sin +	All +
tan +	cos +

PRINTABLE VERSION

Quiz 7

You scored 0 out of 100

Question 1

You did not answer the question. rectangular coordinates

Find the rectangular coordinates of the point

$\left[2, \frac{1}{2}\pi\right] = [r, \theta]$

$\boxed{\begin{array}{l} x = r\cos\theta \\ y = r\sin\theta \end{array}}$

$\boxed{\begin{array}{l} r^2 = x^2 + y^2 \\ \cos\theta = \frac{x}{r} \\ \sin\theta = \frac{y}{r} \end{array}}$ polar coordinates

$\Rightarrow x = 2 \cdot \cos\frac{\pi}{2} = 0$

$y = 2 \cdot \sin\frac{\pi}{2} = 2$

$\Rightarrow (0, 2)$

a) $(-1, 3)$

b) $(0, 2)$

c) $(1, 2)$

d) $(0, -2)$

e) $(-1, 1)$

Question 2

You did not answer the question.

Find the rectangular coordinates of the point

$\left[-4, \frac{3}{4}\pi\right] = [r, \theta]$

$x = -4\cos\frac{3\pi}{4} = -4 \cdot -\frac{\sqrt{2}}{2} = 2\sqrt{2}$

$y = -4\sin\frac{3\pi}{4} = -4 \cdot \frac{\sqrt{2}}{2} = -2\sqrt{2}$

$\Rightarrow (2\sqrt{2}, -2\sqrt{2})$

a) $(4\sqrt{2}, 4)$

b) $(2\sqrt{2}, -2\sqrt{2})$

c) $(\sqrt{2}, -4\sqrt{2})$

d) $(-2\sqrt{2}, 2\sqrt{2})$

e) $(\sqrt{2}, -\sqrt{2})$

Question 3

You did not answer the question.

Give all possible polar coordinates for the point $(4, 4\sqrt{3})$ given in rectangular coordinates. "(x,y)"

a) $\left[8, \frac{1}{3}\pi + 2n\pi\right], \left[-8, \frac{4}{3}\pi + 2n\pi\right]$

b) $\left[16, \frac{1}{3}\pi + 2n\pi\right], \left[-16, \frac{4}{3}\pi + 2n\pi\right]$

c) $\left[4, -\frac{1}{3}\pi + 2n\pi\right], \left[-4, -\frac{4}{3}\pi + 2n\pi\right]$

d) $\left[-8, \frac{1}{3}\pi + 2n\pi\right], \left[8, \frac{4}{3}\pi + 2n\pi\right]$

e) $\left[8, \frac{4}{3}\pi + 2n\pi\right], \left[-8, \frac{1}{3}\pi + 2n\pi\right]$

① $r^2 = x^2 + y^2 = 4^2 + (4\sqrt{3})^2$

$= 16 + 48 = 64$

$\Rightarrow r = 8$

② $\cos\theta = \frac{x}{r} = \frac{4}{8} = \frac{1}{2}$

$\sin\theta = \frac{y}{r} = \frac{4\sqrt{3}}{8} = \frac{\sqrt{3}}{2}$ first Quadrant

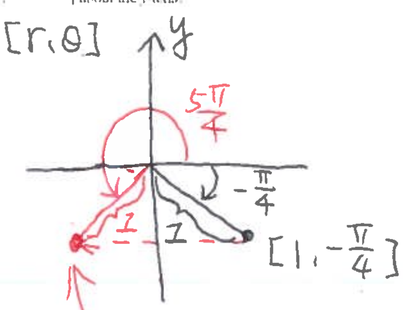
$\Rightarrow \theta = \frac{\pi}{3} + 2n\pi$

$\left[8, \frac{\pi}{3} + 2n\pi\right] = \left[-8, \pi + \frac{\pi}{3} + 2n\pi\right]$

Question 4

You did not answer the question.

Find the point symmetric to $\left[1, -\frac{1}{4}\pi\right]$ about the y-axis. $= [r, \theta]$



$\left[1, \frac{5\pi}{4}\right]$

a) $\left[2, \frac{5}{4}\pi\right]$

b) $\left[1, \frac{5}{4}\pi\right]$

c) $\left[-1, -\frac{1}{4}\pi\right]$

d) $\left[2, \frac{1}{4}\pi\right]$

e) $\left[-1, \frac{3}{4}\pi\right]$

Question 5

You did not answer the question.

Write the equation in polar coordinates.

rectangular: $x = r\cos\theta$, $y = r\sin\theta$ $\Longleftrightarrow$ polar: $r^2 = x^2+y^2$, $\cos\theta = \frac{x}{r}$, $\sin\theta = \frac{y}{r}$

$x^2 + (y-5)^2 = 25$

$(r\cos\theta)^2 + (r\sin\theta - 5)^2 = 25$

$r^2\cos^2\theta + r^2\sin^2\theta - 10r\sin\theta + 25 = 25$

$\Rightarrow r^2(\cos^2\theta + \sin^2\theta) - 10r\sin\theta = 0$

$\Rightarrow r^2 \cdot 1 = 10r\sin\theta$

divide by r $\Rightarrow r = 10\sin\theta$

a) $r = 10\cos(\theta)$

b) $r = 5\sin(\theta) + 25$

c) $r = 25$

d) $r = 5\cos^2(\theta)\sin(\theta)$

e) $\boxed{r = 10\sin(\theta)}$

Question 6

You did not answer the question.

Write the equation in polar coordinates.

