PRINTABLE VERSION

Quiz 9

You scored 0 out of 100

Question 1

You did not answer the question.

Find the least upper bound (if it exists) and the greatest lower bound (if it exists) for the given set.

(2, 5)

a) lub and glb do not exist

b) lub = 5; glb = 2

c) lub = -5; glb = -2

d) lub = 2; glb = 5

e) lub = -2; glb = -5

L.U.B = 5

G.L.B = 2

Question 2

You did not answer the question.

Find the least upper bound (if it exists) and the greatest lower bound (if it exists) for the set $\{x : |x - 1| < 10\}$.

a) no lub ; glb = -9

b) lub = 9 ; glb = -11

c) lub and glb do not exist

d) lub = 11 ; glb = -9

e) lub = 10 ; glb = -10

$\Rightarrow |x-1| < 10 \Rightarrow -10 < x-1 < 10$

$\Rightarrow -9 < x < 11$

g.l.b lub

Question 3

You did not answer the question.

Find the least upper bound (if it exists) and the greatest lower bound (if it exists) for the set $\{-5, -\frac{9}{2}, -\frac{13}{3}, -\frac{17}{4}, \ldots\}$.

Find pattern: Top: $-5, -9, -13, -17. \Rightarrow -5-4(n-1) = -5+4-4n = -1-4n$

bottom $1, 2, 3, 4. \Rightarrow n$

$\Rightarrow \left\{\frac{-1-4n}{n}\right\} = \left\{-\frac{1}{n} - 4\right\}$ which tends to -4 as $n \to \infty$ $\Rightarrow -5 \le \frac{-1-4n}{n} \le -4$

a) lub = -4 ; glb = -5

b) lub and glb do not exist

c) lub = -5 ; glb = -6

d) lub = -3 ; glb = -5

e) no lub ; glb = -5

Q4. $\ln(x) > 7 \Rightarrow x > e^7$

$g.l.b = e^7$

lub DNE (which is "∞")

Question 4

You did not answer the question.

Find the least upper bound (if it exists) and the greatest lower bound (if it exists) for the set $\{x : \ln(x) > 7\}$.

a) lub = e^7 ; glb = 0

b) no glb ; lub = e^7

c) no lub ; glb = e^7

d) lub and glb do not exist

e) no lub ; glb = ln(7)

Q5.

Question 5

You did not answer the question.

The first several terms of a sequence $\{a_n\}$ are given. Assume that the pattern continues as indicated and find an explicit formula for a_n.

$20, 0, 20, 0, 20, \ldots$

$a_1 \; a_2 \; a_3 \; a_4 \; a_5$

$a_1 = 20 = 10-(-10) = 10+(-1)^1(-10)$

$a_2 = 0 = 10+(-10) = 10+(-1)^2(-10)$

$a_3 = 20 = 10-(-10) = 10+(-1)^3(-10)$

$a_n = 10+(-1)^n(-10)$

a) $a_n = -10(-1)^n$

b) $a_n = -10(-1)^{n-1} - 10$

c) $a_n = -10(-1)^n + 10$

d) $a_n = 10(-1)^n + 10$

e) $a_n = -10(-1)^{n+1} - 10$

Question 6

You did not answer the question.

The first several terms of a sequence $\{a_n\}$ are given. Assume that the pattern continues as indicated and find an explicit formula for a_n.

$$\frac{-1}{(6)}, \frac{2}{(12)}, \frac{7}{(18)}, \frac{14}{(24)}, \frac{23}{(30)}, \ldots$$

Top, $-1, 2, 7, 14, 23 \Rightarrow n^2-2$.

bottom $6, 12, 18, 24, 30 \Rightarrow 6n$.

a) $a_n = \dfrac{n^2+2}{(6n)}$

b) $a_n = \dfrac{n^2-2}{(6n)}$

c) $a_n = \dfrac{(n-1)^2-2}{(6n)}$

d) $a_n = \dfrac{(n+1)^2-2}{(6n)}$

e) $a_n = \dfrac{2n-2}{(6n)}$

Question 7

You did not answer the question.

Determine the boundedness and monotonicity of the sequence with a_n as indicated.

$$a_n = \frac{12}{n}$$

$a_n = \frac{12}{n} > \frac{12}{n+1} = a_{n+1}$

which is decreasing.

