PRINTABLE VERSION

Quiz 8

You scored 0 out of 100

Question 1

You did not answer the question.

Express the curve by an equation in x and y.

$x(t) = t^2$
$y(t) = 3t + 5$

$\Rightarrow \frac{y-5}{3} = t$

$\Rightarrow \left(\frac{y-5}{3}\right)^2 = x$

$\Rightarrow 9x = (y-5)^2$

a) $3y = (x+5)^2$

b) $\boxed{9x = (y-5)^2}$

c) $9y = (x-5)^2$

d) $x = \frac{1}{3}(y-5)^2$

e) $x = (y-5)^2 + 9$

Question 2

You did not answer the question.

Express the curve by an equation in x and y.

$x(t) = 4\cos(t)$
$y(t) = 3\sin(t)$

$\Rightarrow \cos(t) = \frac{x}{4}$

$\sin(t) = \frac{y}{3}$

$1 = \cos^2(x) + \sin^2(x) = \left(\frac{x}{4}\right)^2 + \left(\frac{y}{3}\right)^2$

$\Rightarrow 144 = 9x^2 + 16y^2$

a) $9x^2 + 16y^2 = 12$

b) $9x^2 - 16y^2 = 12$

c) $16x^2 + 9y^2 = 144$

d) $16x^2 - 9y^2 = 144$

e) $\boxed{9x^2 + 16y^2 = 144}$

Question 3

You did not answer the question.

Express the curve by an equation in x and y.

$x(t) = e^t$
$y(t) = 5 - e^{3t}$ $= 5 - (e^t)^3$

$e^t > 0 \Rightarrow x > 0$

$\Rightarrow y = 5 - x^3$

a) $y = 5 + x^3$, $x > 0$

b) $\boxed{y = 5 - x^3, \ x > 0}$

c) $x = 5 - y^3$, $y > 0$

d) $x = 5 + y^3$, $x < 0$

e) $y = 5 + x^2$, $x < 0$

Question 4

You did not answer the question.

Express the curve by an equation in x and y and identify the correct sketch of the curve: $(3 + 3t, 5 - 6t)$, $0 \le t \le 2$.

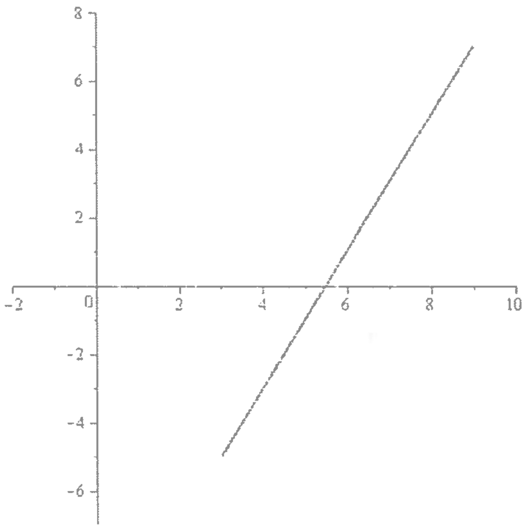
$x = 3 + 3t \Rightarrow t = \frac{x-3}{3}$

$y = 5 - 6t \Rightarrow t = \frac{y-5}{-6}$

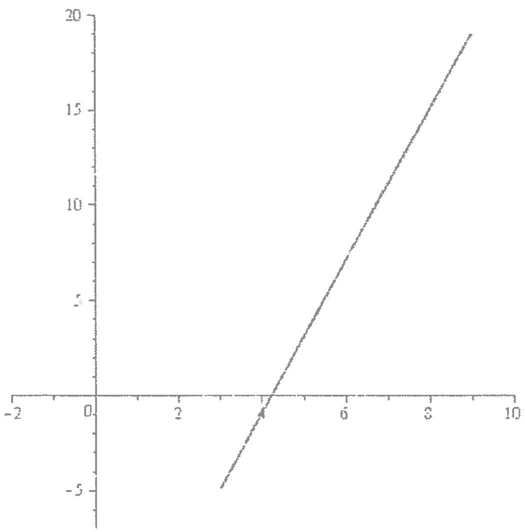
$\Rightarrow \frac{x-3}{3} = \frac{y-5}{-6} \Rightarrow -2(x-3) = y-5$

