PRINTABLE VERSION

Quiz 12

You scored 0 out of 100

Question 1

You did not answer the question.

Express in sigma notation.

$$(5)(6)+(6)(7)+(7)(8)+(8)(9)+\ldots+(15)(16)$$

a) $\sum_{k=0}^{9}(k+5)(k+6)$ $=\underset{k=0}{(5)(6)}+\underset{k=1}{(6)(7)}+\cdots+\underset{k=9}{(14)(15)}$ ✗

b) $\sum_{k=0}^{10}(k+5)(k+7)$ $=(5)(7)+\cdots$ ✗

c) $\boxed{\sum_{k=0}^{10}(k+5)(k+6)}$ $=\underset{k=0}{(5)(6)}+\underset{k=1}{(6)(7)}+\cdots+\underset{k=9}{(14)(15)}+\underset{k=10}{(15)(16)}$ ✓

d) $\sum_{k=0}^{11}(k+5)(k+6)$

e) $\sum_{k=1}^{10}(k+5)(k+7)$

Question 2

You did not answer the question.

Which of the following shows both correct sigma notations for

$$\frac{1}{3^{(2)}}+\frac{1}{3^{(3)}}+\ldots+\frac{1}{3^{(9)}}$$

a) $\left[\sum_{k=3}^{7}\frac{1}{3^k},\ \sum_{i=0}^{10}\frac{1}{3^{i+2}}\right]$

$= \frac{1}{3^3}+\frac{1}{3^4}+\cdots$ ✗

$\frac{1}{3^5}+\frac{1}{3^6}+\cdots$ ✗

$=$

b) $\left[\sum_{k=3}^{10}\frac{1}{3^{k+2}},\ \sum_{i=0}^{7}\frac{1}{3^{i+2}}\right]$

c) $\left[\sum_{k=3}^{10}\frac{1}{3^{k+2}},\ \sum_{i=0}^{7}\frac{1}{3^{i}}\right]$ ✗ $\frac{1}{3^0}+\frac{1}{3^1}+\cdots+\frac{1}{3^7}$ ✗

d) $\left[\sum_{k=0}^{7}\frac{1}{3^k},\ \sum_{i=3}^{10}\frac{1}{3^{i+2}}\right]$ ✗

e) $\boxed{\left[\sum_{k=2}^{9}\frac{1}{3^k},\ \sum_{i=0}^{7}\frac{1}{3^{i+2}}\right]}$ ✓

Question 3

You did not answer the question.

Find the sum of the series.

$$\sum_{k=4}^{\infty}\frac{2}{k^2-k} \overset{\text{partial fraction}}{=} \sum_{k=4}^{\infty}\frac{2}{k(k-1)}$$

$$=\sum_{k=4}^{\infty}\frac{2}{k-1}+\frac{-2}{k}$$

$$=\underset{k=4}{\left(\frac{2}{3}-\cancel{\frac{2}{4}}\right)}+\underset{k=5}{\left(\cancel{\frac{2}{4}}-\cancel{\frac{2}{5}}\right)}+\underset{k=6}{\left(\cancel{\frac{2}{5}}+\cancel{\frac{-2}{6}}\right)}+\cdots$$

$=\frac{2}{3}$ since $\frac{2}{k-1}$ and $\frac{-2}{k}$ tend to 0 as $k\to\infty$

a) 2

b) $\frac{4}{3}$

c) $\boxed{\frac{2}{3}}$

d) 1

e) $\frac{4}{9}$

Question 4

You did not answer the question.

Find the sum of the series.

Formula: $\frac{a}{1-r}$ Geometric series

$$\sum_{k=0}^{\infty}\frac{(-1)^k}{6^k}=\sum_{k=0}^{\infty}\left(-\frac{1}{6}\right)^k=\frac{1}{1-\left(-\frac{1}{6}\right)}=\frac{6}{7}$$

$a=1$, $r=-\frac{1}{6}$

initial (as $k=0$) common ratio

a) $\frac{12}{7}$

b) $\frac{18}{7}$

c) $\frac{4}{7}$

d) $\frac{9}{7}$

e) $\frac{6}{7}$

Question 5

You did not answer the question.

