^{n+1}}$$

Now $n=6$.

We have the error is

$$R_6(\tfrac{1}{3}) = \frac{f^{7}(c)}{(6+1)!} \cdot \left(\tfrac{1}{3}\right)^{6+1}$$

$|f^{(n)}(x)| \leq 1$

$$\leq \frac{1}{7!} \cdot \left(\tfrac{1}{3}\right)^7 = \frac{(\frac{1}{3})^7}{7!}$$

a) $\frac{\left(\frac{1}{3}\right)^{(5)}}{(5)!}$

b) $\frac{1}{(6)!}$

c) $\frac{\left(\frac{1}{3}\right)^{(7)}}{(7)!}$

d) $\frac{1}{\left(\frac{1}{3}\right)^{(7)} (7)!}$

e) $\frac{\left(\frac{1}{3}\right)^{(6)}}{(6)!}$

Question 10

You did not answer the question.

Let P_n be the nth Taylor Polynomial of the function $f(x)$ centered at $x = 0$. Assume that f is a function such that $|f^{(n)}(x)| \leq 1$ for all n and x (the sine and cosine functions have this property.) Find the least integer n for which $P_n(4)$ approximates $f(4)$ to within 0.1

a) 11

b) 14

c) 13

d) 15

e) 9

Q10. Given $|f^{(n)}(x)| \leq 1$ $\forall n$ and x.

Find n s.t. $|R_n(4)| \leq 0.1$

$$\Rightarrow |R_n(4)| = \left|\frac{f^{(n+1)}(c)}{(n+1)!} \cdot 4^{n+1}\right| \leq \frac{4^{n+1}}{(n+1)!} \leq 0.1$$

$$\frac{4^{n+1}}{(n+1)!} \leq \frac{1}{10}$$

let $a_n = \frac{4^{n+1}}{(n+1)!}$

$a_9 = \frac{4^{10}}{10!} = 0.28$

$a_{10} = \frac{4^{11}}{11!} = 0.105$

$a_{11} = \frac{4^{12}}{12!} = 0.035$

by calculator

Question 11

You did not answer the question.

Use a Taylor polynomial centered at $x = 0$ to estimate the following to within 0.01.

$\sqrt{(e^{1.7})}$

a) 2.635

b) 2.035

c) 2.335

d) 2.435

e) 2.235

$\sqrt{e^{1.7}} = e^{\frac{1.7}{2}}$. let $f(x) = e^x$.

Find n s.t. $R_n(\frac{1.7}{2}) < 0.01$. Then find P_n.

But let us use a different way here.

$e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!}$, Now, try to

Find it term by term

$k=0$, $\frac{(\frac{1.7}{2})^0}{0!} = 1$

$k=1$, $\frac{(\frac{1.7}{2})^1}{1!} = \frac{1.7}{2}$

$k=2$, $\frac{(\frac{1.7}{2})^2}{2!} = 0.36125$

$k=3$, $\frac{(\frac{1.7}{2})^3}{3!} = 0.1023$

$k=4$ $\frac{(\frac{1.7}{2})^4}{4!} = 0.02175$

$k=5$, $\frac{(\frac{1.7}{2})^5}{5!} = 0.003$

STOP

$k=6$ $\frac{(\frac{1.7}{2})^6}{6!} = 0.00052$

too small

sum up (1.5) $\boxed{2.3}$

Question 12

You did not answer the question.

Use a Taylor polynomial centered at $x = 0$ to estimate the following to within 0.01

$\ln(1.20)$

a) 0.20

b) 0.18

c) 0.17

$\ln(1+x) = 0 + x - \frac{x^2}{2} + \frac{x^3}{3}$

let $f(x) = \ln(1+x)$.

$f^{(n+1)}(x) = \frac{(-1)^n n!}{(x+1)^{n+1}}$, and $x = 0.2 \Rightarrow c \in [0, 0.2]$

$$\Rightarrow |f^{(n+1)}(c)| \leq n! \Rightarrow R_n(0.2) = \left|\frac{f^{(n+1)}(c)}{(n+1)!} (0.2)^{n+1}\right|$$

$$\leq \frac{1}{n+1} (0.2)^{n+1} \leq 0.01 \underset{\text{try \& error}}{\Rightarrow} n = 3.$$

$P_3 = \ln 1 + \frac{2}{10} - \frac{4}{200} + \frac{8}{3000}$

$= 0.18 + 0.0026$

$$R_n = \frac{f^{(n+1)}(c)}{(n+1)!} x^{n+1}.$$

d) 0.16

e) 0.19

Question 13

You did not answer the question.