$\Rightarrow 2x + y = 11$ a line

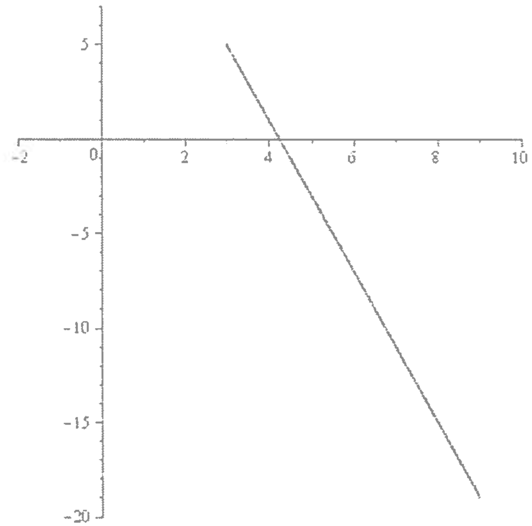
$(3+3t, 5-6t)$, $0 \le t \le 2 \Rightarrow$ as $t=0$ $(3,5) \to$ as $t=2$ $(9,-7)$



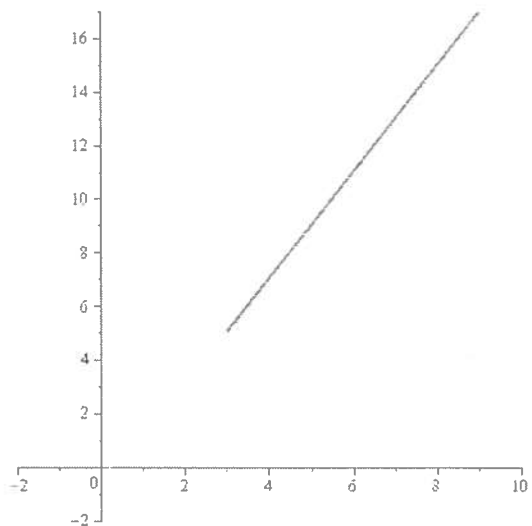
a) $y = -11 + 2x$;



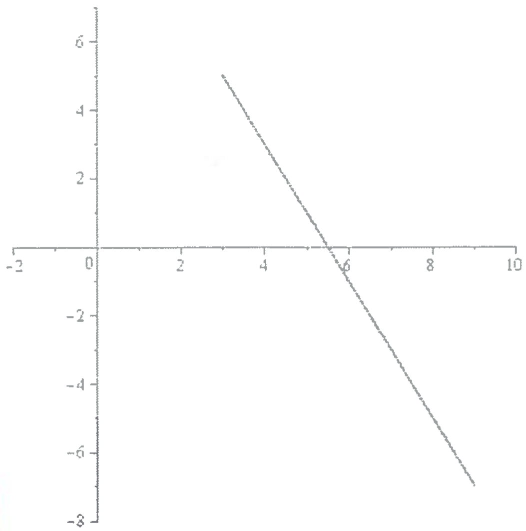
b) $y = -17 + 4x$;



c) $y = 17 - 4x$;



d) $y = -1 + 2x$;



e) $y = 11 - 2x$

Question 5

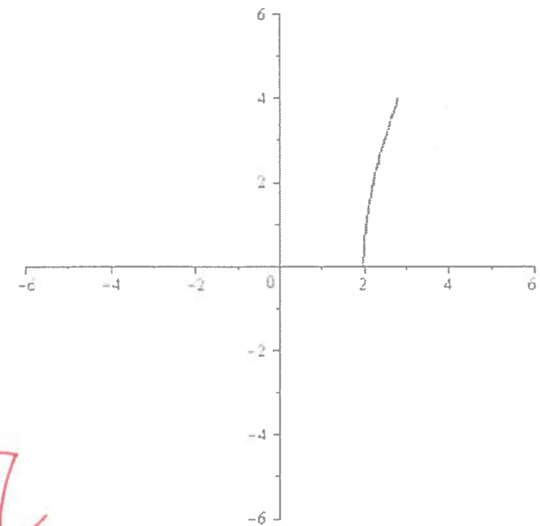
You did not answer the question.