Find the sum of the series.

$$\sum_{k=0}^{\infty} \frac{1-6^k}{8^k} = \sum_{k=0}^{\infty} \frac{1}{8^k} - \frac{6^k}{8^k} = \sum_{k=0}^{\infty}\left(\frac{1}{8}\right)^k - \left(\frac{6}{8}\right)^k = \sum_{k=0}^{\infty}\left(\frac{1}{8}\right)^k - \sum_{k=0}^{\infty}\left(\frac{3}{4}\right)^k$$

By Geometric series

since $\frac{1}{8}$ and $\frac{6}{8} < 1$

So both of them converges

Thus we can separate this sum into two sums.

$$= \frac{1}{1-\frac{1}{8}} - \frac{1}{1-\frac{3}{4}}$$

$$= \frac{8}{7} - 4 = -\frac{20}{7}$$

a) $-\frac{60}{7}$

b) $-\frac{40}{7}$

c) $-\frac{20}{7}$

d) $-\frac{30}{7}$

e) $-\frac{40}{21}$

Question 6

You did not answer the question.

Determine whether the series converges or diverges.

p-series $\sum \frac{1}{k^p} = \begin{cases} \text{converges} & \text{if } p>1 \\ \text{diverges} & \text{if } p\le 1 \end{cases}$

$$\sum \frac{k}{6k^3+3} \sim \sum \frac{k}{k^3} = \sum \frac{1}{k^2} \quad (2>1)$$

which converges by p-series

a) diverges

b) cannot be determined

c) converges

So by limit Comparison, it converges

Question 7

You did not answer the question.

Determine whether the series converges or diverges.

$$\sum \frac{6}{\sqrt{k+1}} \sim \sum \frac{1}{\sqrt{k}} = \sum \frac{1}{k^{\frac{1}{2}}} \quad \left(\frac{1}{2}<1\right)$$

which diverges by p-series

a) cannot be determined

b) diverges

c) converges

So by limit comparison, it diverges

Question 8

You did not answer the question.

Determine whether the series converges or diverges.

$$\sum \frac{1}{\sqrt{4k^2-2k}} \sim \sum \frac{1}{\sqrt{k^2}} = \sum \frac{1}{k^1} \quad (1 \le 1)$$

which diverges by p-series

a) converges

b) cannot be determined

c) diverges

So by limit Comparison, it diverges

Question 9

You did not answer the question.

Determine whether the series converges or diverges.

$$\sum \frac{1}{k(k+3)(k+2)} \sim \sum \frac{1}{k^3} \quad (3>1)$$

which converges by p-series

a) converges

So by limit Comparison, it converge

Ratio Test: for $\sum a_n$ check $\frac{a_{n+1}}{a_n} \to L$ as $n\to\infty$, then $\begin{cases} \sum a_n \text{ converges} & \text{if } L<1 \\ \text{fails} & \text{if } L=1 \\ \sum a_n \text{ diverges} & \text{if } L>1 \end{cases}$

b) diverges

c) cannot be determined

Question 10

You did not answer the question.

Determine whether the series converges or diverges.

$$\sum \frac{4}{k(\ln(k))^2}$$

Find $\int_1^\infty \frac{4}{x(\ln(x))^2}dx = \lim_{b\to\infty}\lim_{a\to 1}\left[\int_a^b \frac{4}{x(\ln(x))^2}dx\right]$

$\lim_{b\to\infty}\lim_{a\to 1}\left[\frac{-4}{\ln(x)}\right]_a^b = \lim_{b\to\infty}\lim_{a\to 1}\left[\frac{-4}{\ln(b)} + \frac{4}{\ln(a)}\right]$

Diverges

So, by Integral test it diverges

a) converges

b) cannot be determined

c) $\boxed{\text{diverges}}$

Question 11

You did not answer the question.