$a_1 = 12, \; a_2 = \frac{12}{2} = 6 \ldots$

$\lim_{n\to\infty} \frac{12}{n} = 0$.

$\Rightarrow 0 < \frac{12}{n} \le 12$

a) decreasing; bounded below by 1 and above by 12.

b) nondecreasing; bounded below by 1 and above by 12.

c) nonincreasing; bounded below by 0 and above by 12.

d) decreasing; bounded below by 0 and above by 12.

e) increasing; bounded below by 0 and above by 12.

Question 8

You did not answer the question.

Determine the boundedness and monotonicity of the sequence with a_n as indicated.

$$a_n = \frac{n^2}{\sqrt{n^3+7}} = \sqrt{\frac{n^4}{n^3+7}} = \frac{P(n)}{Q(n)}$$

$\deg P > \deg Q$

$\frac{4}{2} = 2 \qquad \frac{3}{2}$

$a_n \to \infty$ as $n \to \infty$

$a_1 = \frac{1}{\sqrt{8}} = \frac{1}{4}\sqrt{2}$

increasing.

a) increasing; bounded below by 0 and above by $\frac{1}{4}\sqrt{2}$

b) decreasing; bounded below by $\frac{1}{4}\sqrt{2}$ but not bounded above.

c) nonincreasing; bounded below by 0 and above by $\frac{1}{4}\sqrt{2}$

d) nondecreasing; bounded below by $\frac{1}{4}\sqrt{2}$ but not bounded above.

e) increasing; bounded below by $\frac{1}{4}\sqrt{2}$ but not bounded above.

Question 9

You did not answer the question.

Determine the boundedness and monotonicity of the sequence with a_n as indicated.

$$a_n = \frac{(n+9)^2}{n^2} = \left(\frac{n+9}{n}\right)^2 = \left(1+\frac{9}{n}\right)^2$$

$a_1 = \frac{10^2}{1^2} = 100$

$\lim_{n\to\infty}\left(1+\frac{9}{n}\right)^2 = 1^2 = 1$

$a_n - a_{n+1}$

$= \left(1+\frac{9}{n}\right)^2 - \left(1+\frac{9}{n+1}\right)^2$

$= \left(1+\frac{9}{n}-1-\frac{9}{n+1}\right)\left(1+\frac{9}{n}+1+\frac{9}{n+1}\right)$

$= \left(\frac{9}{n}-\frac{9}{n+1}\right)\left(2+\frac{9}{n}+\frac{9}{n+1}\right) > 0$

$\Rightarrow a_n > a_{n+1}$

$\Rightarrow$ decreasing.

a) nonincreasing; bounded above by 100 but not bounded below.

b) nondecreasing; bounded below by 1 and above by 100

c) decreasing; bounded below by 1 but not bounded above.

d) decreasing; bounded below by 1 and above by 100

e) decreasing; bounded above by 100 but not bounded below.

Question 10

You did not answer the question.

Determine the boundedness and monotonicity of the sequence with a_n as indicated.

$$a_n = (-1)^{2n+1}\sqrt{n+9}$$

$a_n - a_{n+1} = (-1)^{2n+1}\sqrt{n+9} - (-1)^{2(n+1)+1}\sqrt{(n+1)+9}$

$= (-1)^{2n+1}\sqrt{n+9} - (-1)^{2n+3}\sqrt{n+10}$

$n \in \mathbb{N}$, $(-1)^{2n+1} = (-1)^{2n+3} = -1 \Rightarrow\; = -\sqrt{n+9} - (-1)\sqrt{n+10} > 0$

$\Rightarrow a_n > a_{n+1} \Rightarrow$ decreasing

$a_1 = (-1)^3\sqrt{10} = -\sqrt{10}$ above

$\lim_{n\to\infty} (-1)^{2n+1}\sqrt{n+9} = -\infty$ (DNE) no below

a) nonincreasing; bounded below by $-\sqrt{10}$ but not bounded above.

b) nondecreasing; bounded above by $-\sqrt{10}$ but not bounded below.

c) not monotonic; bounded above by $-\sqrt{10}$ but not bounded below.

d) decreasing; bounded above by $-\sqrt{10}$ but not bounded below.

e) increasing; bounded below by $-\sqrt{10}$ but not bounded above.

Question 11

You did not answer the question.