Express the curve by an equation in x and y and identify the correct sketch of the curve: $(3\sec(t), 5\tan(t))$, $0 \le t \le \frac{1}{4}\pi$.

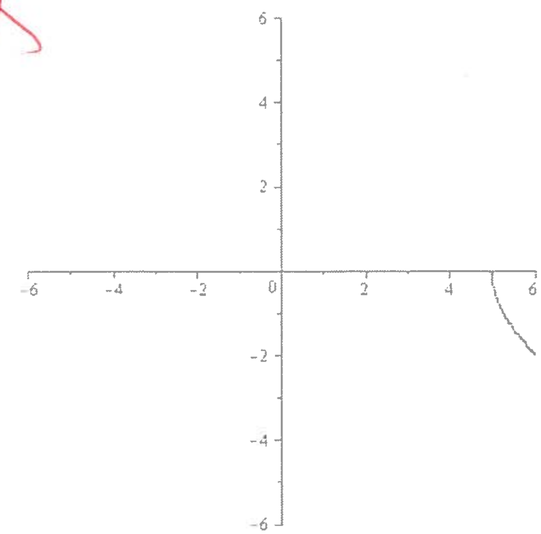
$$x = 3\sec(t) \Rightarrow \frac{x}{3} = \sec(t)$$
$$y = 5\tan(t) \quad \frac{y}{5} = \tan(t)$$

$$1 = \sec^2(t) - \tan^2(t) = \left(\frac{x}{3}\right)^2 - \left(\frac{y}{5}\right)^2 \Rightarrow \text{hyperbolic}$$

$$(3\sec t, 5\tan t),\ 0 \le t \le \frac{\pi}{4} \Rightarrow \underset{\text{as } t \to 0}{(3,0)} \to \underset{t = \frac{\pi}{4}}{(3\sqrt{2}, 5)}$$



a) $1 + \frac{1}{25}y^2 = \frac{1}{9}x^2$



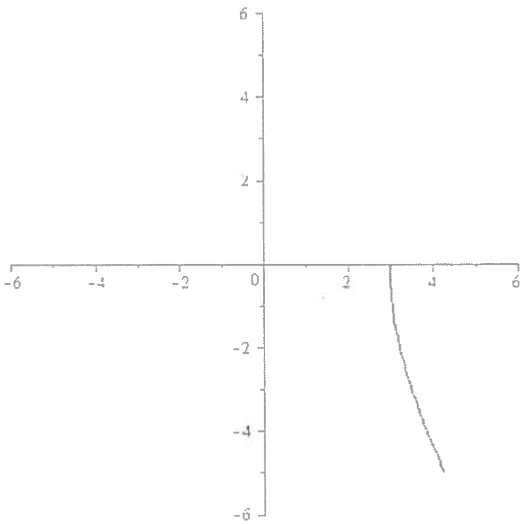
b) $1 + \frac{1}{9}y^2 = \frac{1}{25}x^2$

Formula for parametrization of circle.

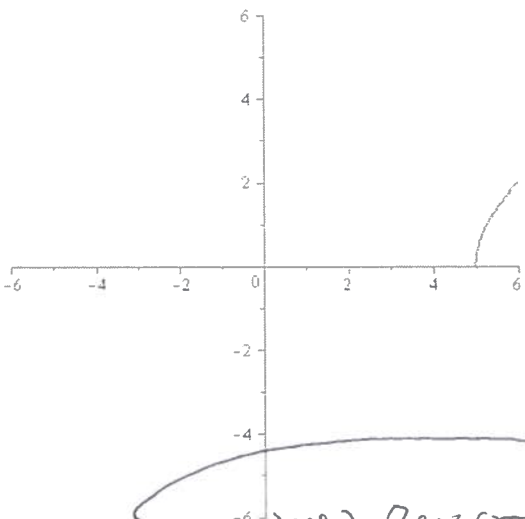
Counterclockwise : $x(t)=r\cos(at+\theta_0)$, $y(t)=r\sin(at+\theta_0)$