Determine whether the series converges or diverges.

$$\sum \frac{k^6-1}{2k^4+5}$$

a) $\boxed{\text{diverges}}$

b) converges

c) cannot be determined

Since $\frac{k^6-1}{2k^4+5} \not\to 0$ as $k\to\infty$

By B.D.T (Basic Divergence Test), it diverges

Question 12

You did not answer the question.

Determine whether the series converges or diverges.

$$\sum \frac{5+\cos(k)}{\sqrt{k+5}}$$

$\cos(k)\geq -1$

$\frac{5+\cos(k)}{\sqrt{k+5}} > \frac{4}{\sqrt{k+5}} \sim \frac{1}{\sqrt{k}}$ which diverges by p-series

a) converges

b) $\boxed{\text{diverges}}$

c) cannot be determined

So by Basic Comparison test, it diverges

Question 13

You did not answer the question.

Determine whether the series converges or diverges.

$$\sum \frac{1}{k\,3^k}$$

let $a_k = \frac{1}{k3^k}$, $\frac{a_{k+1}}{a_k} = \frac{1}{(k+1)3^{k+1}}\cdot\frac{k\cdot 3^k}{1} = \underbrace{\frac{k}{k+1}}_{\downarrow\, 1 \text{ as } k\to\infty}\cdot\frac{1}{3} \to \frac{1}{3}$

a) diverges

b) $\boxed{\text{converges}}$

c) cannot be determined

Since $\frac{1}{3}<1$, by Ratio Test, it converges

Question 14

You did not answer the question.

Determine whether the series converges or diverges.

$$\sum \left(\frac{6k}{12k+2}\right)^k$$

Root test for $\sum a_k$: check $\sqrt[k]{a_k}\to L$ as $k\to\infty$ then $\begin{cases}\sum a_k \text{ conv.} & \text{if } L<1\\ \text{fails} & \text{if } L=1\\ \sum a_k \text{ diver.} & \text{if } L>1\end{cases}$

let $a_k = \left(\frac{6k}{12k+2}\right)^k$.

$\sqrt[k]{a_k} = \frac{6k}{12k+2} \to \frac{1}{2} < 1$ (leading coefficient)

By Root test it converges

a) cannot be determined

b) $\boxed{\text{converges}}$

c) diverges

Question 15

You did not answer the question.

Determine whether the series converges or diverges.

$$\sum k\left(\frac{7}{9}\right)^k$$

Note: $\sqrt[k]{k}\to 1$ as $k\to\infty$ (check it by L'HOPITAL'S RULE)

let $a_k = k\cdot\left(\frac{7}{9}\right)^k$.

$\sqrt[k]{a_k} = \sqrt[k]{k}\cdot\frac{7}{9} \to \frac{7}{9} < 1$

By Root test, it converges

a) diverges

b) $\boxed{\text{converges}}$

c) cannot be determined

Question 16

You did not answer the question.

Determine whether the series converges or diverges.

$$\sum \frac{k!}{27^{10k}}$$

let $a_k = \frac{k!}{27^{10k}}$, $\frac{a_{k+1}}{a_k} = \frac{(k+1)!}{27^{10(k+1)}}\cdot\frac{27^{10k}}{k!} = \underbrace{(k+1)}_{\text{Diverges}}\underbrace{\frac{1}{27^{10}}}_{\text{Fixed}} \to \infty > 1$

By Ratio test, it diverges.

a) converges

b) cannot be determined

c) diverges

Question 17

You did not answer the question.

Determine whether the series converges or diverges.

$$\sum \frac{3k!}{(k+3)!}$$

$a_k = \frac{3(k)!}{(k+3)!} = \frac{3}{(k+3)(k+2)(k+1)} \sim \frac{1}{k^3}$

$\sim \sum \frac{1}{k^3}$ which converges by p-series

So by limit Comparison, it converges

a) cannot be determined

b) converges

c) diverges

Question 18

You did not answer the question.

Determine whether the series converges or diverges.

$$\sum \left(\frac{k}{k+5}\right)^k$$

let $a_k = \left(\frac{k}{k+5}\right)^k$.

