$\ln n < n < e^n < n! < n^n$ } L'HÔPITAL'S Rule $\frac{0}{0}, \frac{\infty}{\infty}, 0\cdot\infty \quad 1^\infty, \infty^0, 0^0 \quad \infty - \infty$

PRINTABLE VERSION

$2^n < e^n < 3^n < 4^n < \cdots$

Quiz 10

You scored 0 out of 100

Question 1

You did not answer the question.

State whether the sequence converges as $n \to \infty$; if it does, find the limit

$$\left(\frac{12}{n}\right)^n$$

let $y = \left(\frac{12}{n}\right)^n = \frac{12^n}{n^n} \to 0$ as $n \to \infty$

a) diverges

b) converges to 0

c) converges to 12

d) converges to 1

e) converges to 11

Question 2

You did not answer the question.

State whether the sequence converges as $n \to \infty$; if it does, find the limit

$$\frac{7\ln(n)}{n}$$

$$\lim_{n\to\infty} \frac{7\ln(n)}{n} \underset{\left(\frac{\infty}{\infty}\right)}{\overset{(L')}{=}} \lim_{n\to\infty} \frac{\frac{7}{n}}{1} = \lim_{n\to\infty} \frac{7}{n} = 0$$

By L'HÔPITAL'S RULE

a) converges to 1

b) diverges

c) converges to 7

d) converges to $\ln(7)$

e) converges to 0

Question 3

You did not answer the question.

State whether the sequence converges as $n \to \infty$; if it does, find the limit

$$\frac{7^{n+1}}{8^{n-1}} = 7^2 \cdot \frac{7^{n-1}}{8^{n-1}} = 49\left(\frac{7}{8}\right)^{n-1}$$

$\Rightarrow \frac{7}{8} < 1, \ \left(\frac{7}{8}\right)^n \to 0$ as $n \to \infty$

$\therefore \lim_{n\to\infty} \frac{7^{n+1}}{8^{n-1}} = 49 \cdot \left(\lim_{n\to\infty} \left(\frac{7}{8}\right)^n\right) = 0$

a) converges to 1

b) diverges

c) converges to $\frac{1}{8}$

d) converges to $\frac{7}{8}$

e) converges to 0

Question 4

You did not answer the question.

State whether the sequence converges as $n \to \infty$; if it does, find the limit

$$\int_0^n e^{-5x}\,dx = -\frac{e^{-5x}}{5}\Big|_0^n = \frac{1}{5} - \frac{e^{-5n}}{5}$$

$$\Rightarrow \lim_{n\to\infty}\left(\int_0^n e^{-5x}dx\right) = \lim_{n\to\infty}\left[\frac{1}{5} - \frac{e^{-5n}}{5}\right] = \frac{1}{5} - 0 = \frac{1}{5}$$

$e^{-5n} = \frac{1}{e^{5n}} \to 0$ as $n\to\infty$

a) converges to $\frac{1}{5}$

b) converges to e^{-5}

c) converges to 1

d) converges to 0

e) diverges

Question 5

You did not answer the question.

State whether the sequence converges as $n \to \infty$; if it does, find the limit

$$\int_{-n}^{n} \frac{9}{1+x^2}\,dx = 9\cdot \arctan x \Big|_{-n}^{n} = 9\left[\arctan(n) - \arctan(-n)\right] \xrightarrow{n\to\infty} 9\left[\frac{\pi}{2} - \left(-\frac{\pi}{2}\right)\right] = 9\pi$$

Q6. $n^9 \sin(n\pi)$

~~Since~~ ~~See graph $\Rightarrow$ diverges~~

a) diverges

b) converges to 0

c) converges to $-\frac{9}{7}\pi$

d) converges to $9-\pi$

e) converges to 1

If this is a sequence, sin(n*(pi))=0 for all integer n.
Thus this is a zero sequense which converges to 0.
The answer should be (c).

Question 6

You did not answer the question.

State whether the sequence converges as $n \to \infty$; if it does, find the limit

$n^9 \sin(n\pi)$

a) converges to -1

b) converges to 1

c) converges to 0

d) converges to 9

e) ~~diverges~~

Question 7

You did not answer the question.

State whether the sequence converges as $n \to \infty$; if it does, find the limit

$$\int_{\frac{1}{n}}^{1} \frac{1}{x^{9/10}}\,dx = 10x^{\frac{1}{10}}\Big|_{\frac{1}{n}}^{1}$$

$$= 10 - 10\left(\frac{1}{n}\right)^{\frac{1}{10}}$$

$$\to 10 - 0 = 10 \text{ as } n \to \infty$$

a) diverges

b) converges to 9

c) converges to $\frac{10}{9}$

d) converges to 10

e) converges to 1

Question 8

You did not answer the question.