clockwise : $x(t)=r\cos(-at+\theta_0)$, $y(t)=r\sin(-at+\theta_0)$

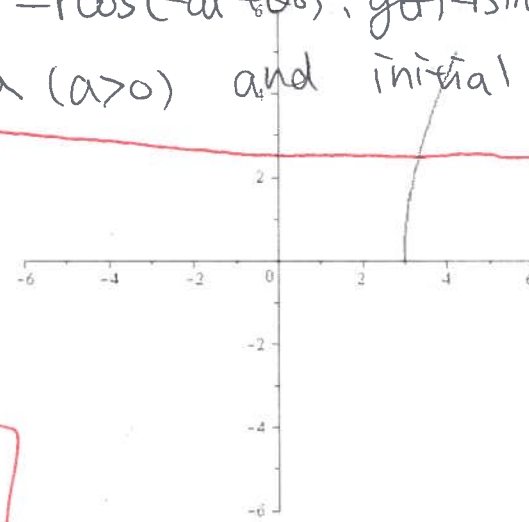
with period a $(a>0)$ and initial θ_0



c) $1+\frac{1}{25}y^2=\frac{1}{9}x^2$;



d) $1+\frac{1}{9}y^2=\frac{1}{25}x^2$;



e) $1+\frac{1}{25}y^2=\frac{1}{9}x^2$

Question 6

You did not answer the question.

A particle with position given by the equations $x(t)=7\sin(2\pi t)$, $y(t)=7\cos(2\pi t)$, $t\in[0,1]$ starts at the point (0, 7) and traverses the unit circle $x^2+y^2=49$ once in a clockwise manner. Write equations of the form $x(t)=f(t)$, $y(t)=g(t)$, $t\in[0,1]$ so that the particle begins at (0, 7) and traverses the circle once in a counterclockwise manner.

period : $\frac{2\pi}{1-0}\cdot\cancel{1}=2\pi$

a) $x(t)=7\sin(2\pi t)$, $y(t)=-7\cos(2\pi t)$

b) $x(t)=-7\cos(2\pi t)$, $y(t)=7\sin(2\pi t)$

c) $x(t)=-7\sin(2\pi t)$, $y(t)=-7\cos(2\pi t)$

d) $x(t)=7\cos(2\pi t)$, $y(t)=7\sin(2\pi t)$

e) $x(t)=-7\sin(2\pi t)$, $y(t)=7\cos(2\pi t)$

$x(t)=r\cos(2\pi t+\theta_0)$, $y(t)=r\sin(2\pi t+\theta_0)$ $\Rightarrow$ $x(0)=0$, $y(0)=7$

$\Rightarrow \cos(\theta_0)=0$, $\sin(\theta_0)=1$ $\Rightarrow \theta_0=\frac{\pi}{2}$

$\Rightarrow x(t)=7\cos(2\pi t+\frac{\pi}{2})=$

$x(t)=7\cos(2\pi t+\frac{\pi}{2})$
$=7\cos(2\pi t)\cos(\frac{\pi}{2})-7\sin 2\pi t\cdot\sin\frac{\pi}{2}$
$=-7\sin(2\pi t)$.

$y(t)=7\sin(2\pi t+\frac{\pi}{2})=7\cos(2\pi t)$

Question 7

You did not answer the question.

Find a parametrization $x=x(t)$, $y=y(t)$, $t\in[0,1]$, for the line segment from $\underset{A}{(7,9)}$ to $\underset{B}{(4,10)}$.