Find the Lagrange form of the remainder R_n for given function and the indicated integer n.

$f(x) = e^{5x}$

$n = 5$

$n=5$. To Find $f^{(6)}(x)$, we have

$f(x) = e^{5x}$,

$f'(x) = 5e^{5x}$

$f''(x) = 25e^{5x}$

$f'''(x) = 125e^{5x}$

$f^{(4)}(x) = 625e^{5x}$

$f^{(5)}(x) = 3125e^{5x}$

$f^{(6)}(x) = 5 \times 3125 e^{5x}$

$$R_6 = \frac{5 \cdot 3125 e^{5c}}{6!} \cdot x^6$$

$$= \frac{3125}{144} e^{5c} x^6$$

$|c| < |x|$

a) $\frac{125}{24} e^{5c} x^5, |c| < |x|$

b) $\frac{625}{144} e^{5c} x^6, |c| < |x|$

c) $\frac{625}{24} e^{5c} x^5, |c| < |x|$

d) $\frac{3125}{1008} e^{5c} x^7, |c| < |x|$

e) $\frac{3125}{144} e^{5c} x^6, |c| < |x|$

Question 14

You did not answer the question.

Find the Lagrange form of the remainder R_n for given function and the indicated integer n.

$f(x) = \cos(2x)$

$n = 4$

$n=4$. To find $f^{(5)}(x)$, we have

$f(x) = \cos(2x)$

$f'(x) = -2\sin(2x)$

$f''(x) = -4\cos(2x)$

$f'''(x) = +8\sin(2x)$

$f^{(4)}(x) = 16\cos(2x)$

$$R_4(x) = \frac{16\cos(2c)}{5!} \cdot x^5$$

$$= \frac{2\cos(2c)}{15} x^5$$

$|c| < |x|$

a) $\frac{2}{15}\cos(2c)x^5, |c| < |x|$

b) $\frac{2}{3}\cos(2c)x^5, |c| < |x|$

c) $-\frac{2}{45}\sin(2c)x^5, |c| < |x|$

d) $-\frac{4}{15}\sin(2c)x^5, |c| < |x|$

e) $\frac{1}{3}\sin(2c)x^5, |c| < |x|$

Q15, Given $x=0.3$, and $\varepsilon = 0.0001$,

To find n s.t. $R_n(0.3) \le 0.0001$

$f(x) = \ln(1+x)$.

$f'(x) = \frac{1}{1+x} = (1+x)^{-1}$

$f''(x) = -1(1+x)^{-2}$

$f'''(x) = (-1)(-2)(1+x)^{-3}$

$\vdots$

$f^{(n)}(x) = (-1)^{n-1}(n-1)!(1+x)^{-n}$

Question 15

You did not answer the question.

Assume that f is the given function and that P_n represents the nth Taylor Polynomial centered at $c = 0$. Find the least integer n for which $P_n(0.3)$ approximates $\ln(1.3)$ to within 0.0001

$f(x) = \ln(1+x)$

$c \in [0, 0.3]$

$\Rightarrow f^{(n+1)}(x) = (-1)^n n!(1+x)^{-n-1} = \frac{(-1)^n n!}{(1+x)^{n+1}} \Rightarrow |f^{(n+1)}(c)| \le n!$

$\Rightarrow |R_n(0.3)| = \left|\frac{f^{(n+1)}(c)}{(n+1)!}(0.3)^{n+1}\right| \le \frac{n!}{(n+1)!}(0.3)^{n+1} \le \frac{1}{10000}$

$\frac{(0.3)^{n+1}}{n+1} \le \frac{1}{10000}$

$n=4$. $\frac{(0.3)^5}{5} = 0.000486 \ge 0.0001$

$n=6$. $\frac{(0.3)^7}{7} = 0.00003 \le 0.0001$

a) 9

b) 8

c) 4

d) 6

e) 10

Question 16

You did not answer the question.

Find the Taylor polynomial of the function f for the given values of a and n, and give the Lagrange form of the remainder.

$f(x) = \sqrt{3x}$

$a = 3$

$n = 3$

$f(x) = (3x)^{\frac{1}{2}}$

$f'(x) = \left(\frac{3}{2}\right)(3x)^{-\frac{1}{2}}$

$f''(x) = -\frac{9}{4}(3x)^{-\frac{3}{2}}$

$f'''(x) = \frac{3^4}{2^3}(3x)^{-\frac{5}{2}}$ ...