Check $\lim_{k\to\infty} \left(\frac{k}{k+5}\right)^k \overset{(L')}{=} \lim_{k\to\infty} e^{\ln\left(\frac{k}{k+5}\right)^k} = \lim_{k\to\infty} e^{k \ln\left(\frac{k}{k+5}\right)}$

(1^∞)

$= e^{\lim_{k\to\infty} k \ln\left(\frac{k}{k+5}\right)} = e^{-1} \not\to 0$ by B.D.T.

$\lim_{k\to\infty} k \cdot \ln\left(\frac{k}{k+5}\right) \overset{(L')}{=} \lim_{k\to\infty} \frac{\ln\left(\frac{k}{k+5}\right)}{\frac{1}{k}} \overset{(L')}{=} \lim_{k\to\infty} \frac{\frac{1}{k} - \frac{1}{k+5}}{-\frac{1}{k^2}} = -1$

$(\infty \cdot 0)$ $\left(\frac{0}{0}\right)$

a) converges

b) diverges

c) cannot be determined

Question 19

You did not answer the question.

Determine whether the series converges or diverges.

$$\sum \frac{5(k!)}{k^k}$$

Let $a_k = \frac{5(k!)}{k^k}$, then $\frac{a_{k+1}}{a_k} = \frac{5[(k+1)!]}{(k+1)^{k+1}} \cdot \frac{k^k}{5 \cdot k!}$

$= \frac{(k+1)!}{k!} \cdot \frac{k^k}{(k+1)^{k+1}} = k+1 \cdot \frac{k^k}{(k+1)(k+1)^k} = \left(\frac{k}{k+1}\right)^k \to \frac{1}{e} < 1$

By Ratio Test, it converges

(Check! See the last page)

a) diverges

b) converges

c) cannot be determined

Question 20

You did not answer the question.

Determine whether the series converges absolutely, converges conditionally or diverges.

$$\frac{1}{(4)} - \frac{2}{(5)} + \frac{3}{(6)} - \frac{4}{(7)} + \ldots + (-1)^{k+1}\left(\frac{k}{k+3}\right) + \ldots$$

pattern

$\sum_{k=1}^{\infty} (-1)^{k+1}\left(\frac{k}{k+3}\right)$

since $(-1)^k\left(\frac{k}{k+3}\right) \not\to 0$ as $k\to\infty$

By B.D.T. $\sum (-1)^{k+1}\left(\frac{k}{k+3}\right)$ diverges

a) cannot be determined

b) diverges

c) converges conditionally

d) converges absolutely

Question 21

You did not answer the question.

Determine whether the series converges absolutely, converges conditionally or diverges.

$$\sum \frac{k^5}{3^k}$$

$\sum \left|\frac{k^5}{3^k}\right| = \sum \frac{k^5}{3^k}$

Check absolutely converges

let $a_k = \frac{k^5}{3^k}$. $\frac{a_{k+1}}{a_k} = \frac{(k+1)^5}{3^{k+1}} \cdot \frac{3^k}{k^5}$

$= \frac{(k+1)^5}{k^5} \cdot \frac{1}{3} \to \frac{1}{3} < 1$ as $k\to\infty$

By Ratio test, it A.C.

a) converges absolutely

b) cannot be determined

c) diverges

d) converges conditionally

Question 22

You did not answer the question.

Determine whether the series converges absolutely, converges conditionally or diverges.

$$\sum \frac{(-1)^k}{3k+4}$$

check A.C. $\sum \left|\frac{(-1)^k}{3k+4}\right| = \sum \frac{1}{3k+4}$ diverges by p-series

Check conditionally converges

let $b_k = \frac{1}{3k+4}$, since $b_k \to 0$ and decreases $(a_{k+1} < a_k)$

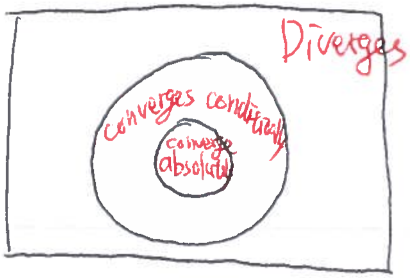
Then by Alternative series test, it converges conditionally.

a) converges absolutely

b) converges conditionally

c) cannot be determined

d) diverges



c) diverges

d) converges absolutely

Question 23

You did not answer the question.