$A(1-t)+Bt \overset{\text{or}}{=} (B-A)t+A$ formula $t\in[0,1]$

$[(4,10)-(7,9)]t+(7,9)=(7-3t,\ 9+t)$

a) $x(t) = 7 + 3t$, $y(t) = 9 - t$

b) $x(t) = 7 - 3t$, $y(t) = 9 + 2t$

c) $x(t) = 7 + 2t$, $y(t) = 9 + t$

d) $x(t) = 7 - t$, $y(t) = 9 + 3t$

e) $x(t) = 7 - 3t$, $y(t) = 9 + t$

Question 8

You did not answer the question.

Find an equation in x and y for the line tangent to the curve at $t = 3$
$x(t) = t - 2$
$y(t) = t^4$

Q8, Slope: $\boxed{\dfrac{dy}{dx} = \dfrac{\frac{dy}{dt}}{\frac{dx}{dt}}}$

$= \dfrac{4t^3}{1} = 4t^3$

Slope @ $t=3$ $\Rightarrow 4 \cdot 3^3 = 108$

Point @ $t=3$

$\Rightarrow x(3) = 1$

$y(3) = 81$

$\Rightarrow$ line equation:

$y - 81 = 108(x - 1)$

$\Rightarrow 108x - y = 27$

a) $108x + 459 + y = 0$

b) $27x + 135 - y = 0$

c) $-108x - 27 + y = 0$

d) $108x - 27 - y = 0$

e) $108x - 27 + y = 0$

Question 9

You did not answer the question.

Find an equation in x and y for the line tangent to the curve at $t = 3$
$x(t) = \frac{2}{t}$
$y(t) = t^2 - 2$

Q9, Slope: $\dfrac{\frac{dy}{dt}}{\frac{dx}{dt}} = \dfrac{2t}{\frac{-2}{t^2}} = -t^3$

Slope @ $t=3$ $\Rightarrow -27$

Point @ $t=3$, $x(3) = \frac{2}{3}$

$y(3) = 9 - 2 = 7$

$y - 7 = -27\left(x - \frac{2}{3}\right)$

$\Rightarrow y - 7 = -27x + 18$

a) $-6x + \frac{10}{3} = 0$

b) $3x - \frac{20}{9} = 0$

c) $2x + \frac{14}{9} = 0$

d) $6x - \frac{50}{9} + \frac{2}{9}y = 0$ $\Rightarrow 54x + 2y = 50 \Rightarrow 27 + y = 25$

e) $6x - \frac{43}{9} + \frac{1}{9}y = 0$

Question 10

You did not answer the question.

Find an equation in x and y for the line tangent to the polar curve at $\theta = \frac{1}{2}\pi$.
$r = 11\cos(2\theta)$

$x = r\cos\theta = 11\cos(2\theta)\cos\theta$

$y = r\sin\theta = 11\cos(2\theta)\sin\theta$

$\Rightarrow$ slope: $\left.\dfrac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}\right|_{\theta=\frac{\pi}{2}} = \dfrac{-22\sin(2\theta)\sin\theta + 11\cos(2\theta)\cos\theta}{-22\sin(2\theta)\cos\theta - 11\cos(2\theta)\sin\theta}$

$= \dfrac{-11 \cdot 0}{11} = 0$ and point @ $\theta = \frac{\pi}{2}$

horizontal line. $(0, -11) \Rightarrow y = -11$.

a) $x - 11 = 0$

b) $y + 11 = 0$

c) $y = 2x - 11$

d) $y = x + 11$

e) $y = -12$

Question 11

You did not answer the question.

Parametrize the curve by a pair of differentiable functions $x = x(t)$, $y = y(t)$ with $[x'(t)]^2 + [y'(t)]^2 \neq 0$, then determine the tangent line at the orgin.

$y = -2x^3$

let $x = t$, $y = -2t^3$

[check $(x')^2 + (y')^2 = |-6t^2| \neq 0, \forall t$]

Slope @ $(0,0) \Rightarrow$ Slope @ $t=0$

$\left.\dfrac{dy}{dx}\right|_{t=0} = \left.\dfrac{\frac{dy}{dt}}{\frac{dx}{dt}}\right|_{t=0} = \left.\dfrac{-6t^2}{1}\right|_{t=0} = 0$

$\Rightarrow$ horizontal line $\Rightarrow y = 0$
which goes through $(0,0)$

a) $x(t) = -2t^3$, $y(t) = t$; tangent line $y = -1$

b) $x(t) = t$, $y(t) = -2t^3$; tangent line $y = 0$

c) $x(t) = t^2$, $y(t) = -2t^3$; tangent line $x = 0$

d) $x(t) = t$, $y(t) = -2t^3$; tangent line $x = 0$

e) $x(t)=-2t^3$, $y(t)=t$; tangent line $y=0$

Question 12

You did not answer the question.