Give the first six terms of the sequence and then give the nth term.

$$a_1 = 1; \quad a_{n+1} = \frac{10}{n+1} a_n$$

$a_1 = 1$

as $n=1 \Rightarrow a_2 = a_{1+1} = \frac{10}{1+1} a_1 = \frac{(10)}{(2)}$

as $n=2 \Rightarrow a_3 = a_{2+1} = \frac{10}{2+1} a_2 = \frac{10}{3}\cdot\frac{10}{2} = \frac{100}{6}$

as $n=3$ $a_4 = \frac{10}{3+1} a_3 = \frac{10}{4}\cdot\frac{10}{3}\cdot\frac{10}{2} = \frac{1000}{4!}$ (10^3)

$n=4$, $a_5 = \frac{10}{4+1} a_4 = \frac{10}{5}\cdot\frac{10}{4}\cdot\frac{10}{3}\cdot\frac{10}{2} = \frac{10^4}{5!}$

$\vdots$

$a_n = \frac{10^{n-1}}{n!}$

a) $a_1 = 1$, $a_2 = \frac{(10)}{(2)}$, $a_3 = \frac{(100)}{(6)}$, $a_4 = \frac{(900)}{(24)}$, $a_5 = \frac{(9900)}{(120)}$, $a_6 = \frac{(99900)}{(720)}$; $a_n = \frac{10^{n+1}}{n!}$

b) $a_1 = 1$, $a_2 = \frac{(10)}{(2)}$, $a_3 = \frac{(100)}{(6)}$, $a_4 = \frac{(1000)}{(24)}$, $a_5 = \frac{(10000)}{(120)}$, $a_6 = \frac{(100000)}{(720)}$; $a_n = \frac{10^{n+1}}{n!}$

c) $a_1 = 1$, $a_2 = \frac{(10)}{(2)}$, $a_3 = \frac{(100)}{(6)}$, $a_4 = \frac{(1000)}{(24)}$, $a_5 = \frac{(10000)}{(120)}$, $a_6 = \frac{(100000)}{(720)}$; $a_n = \frac{10^{n-1}}{n!}$

d) $a_1 = 1$, $a_2 = \frac{(10)}{(2)}$, $a_3 = \frac{(100)}{(6)}$, $a_4 = \frac{(1000)}{(24)}$, $a_5 = \frac{(10000)}{(120)}$, $a_6 = \frac{(100000)}{(720)}$; $a_n = \frac{10^{n}}{n!}$

e) $a_1 = \frac{(10)}{(1)}$, $a_2 = \frac{(100)}{(2)}$, $a_3 = \frac{(1000)}{(6)}$, $a_4 = \frac{(10000)}{(24)}$, $a_5 = \frac{(100000)}{(120)}$, $a_6 = \frac{(1000000)}{(720)}$; $a_n = \frac{10^{n}}{n!}$

Question 12

You did not answer the question.

State whether the sequence converges and, if it does, find the limit.

$$\sqrt{n+9}$$

$a_n = \sqrt{n+9}$

as $n\to\infty$ $a_n \to \infty$

a) converges to 0

b) converges to -1

c) converges to $\sqrt{10}$

d) diverges

e) converges to 1

Question 13

You did not answer the question.

State whether the sequence converges and, if it does, find the limit.

$$\frac{n+5}{n^2}$$

$\frac{n+5}{n^2} = \frac{n}{n^2} + \frac{5}{n^2} = \frac{1}{n} + \frac{5}{n^2} \to 0$ as $n\to\infty$

a) converges to 0

b) converges to 6

c) diverges

d) converges to -1

e) converges to 1

Question 14

You did not answer the question.

State whether the sequence converges and, if it does, find the limit.

$$\frac{2^n}{4^n+4}$$

$0 < \frac{2^n}{4^n+4} < \frac{2^n}{4^n} = \left(\frac{1}{2}\right)^n$

$\downarrow$ as $n\to\infty$ 0 $\qquad$ $\downarrow$ as $n\to\infty$ 0

By sequeeze Thm, $\frac{2^n}{4^n+4} \to 0$ as $n\to\infty$

a) diverges

b) converges to 0

c) converges to 1

d) converges to -1

e) converges to $^1/_4$

Question 15

You did not answer the question.