Determine whether the series converges absolutely, converges conditionally or diverges.

$$\sum \frac{(-1)^k (2k)}{5^k}$$

a) diverges

b) converges absolutely

c) converges conditionally

d) cannot be determined

(a) Check absolutely converges:

$$\sum \left|\frac{(-1)^k(2k)}{5^k}\right| = \sum \frac{2k}{5^k}, \quad \text{let } a_k = \frac{2k}{5^k}. \quad \frac{a_{k+1}}{a_k} = \frac{2(k+1)}{5^{k+1}} \cdot \frac{5^k}{2k} = \frac{1}{5} \cdot \frac{2k+2}{2k} \to \frac{1}{5} < 1$$

By Ratio test, $\sum \frac{(-1)^k(2k)}{5^k}$ is absolutely converges

Question 24

You did not answer the question.

Determine whether the series converges absolutely, converges conditionally or diverges.

$$\sum (-1)^k (k) e^{-k}$$

a) converges absolutely

b) cannot be determined

c) diverges

d) converges conditionally

Check absolutely converges:

$$\sum |(-1)^k k e^{-k}| = \sum \frac{k}{e^k} \quad \text{let } a_k = \frac{k}{e^k} \quad \frac{a_{k+1}}{a_k} = \frac{k+1}{e^{k+1}} \cdot \frac{e^k}{k} = \frac{k+1}{k} \cdot \frac{1}{e} \to \frac{1}{e} < 1$$

By Ratio test, $\sum (-1)^k k e^{-k}$ is absolutely converges $(e \doteq 2.71)$

Question 25

You did not answer the question.

Determine whether the series converges absolutely, converges conditionally or diverges.

$$\sum \frac{(-1)^k \cos(\pi k)}{6k+5}$$

a) converges conditionally

b) cannot be determined

$$\boxed{\cos(\pi k) = (-1)^k \Rightarrow |\cos(\pi k)| = 1}$$

Check absolutely converges (A.C.)

$$\sum \left|\frac{(-1)^k \cos(\pi k)}{6k+5}\right| = \sum \frac{|\cos(\pi k)|}{6k+5} = \sum \frac{1}{6k+5} \text{ which diverges by p-series}$$

NOT A.C.

check conditionally converges

$$\boxed{[(-1)^2]^k = 1^k = 1}$$

$$\sum \frac{(-1)^k \cos(\pi k)}{6k+5} = \sum \frac{(-1)^k(-1)^k}{6k+5} = \sum \frac{(-1)^{2k}}{6k+5} = \sum \frac{1}{6k+5} \text{ which diverges}$$

$\Rightarrow$ Diverges

check

$$\lim_{k\to\infty}\left(\frac{k}{k+1}\right)^k \underset{(1^\infty)}{\overset{L'}{=}} \lim_{k\to\infty} e^{\ln\left(\frac{k}{k+1}\right)^k}$$

$$= \lim_{k\to\infty} e^{k\ln\left(\frac{k}{k+1}\right)}$$

$$= e^{\lim_{k\to\infty} k\ln\left(\frac{k}{k+1}\right)}$$

$$= e^{-1} = \frac{1}{e}$$

$$\lim_{k\to\infty} k\ln\left(\frac{k}{k+1}\right) \underset{(\infty\cdot 0)}{\overset{(L')}{=}} \lim_{k\to\infty} \frac{\ln\left(\frac{k}{k+1}\right)}{\frac{1}{k}}$$

$$\underset{\left(\frac{0}{0}\right)}{\overset{(L')}{=}} \lim_{k\to\infty} \frac{\frac{1}{k(k+1)}}{\frac{-1}{k^2}} = \lim_{k\to\infty} -\frac{k^2}{k(k+1)} = -1$$