Find the points (x, y) at which the curve has a horizontal tangent.

$$x(t)=5-2\sin(t)$$
$$y(t)=5+6\cos(t)$$

a) (5, 2) and (2, 5)

b) (5, 3) and (−2, −5)

c) (5, 11) and (5, −1)

d) (4, −3) and (3, 5)

e) (−4, 1) and (4, 3)

horizontal line $\Leftrightarrow$ its slope is 0 $\Leftrightarrow \frac{dy}{dx}=0$

$\Leftrightarrow \frac{dy}{dx}=\frac{\frac{dy}{dt}}{\frac{dx}{dt}}=0 \Leftrightarrow 0=\frac{\frac{dy}{dt}}{\frac{dx}{dt}}=\frac{-6\sin t}{-2\cos t}$

$\Leftrightarrow \sin(t)=0 \Leftrightarrow t=0$ or π

$\Leftrightarrow \underset{t=0}{(x(0),y(0))}=(5,11)$, $\underset{t=\pi}{(x(\pi),y(\pi))}=(5,-1)$

Question 13

You did not answer the question.

Find the points (x, y) at which the curve has a horizontal tangent.

$$x(t)=t^2-5t$$
$$y(t)=t^3-3t^2-24t$$

HORIZONTAL LINE $\Leftrightarrow \frac{dy}{dx}=0 \Leftrightarrow 0=\frac{dy}{dx}=\frac{\frac{dy}{dt}}{\frac{dx}{dt}}$

a) (28,14) and (−80,−4)

b) (−6,0) and (4,0)

c) (0,−6) and (0,4)

d) (14,28) and (−4,−80)

e) (5,0) and (−5,0)

$\Rightarrow 0=\frac{\frac{dy}{dt}}{\frac{dx}{dt}}=\frac{3t^2-6t-24}{2t-5} \Leftrightarrow 3t^2-6t-24=0,\ t\neq\frac{5}{2}$

$\Rightarrow t^2-2t-8=0$

$\Leftrightarrow (t+2)(t-4)=0,\ t\neq\frac{5}{2} \Leftrightarrow t=-2$ or 4.

$(x(-2),y(-2))=(14,28)$, or $(x(4),y(4))=(-4,-80)$

Question 14

You did not answer the question.

Find the points (x, y) at which the curve has a vertical tangent.

$$x(t)=2-3\sin(t)$$
$$y(t)=2+\cos(t)$$

Q14. vertical line $\Leftrightarrow \frac{dy}{dx}=\infty \Leftrightarrow \frac{dx}{dt}=0$

$0=\frac{dx}{dt}=-3\cos(t) \Rightarrow t=\frac{\pi}{2}, \frac{3\pi}{2}$

a) (2, 3) and (3, 2)

b) (1, −4) and (4, 2)

c) (2, 4) and (−3, −2)

d) (−1, 2) and (5, 0)

e) (−1, 2) and (5, 2)

$t=\frac{\pi}{2}$

$(x(\frac{\pi}{2}),y(\frac{\pi}{2}))=(-1,2)$

$(x(\frac{3\pi}{2}),y(\frac{3\pi}{2}))=(5,2)$

Question 15

You did not answer the question.