State whether the sequence converges and, if it does, find the limit.

$$\frac{5n}{\sqrt{n^2+2n+1}} < \frac{5n}{\sqrt{n^2+1}} < \frac{5n}{\sqrt{n^2}}$$

$$\frac{5n}{\sqrt{n^2+2n+1}} = \frac{5n}{\sqrt{(n+1)^2}} = \frac{5n}{n+1} \to 5 \text{ as } n\to\infty$$

$$\frac{5n}{\sqrt{n^2}} = \frac{5n}{n} = 5 \to 5 \text{ as } n\to\infty$$

By Squeeze $\Rightarrow \frac{5n}{\sqrt{n^2+1}} \to 5$ as $n\to\infty$

See last page

a) diverges

b) converges to 7

c) converges to 6

d) converges to 0

e) converges to 5

Question 16

You did not answer the question.

State whether the sequence converges and, if it does, find the limit.

$$\ln\left(\frac{7n}{n+1}\right)$$

Since $\ln x$ is continuous, we have

$$\lim_{n\to\infty} \ln\left(\frac{7n}{n+1}\right) = \ln\left(\lim_{n\to\infty}\frac{7n}{n+1}\right) = \ln(7)$$

See on last page

a) diverges

b) converges to 2

c) converges to ln(7/2)

d) converges to 1

e) converges to ln(7)

Question 17

You did not answer the question.

State whether the sequence converges and, if it does, find the limit.

$$\frac{n^2}{\sqrt{6n^4+1}} = \frac{P(n)}{Q(n)}$$

degree of $P(n) = 2$.

degree of $Q(n) = 4 \times \frac{1}{2}$

$$\Rightarrow \lim_{n\to\infty}\frac{P(n)}{Q(n)} = \text{leading coefficient} = \frac{1}{\sqrt{6}}$$

See on last page

a) converges to $\frac{1}{6}\sqrt{6}$

b) converges to 1

c) diverges

d) converges to $\frac{1}{7}\sqrt{7}$

e) converges to $^1/_8$

Q18, since e^x is continuous

$$\lim_{n\to\infty} e^{\left(\frac{9}{\sqrt{n}}\right)} = e^{\left(\lim_{n\to\infty}\frac{9}{\sqrt{n}}\right)} = e^0 = 1$$

Question 18

You did not answer the question.

State whether the sequence converges and, if it does, find the limit.

$$e^{\left(\frac{9}{\sqrt{n}}\right)}$$

a) converges to 0

b) converges to e^9

c) diverges

d) converges to 1

e) converges to e

$$\frac{1}{n} - \frac{1}{n+7} = \frac{7}{n(n+7)} \to 0 \text{ as } n\to\infty$$

Question 19

You did not answer the question.

State whether the sequence converges and, if it does, find the limit.

$$\frac{1}{n} - \frac{1}{n+7}$$

a) converges to $^1/_{93}$

b) converges to 0

c) converges to $^7/_8$

d) converges to 1

e) diverges

Question 20

You did not answer the question.

State whether the sequence converges and, if it does, find the limit.

$$a_1 = 1, \quad a_{n+1} = 11 - a_n$$

a) converges to 11

b) converges to 1

c) diverges

d) converges to 12

e) converges to 10

$a_1 = 1, \quad a_2 = 11 - a_1 = 10. \quad a_3 = 11 - a_2 = 1. \quad a_4 = 11 - 1 = 10$

$\Rightarrow \{1, 10, 1, 10, 1, 10, \cdots\} \Rightarrow$ diverges.

The limit of this patter $\frac{P(n)}{Q(n)}$ with two polynomials P, Q.
Let the highest degree of P, Q be $\deg(P)$, $\deg(Q)$, respectively.

① $\deg(P) > \deg(Q)$, $\lim_{n \to \infty} \frac{P(n)}{Q(n)} = \infty$ (DNE)

② $\deg(P) = \deg(Q)$, $\lim_{n \to \infty} \frac{P(n)}{Q(n)} = \frac{\text{the coefficient of the highest term of } P(n)}{\text{the coefficient of the highest term of } Q(n)}$

③ $\deg(P) < \deg(Q)$, $\lim_{n \to \infty} \frac{P(n)}{Q(n)} = 0$

Squeeze's rule

$$g(x) \le h(x) \le f(x)$$

If $\lim_{x\to\infty} f(x) = L$ and $\lim_{x\to\infty} g(x) = L$,

Then we have $\lim_{x\to\infty} h(x) = L$

If $f(x)$ is a continuous function,

we have

$$\lim_{x\to\infty} f(x) = f\left(\lim_{x\to\infty} x\right).$$