$f^{(4)}(x) = -\frac{3^5 \cdot 5}{16}(3x)^{-\frac{7}{2}}$

a) $P_3(x) = \frac{9}{4} + \frac{1}{4}x - \frac{1}{48}(x-3)^2 - \frac{1}{288}(x-3)^3$; $R_3(x) = \frac{1}{64}\frac{\sqrt{3}(x-3)^4}{c^{5/2}}$, $|c-3| < |x-3|$

b) $P_3(x) = \frac{9}{2} + \frac{1}{2}x - \frac{1}{24}(x+3)^2 + \frac{1}{144}(x-3)^3$; $R_3(x) = \frac{1}{64}\frac{\sqrt{3}(x-3)^4}{c^{5/2}}$, $|c-3| < |x-3|$

c) $P_3(x) = \frac{3}{2} + \frac{1}{2}x - \frac{1}{24}(x-3)^2 + \frac{1}{144}(x-3)^3$; $R_3(x) = \frac{5}{128}\frac{\sqrt{3}(x-3)^4}{c^{7/2}}$, $|c-3| < |x-3|$

d) $P_3(x) = \sqrt{3} + \frac{1}{2}x - \frac{3}{2} + \frac{1}{24}(x-3)^2 + \frac{1}{72}(x-3)^3$; $R_3(x) = -\frac{5}{128}\frac{\sqrt{3}(x-3)^4}{c^{7/2}}$, $|c-3| < |x-3|$

$$P_3(x) = f(3) + \frac{f'(3)}{1!}(x-3) + \frac{f''(3)}{2!}(x-3)^2 + \frac{f'''(3)}{3!}(x-3)^3$$

$$= 3 + \frac{(x-3)}{2} - \frac{(x-3)^2}{24} + \frac{(x-3)^2}{72}$$

Q17. $f(x) = \cos(4x)$, $f'(x) = -4\sin(4x)$, $f''(x) = -16\cos(4x)$, $f'''(x) = 64\sin(4x)$, $f^{(4)}(x) = 256\cos(4x)$

e) $P_3(x) = \frac{15}{4} - \frac{1}{4}x + \frac{1}{24}(x-3)^2 - \frac{1}{288}(x-3)^3$ $R_3(x) = -\frac{5}{32}\frac{\sqrt{3}(x-3)^4}{c^{7/2}}$, $|c-3| < |x-3|$

Question 17

$$\Rightarrow P_3 = \cos(20) - \frac{4\sin(20)}{1!}(x-5) - 8\cos(20)(x-5)^2 + \frac{32}{3}\sin(20)(x-5)^3$$

You did not answer the question.

Find the Taylor polynomial of the function f for the given values of a and n, and give the Lagrange form of the remainder.

$$f(x) = \cos(4x)$$
$$a = 5$$
$$n = 3$$

$$R_3 = \frac{256\cos(4c)}{4!}(x-5)^4$$

a) $P_3(x) = \sin(20) - 4\cos(20)(x-5) - 8\sin(20)(x-5)^2 + \frac{32}{3}\cos(20)(x-5)^3$;
$R_3(x) = -\frac{32}{3}\cos(4c)(x-5)^4$, $|c-5| < |x-5|$

b) $P_3(x) = \cos(20) - 4\sin(20)(x-5) - 8\cos(20)(x-5)^2 + \frac{32}{3}\sin(20)(x-5)^3$;
$R_3(x) = \frac{32}{3}\cos(4c)(x-5)^4$, $|c-5| < |x-5|$

c) $P_3(x) = \cos(20) - 4\sin(20)(x-5) - 8\cos(20)(x-5)^2 + \frac{32}{3}\sin(20)(x-5)^3$;
$R_3(x) = \frac{64}{3}\cos(c)(x-5)^4$, $|c-5| < |x-5|$

d) $P_3(x) = \cos(20) - 4\sin(20)(x-5) - 8\cos(20)(x-5)^2 + \frac{32}{3}\sin(20)(x-5)^3$;
$R_3(x) = \frac{16}{3}\cos(4c)(x-5)^4$, $|c-5| < |x-5|$

e) $P_3(x) = \cos(20) + 4\sin(20)(x-5) + 8\cos(20)(x-5)^2 + \frac{32}{3}\sin(20)(x-5)^3$;
$R_3(x) = -\frac{64}{3}\cos(c)(x-5)^4$, $|c-5| < |x-5|$

Question 18

You did not answer the question.