Find the points (x, y) at which the curve has a vertical tangent. $\Leftrightarrow \frac{dx}{dt}=0 \Leftrightarrow 2t-2=0$

$$x(t)=t^2-2t$$
$$y(t)=t^3-4t^2+t$$

$\Leftrightarrow t=1$

$(x(1),y(1))=(-1,-2)$

a) (−1, −4)

b) (−1, −2)

c) $(-\frac{1}{2}, -1)$

d) (−2, −4)

e) (−2, −2)

Q16. $\boxed{\text{Method 1}}$ $y=5x+4$ is a line between (0,4) and (2,14), the distance between (0,4) and (2,14) is

$\sqrt{(14-4)^2+(2-0)^2}=\sqrt{104}=2\sqrt{26}$

Question 16

You did not answer the question.

Find the length of the graph.

$$f(x)=5x+4$$
$$x\in[0,2]$$

a) $6\sqrt{26}$

b) $2\sqrt{26}$

$\boxed{\text{Method 2}}$ By formula on last page

Now "f(x)" is given $\Rightarrow$ Using item ①

we have $f'(x)=5$

lengeh $=\int_0^2\sqrt{1+(5)^2}\,dx = x\sqrt{26}\Big|_0^2 =$

$2\sqrt{26}-0=2\sqrt{26}$

c) $3\sqrt{26}$

d) $4\sqrt{26}$

e) $\frac{4}{3}\sqrt{26}$

Q17, By formula ①, f(x) is given

$$f'(x)=\frac{x}{4}-\frac{1}{x} \quad \text{for } x\in[1,5]$$

The length is

$$\int_1^5\sqrt{1+\left(\frac{x}{4}-\frac{1}{x}\right)^2}\,dx=\int_1^5\sqrt{1+\frac{x^2}{16}-\frac{1}{2}+\frac{1}{x^2}}\,dx$$

$$=\int_1^5\sqrt{\frac{x^2}{16}+\frac{1}{2}+\frac{1}{x^2}}\,dx$$

$$=\int_1^5\sqrt{\left(\frac{x}{4}+\frac{1}{x}\right)^2}\,dx$$

$$=\int_1^5\frac{x}{4}+\frac{1}{x}\,dx=\frac{x^2}{8}+\ln x\Big|_1^5$$

$$=\frac{25-1}{8}+\ln5-\ln1$$

$$=3+\ln5$$

Question 17

You did not answer the question.

$\frac{x^2}{16}-\frac{1}{2}+\frac{1}{x^2}$

Find the length of the graph

$f(x)=\frac{1}{8}x^2-\ln(x)$

$x\in[1,5]$

a) $2+\frac{2}{3}\ln(5)$

b) $9+3\ln(5)$

c) $\frac{9}{2}+\frac{3}{2}\ln(5)$

d) $\boxed{3+\ln(5)}$

e) $6+2\ln(5)$

Question 18

You did not answer the question.

$$=\sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2} \quad @\, t$$

The equations below give the position of a particle at each time t from $t=0$ to $t=\pi$. Find the initial speed of the particle, the terminal speed, and the distance traveled.

$x(t)=10\,e^t\sin(t)$

$y(t)=10\,e^t\cos(t)$

$$\frac{dx}{dt}=10e^t\sin t+10e^t\cos t$$

$$\frac{dy}{dt}=10e^t\cos t-10e^t\sin t$$

speed $\Rightarrow \left|\frac{ds}{dt}\right|$, distance $\Rightarrow$ length

a) initial speed $=20\sqrt{2}$, terminal speed $=20\sqrt{2}\,e^{\pi}$; distance traveled $=20\sqrt{2}\,(-1+e^{\pi})$

b) initial speed $=\frac{20}{3}\sqrt{2}$, terminal speed $=\frac{20}{3}\sqrt{2}\,e^{\pi}$; distance traveled $=10\sqrt{2}\,(-1+e^{\pi})$

c) initial speed $=10\sqrt{2}$, terminal speed $=10\sqrt{2}\,e^{\pi}$; distance traveled $=10\sqrt{2}\,(-1+e^{\pi})$

d) initial speed $=5\sqrt{2}$, terminal speed $=5\sqrt{2}\,e^{\pi}$; distance traveled $=5\sqrt{2}\,(-1+e^{\pi})$

e) initial speed $=15\sqrt{2}$, terminal speed $=15\sqrt{2}\,e^{\pi}$; distance traveled $=10\sqrt{2}\,(-1+e^{\pi})$