State whether the sequence converges as $n \to \infty$; if it does, find the limit

$$\frac{n!}{(12n)} = \frac{(n-1)!}{12} \to \infty \text{ as } n \to \infty$$

a) converges to 0

b) diverge

c) converges to -1

d) converges to 1

e) converges to 1200

Question 9

You did not answer the question.

State whether the sequence converges as $n \to \infty$; if it does, find the limit

$$\frac{n^n}{3^{n^2}} = \left(\frac{n}{3^n}\right)^n$$

a) diverges

b) converges to $\frac{1}{3}$

c) converges to $\frac{1}{4}$

d) converges to 0

e) converges to 1

$$0 < \frac{n}{3^n} \le \frac{1}{3} < 1, \forall n.$$

$$\Rightarrow 0 \le \left(\frac{n}{3^n}\right)^n \le \left(\frac{1}{3}\right)^n$$

as $n \to \infty$: 0 and $\left(\frac{1}{3}\right)^n \to 0$ as $n \to \infty$. By Squeeze's

$$\Rightarrow \left(\frac{n}{3^n}\right)^n \to 0 \text{ as } n \to \infty$$

Question 10

You did not answer the question.

State whether the sequence converges as $n \to \infty$; if it does, find the limit

$$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{n}=e$$

$$\left(1+\frac{3}{(5n)}\right)^{5n}=\left(1+\frac{1}{\left(\frac{5}{3}n\right)}\right)^{\frac{5n}{3}\cdot 3}$$

$$=\left[\left(1+\frac{1}{\left(\frac{5}{3}n\right)}\right)^{\frac{5n}{3}}\right]^{3}$$

a) converges to 1

b) converges to e^5

c) converges to $e^{\frac{3}{5}}$

d) diverges

e) converges to e^3

$$\Rightarrow \lim_{n\to\infty}\left(1+\frac{3}{5n}\right)^{5n}=\left[\lim_{n\to\infty}\left(1+\frac{1}{\frac{5}{3}n}\right)^{\frac{5n}{3}}\right]^{3}=e^{3}$$

$[\ \ ]^3$ is conti.

Question 11

You did not answer the question.

Calculate the limit.

$$\left(\frac{0}{0}\right)\qquad \lim_{x\to 0^+}\frac{\sin(x)}{(5\sqrt{x})}\overset{L'}{=}\lim_{x\to 0^+}\frac{\cos(x)}{\frac{5}{2}\frac{1}{\sqrt{x}}}$$

$$=\lim_{x\to 0^+}\frac{2}{5}\underbrace{\sqrt{x}}_{0}\,\underbrace{\cos(x)}_{1}=0$$

a) 0

b) -5

c) 5

d) -1

e) 1

Question 12

You did not answer the question.

Calculate the limit.

$\boxed{\text{Method 1}}$

$$\lim_{x\to 2}\frac{x-2}{x^2-4}=\lim_{x\to 2}\frac{(x-2)}{(x+2)(x-2)}$$

$$=\lim_{x\to 2}\frac{1}{(x+2)}=\frac{1}{4}$$

a) 0

b) 1

$\boxed{\text{Method 2}}$

$$\lim_{x\to 2}\frac{x-2}{x^2-4}\underset{\left(\frac{0}{0}\right)}{\overset{L'}{=}}\lim_{x\to 2}\frac{1}{2x}=\frac{1}{4}$$

c) $-\frac{1}{4}$

d) -1

e) $\frac{1}{4}$

Question 13

You did not answer the question.

Calculate the limit.

$$\lim_{x\to 0}\frac{10^x-1}{x}\underset{\left(\frac{0}{0}\right)}{\overset{L'}{=}}\lim_{x\to 0}\frac{\ln 10\cdot 10^x}{1}$$

$$=\ln 10$$

a) 1

b) -1

c) $-\ln(10)$

d) $\ln(10)$

e) 0

Question 14

You did not answer the question.

Calculate the limit.

$$\lim_{x\to 0}\frac{e^x+e^{-x}-2}{(1-\cos(12x))}\underset{\left(\frac{0}{0}\right)}{\overset{L'}{=}}\lim_{x\to 0}\frac{e^x-e^{-x}}{12\sin(12x)}$$

$$\underset{\left(\frac{0}{0}\right)}{\overset{L'}{=}}\lim_{x\to 0}\frac{e^x+e^{-x}}{144\cos(12x)}=\frac{2}{144}=\frac{1}{72}$$

a) 1

b) $\frac{1}{72}$

c) $\frac{1}{36}$

d) 0

e) 72

Question 15

You did not answer the question.

Calculate the limit.

$$\lim_{x\to 0}\frac{(3+3x-3e^x)}{(4x(e^x-1))} \underset{(\frac{0}{0})}{\overset{(L')}{=}} \lim_{x\to 0}\frac{3-3e^x}{4(e^x-1)+4xe^x}$$

$$\underset{(\frac{0}{0})}{\overset{L'}{=}} \lim_{x\to 0}\frac{-3e^x}{4e^x+4e^x+4xe^x} = \frac{-3}{8}$$

a) $-\frac{3}{4}$

b) $\frac{3}{8}$

c) $-\frac{8}{3}$

d) $-\frac{3}{8}$

e) 0

Question 16

You did not answer the question.