Expand $g(x) = 6x^{-1}$ in powers of $(x-1)$.

$\frac{a}{1-r}$ (geometric series)

$$\frac{6}{x} = \frac{6}{1-[-(x-1)]}$$

a) $\sum_{k=0}^{\infty} 6(-1)^k (x-1)^k$

$$= \sum_{k=0}^{\infty} 6\cdot[-(x-1)]^k = \sum_{k=0}^{\infty} 6\,(-1)^k (x-1)^k$$

b) $\sum_{k=0}^{\infty} 6(-1)^{k-1}(x-1)^k$

c) $\sum_{k=0}^{\infty} 6(-1)^{k+1}(x-1)^k$

d) $\sum_{k=0}^{\infty} (-1)^{k-1}(x-1)^k$

e) $\sum_{k=0}^{\infty} 36(-1)^k(x-1)^k$

Question 19

You did not answer the question.

Expand $g(x) = e^{-3x}$ in powers of $(x+1)$.

Q19. $x+1=0 \Rightarrow x=-1$

$g(x) = e^{-3x} \Rightarrow g(-1) = e^3$

$g'(x) = -3e^{-3x} \Rightarrow g'(-1) = -3e^3$

$g''(x) = (-3)^2 e^{-3x} \Rightarrow g''(-1) = (-3)^2 e^3$

$g'''(x) = (-3)^3 e^{-3x} \Rightarrow g'''(-1) = (-3)^3 e^3$

$\vdots$

$g^{(k)}(x) = (-3)^k e^{-3x} \Rightarrow g^{(k)}(-1) = (-3)^k e^3$

$$\Rightarrow g(x) = e^{-3x} = \sum_{k=0}^{\infty} \frac{(-3)^k e^3}{k!}(x+1)^k$$

a) $\sum_{k=0}^{\infty} \frac{(-1)^{k-1} 3^k (x+1)^k}{k!}$

b) $\sum_{k=0}^{\infty} \frac{(-1)^{k+1} e^3 3^{k+1} (x+1)^k}{k!}$

c) $\sum_{k=0}^{\infty} \frac{(-1)^k e^3 3^k (x+1)^k}{k!}$

d) $\sum_{k=0}^{\infty} \frac{(-1)^k e^3 (x+1)^k}{k!}$

e) $\sum_{k=0}^{\infty} \frac{(-1)^{k+1} e^3 (x+1)^k}{k!}$

Question 20

You did not answer the question.

Expand $g(x) = 9\ln(1+2x)$ in powers of $(x-1)$.

Q20. $g(x) = 9\ln(1+2x)$

$g'(x) = \frac{9}{1+2x}\cdot 2 = 18(1+2x)^{-1}$

$g''(x) = (-1)\cdot 9\cdot 2^2(1+2x)^{-2}$

$g'''(x) = (-1)(-2)\cdot 9\cdot 2^3(1+2x)^{-3}$

$\vdots$

$g^{(k)}(x) = (-1)(-2)\cdots[-(k-1)]\cdot 9\cdot 2^k(1+2x)^{-k}$

$g^k(1) = (-1)^{k-1}(k-1)!\cdot 9\cdot 2^k(3)^{-k}$

$\Downarrow$

a) $9\ln(3) + \sum_{k=1}^{\infty} \frac{9(-1)^{k+1}\left(\frac{3}{4}\right)^k (x-1)^k}{k}$

$$g(x) = 9\ln 3 + \sum_{k=1}^{\infty} \frac{(-1)^{k-1}(k-1)!\,9\cdot 2^k\cdot(3)^{-k}}{k!}(x-1)^k$$

$$= 9\ln(3) + \sum_{k=1}^{\infty} \frac{9\cdot(-1)^{k-1}\left(\frac{2}{3}\right)^k (x-1)^k}{k}$$

b) $9\ln(6) + \sum_{k=1}^{\infty} \frac{(-1)^{k-1}\left(\frac{2}{3}\right)^k (x-1)^k}{k}$

c) $9\ln(3) + \sum_{k=1}^{\infty} \frac{9(-1)^{k+1}\left(\frac{2}{3}\right)^k (x-1)^k}{k}$

d) $\ln(3) + \sum_{k=1}^{\infty} \frac{9(-1)^{k-1}\left(\frac{2}{3}\right)^k (x-1)^k}{k}$

e) $\ln(3) + \sum_{k=1}^{\infty} \frac{9(-1)^{k+1}\left(\frac{2}{3}\right)^k (x-1)^k}{k}$