Initial speed:

$$\sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}=\sqrt{(10e^t\sin t+10e^t\cos t)^2+(10e^t\sin t-10e^t\cos t)^2}$$

$$=\sqrt{200e^{2t}\sin^2 t+200e^{2t}\cos^2 t}=10\sqrt{2}\,e^t$$

$$\text{distance}=\int_0^{\pi}\sqrt{[x'(t)]^2+[y'(t)]^2}\,dt=\int_0^{\pi}\sqrt{200e^{2t}}\,dt=10\sqrt{2}\int_0^{\pi}e^t\,dt=\boxed{10\sqrt{2}(e^{\pi}-1)}$$

initial speed @ $t=0 \Rightarrow 10\sqrt{2}$

terminal @ $t=\pi \Rightarrow 10\sqrt{2}\cdot e^{\pi}$

Question 19

You did not answer the question.

Find the length of $r=2$ from $\theta=0$ to $\theta=2\pi$.

By Formula ③. $\rho(\theta)$ is given which is "$r=2$"

a) 3π

b) $\boxed{4\pi}$

c) $\frac{8}{3}\pi$

d) 12π

e) 6π

$$\Rightarrow \text{The length}=\int_0^{2\pi}\sqrt{2^2+0^2}\,d\theta$$

$$=\int_0^{2\pi}2\,d\theta=2\cdot2\pi=4\pi$$

Q20. By formula ③.

Question 20

You did not answer the question.

Find the length of $r=2+2\cos(\theta)$ from $\theta=0$ to $\theta=\pi$.

$$\rho(\theta)=2+2\cos\theta$$

$$\rho'(\theta)=-2\sin\theta$$

a) 12

b) 24

c) $\frac{16}{3}$

d) 16

e) $\boxed{8}$

$$\text{The length}=\int_0^{\pi}\sqrt{(2+2\cos\theta)^2+(-2\sin\theta)^2}\,d\theta$$

$$=\int_0^{\pi}\sqrt{4+8\cos\theta+\underbrace{4\cos^2\theta+4\sin^2\theta}_{=4}}\,d\theta$$

$$=\int_0^{\pi}\sqrt{8+8\cos\theta}=\sqrt{16}\int_0^{\pi}\sqrt{\cos^2\frac{\theta}{2}}\,d\theta=4\int_0^{\pi}\cos\frac{\theta}{2}\,d\theta$$

$$\boxed{\frac{1}{2}+\frac{1}{2}\cos\theta=\cos^2\frac{\theta}{2}}$$

$$4\cdot2\cdot\sin\frac{\theta}{2}\Big|_0^{\pi}$$

$$8[1-0]=8$$

Formula of length of graph

① $f(x)$ is given for $x \in [a,b]$.

The length of $f(x)$ for $x \in [a,b]$ is

$$\boxed{\int_a^b \sqrt{1+[f'(x)]^2}\,dx.}$$

② A parametrization of a function is given:

$x(t)$, $y(t)$ for $t \in [a,b]$.

The length of this graph for $t \in [a,b]$ is

$$\int_a^b \sqrt{[x'(t)]^2+[y'(t)]^2}\,dt$$

③ A polar equation $\rho(\theta)$ is given:

$\rho(\theta)$ for $\theta \in [a,b]$.

The length of $\rho(\theta)$ for $\theta \in [a,b]$ is

$$\int_a^b \sqrt{[\rho(\theta)]^2+[\rho'(\theta)]^2}\,d\theta$$

Formula for parametrization of line $\overline{AB}$

From A to B. we have

$(B-A)t+A$, $t \in [0,1]$

Formula for parametrization of circle with radius "r"

Counter clockwise:

$x(t)=r\cos(at+\theta_0)$, $y(t)=r\sin(at+\theta_0)$

clockwise

$x(t)=r\cos(-at+\theta_0)$, $y(t)=r\sin(-at+\theta_0)$

with period a $(a>0)$ and initial θ_0 angle