Calculate the limit.

$$\lim_{x\to 0}\frac{(5x-5\tan(x))}{(4x-4\sin(x))} \underset{(\frac{0}{0})}{\overset{L'}{=}} \lim_{x\to 0}\frac{5-5\sec^2(x)}{4-4\cos(x)}$$

$$\underset{(\frac{0}{0})}{\overset{L'}{=}} \lim_{x\to 0}\frac{-10\sec(x)\sec(x)\tan(x)}{+4\sin(x)} \rightarrow \frac{\sin(x)}{\cos(x)}$$

$$= \lim_{x\to 0}\frac{-10\sec^2(x)}{4\cos(x)}$$

$$= -\frac{10}{4} = -\frac{5}{2}$$

a) $\frac{5}{2}$

b) -5

c) 0

d) $-\frac{2}{5}$

e) $-\frac{5}{2}$

Question 17

You did not answer the question.

Calculate the limit.

$$\lim_{x\to 0}\frac{(\cos(x)-\cos(5x))}{(\sin(x^2))} \underset{(\frac{0}{0})}{\overset{(L')}{=}} \lim_{x\to 0}\frac{-\sin(x)+5\sin(5x)}{2x\cdot\cos(x^2)}$$

$$\underset{(\frac{0}{0})}{\overset{(L')}{=}} \lim_{x\to 0}\frac{-\cos(x)+25\cos(5x)}{2\cos(x^2)-4x^2\sin(x^2)}$$

$$= \frac{-1+25}{2-0} = 12$$

a) 24

b) 12

c) -12

d) 1

e) 0

Question 18

You did not answer the question.

Calculate the limit.

$$\lim_{x\to\infty}\frac{\left(\frac{1}{2}\pi-\arctan(x)\right)}{\left(\frac{9}{x}\right)} \underset{(\frac{0}{0})}{\overset{(L')}{=}} \lim_{x\to\infty}\frac{-\frac{1}{1+x^2}}{-\frac{9}{x^2}}$$

$$= \lim_{x\to\infty}\frac{x^2}{9(1+x^2)} = \frac{1}{9}$$

leading coefficient

deg p = deg q

a) 1

b) $-\frac{1}{9}$

c) 0

d) $\frac{2}{9}$

e) $\frac{1}{9}$

Question 19

You did not answer the question.

Calculate the limit.

$$\lim_{x \to \infty} \frac{6}{x(\ln(x+5) - \ln(x))} = \lim_{x \to \infty} \frac{\frac{6}{x}}{\ln\left(\frac{x+5}{x}\right)}$$

$$\overset{L'}{\underset{(\frac{0}{0})}{=}} \lim_{x \to \infty} \frac{-\frac{6}{x^2}}{\frac{1}{x+5} - \frac{1}{x}} \qquad \frac{1}{x+5} - \frac{1}{x} \longrightarrow \frac{-5}{x(x+5)}$$

$$= \lim_{x \to \infty} -\frac{6}{x^2} \cdot \frac{x(x+5)}{-5}$$

$$= \frac{6}{5}$$

leading coefficient

a) 1

b) $\frac{5}{6}$

c) $\boxed{\frac{6}{5}}$

d) 0

e) $-\frac{6}{5}$

Question 20

You did not answer the question.

Find values for a and b such that

$$\lim_{x \to 0} \frac{\cos(ax) - b}{(2x^2)} = -100$$

a) $[a = (20, -20), b = 0]$

b) $[a = (-10, 10), b = 1]$

c) $\boxed{[a = (-20, 20), b = 1]}$

d) $[a = (-40, 40), b = 2]$

e) $[a = (-20, 20), b = -1]$

as $x=0$, we have $\frac{\cos(0) - b}{0} = \frac{1-b}{0}$ since we known this limit exists.

then, we let $1-b=0 \Rightarrow \underline{b=1}$.

That is, we have $(\frac{0}{0})$-form. by L'HÔPITAL'S RULE.

we get $\lim_{x \to 0} \frac{\cos(ax) - 1}{2x^2} \overset{(L')}{\underset{(\frac{0}{0})}{=}} \lim_{x \to 0} \frac{-a\sin(ax)}{4x} \overset{L'}{\underset{(\frac{0}{0})}{=}} \lim_{x \to 0} \frac{-a^2\cos(ax)}{4} = -\frac{a^2}{4}$ which is -100

$\Rightarrow -\frac{a^2}{4} = -100 \Rightarrow a^2 = 400 \Rightarrow \underline{a = \pm 20}$