$(x-7)^2 + y^2 = 49$

$(r\cos\theta - 7)^2 + (r\sin\theta)^2 = 49$

$r^2\cos^2\theta - 14r\cos\theta + 49 + r^2\sin^2\theta = 49$

$\Rightarrow r^2(\cos^2\theta + \sin^2\theta) - 14r\cos\theta = 0$

$\Rightarrow r^2 = 14r\cos\theta$

divide by r $\Rightarrow r = 14\cos\theta$

a) $r = 7\sin(\theta) + 49$

b) $r = 14\sin(\theta)$

c) $\boxed{r = 14\cos(\theta)}$

d) $r = 7\cos^2(\theta)\sin(\theta)$

e) $r = 49$

Question 7

You did not answer the question.

Write the equation in rectangular coordinates.

$2r\cos(\theta) = 9$ $\quad (\cos\theta = \frac{x}{r})$

$\Rightarrow 2\cancel{r} \cdot \frac{x}{\cancel{r}} = 9$

$\Rightarrow 2x = 9 \Rightarrow x = \frac{9}{2}$ (a vertical line)

a) $x^2 = 9$

b) $y = \frac{9}{2}$

c) $x = \frac{2}{9}$

d) $y = \frac{3}{2}$

e) $\boxed{x = \frac{9}{2}}$

Q8 ① since $\sin\theta = \frac{y}{r}$

we have $r = 6 \cdot \frac{y}{r} \Rightarrow r^2 = 6y$

② since $r^2 = x^2+y^2$,

we have

$x^2+y^2 = r^2 = 6y$

$\Rightarrow x^2+y^2 = 6y$

Question 8

You did not answer the question.

Write the equation in rectangular coordinates.

$r = 6\sin(\theta)$

a) $x^2 + y^2 = 6$

b) $x^2 + y^2 = 36$

c) $y = x^2 + 6$

d) $x = y + 6$

e) $\boxed{x^2 + y^2 = 6y}$

See "Graph" in last page.

Q9: $r = \frac{9}{2} - \frac{9}{2}\cos\theta$

$\Rightarrow$ item 4. ① $a = \frac{9}{2}$, $b = \frac{-9}{2}$.

$|a| = |b| \Rightarrow$ Cardioid $\Rightarrow$ (b) or (c)

Question 9

You did not answer the question.

Which of the following shows the correct sketch of the given polar curve?

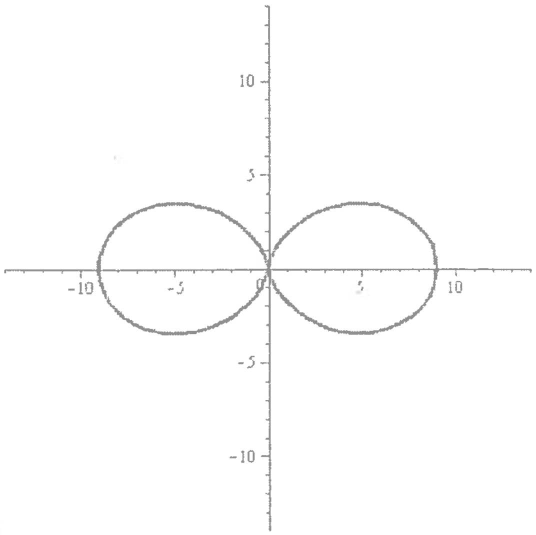
$r = \frac{9}{2} - \frac{9}{2}\cos(\theta)$

Then check the point as $\theta = \pi$.

As $\theta = \pi$, $r = \frac{9}{2} - \frac{9}{2}\cos\pi = \frac{9}{2} - \frac{9}{2}(-1)$

$= \frac{9}{2} + \frac{9}{2} = 9.$

$\Rightarrow$ This graph goes through $[9, \pi]$. $\Rightarrow$ (c).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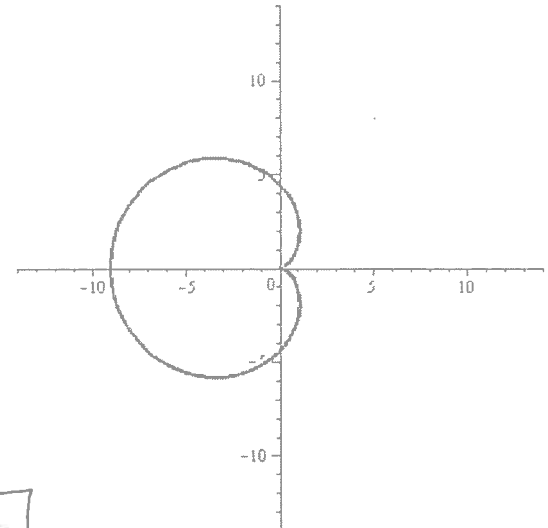
a)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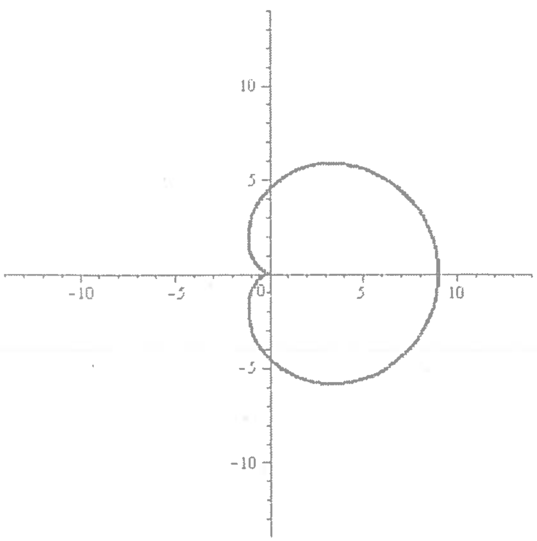
c)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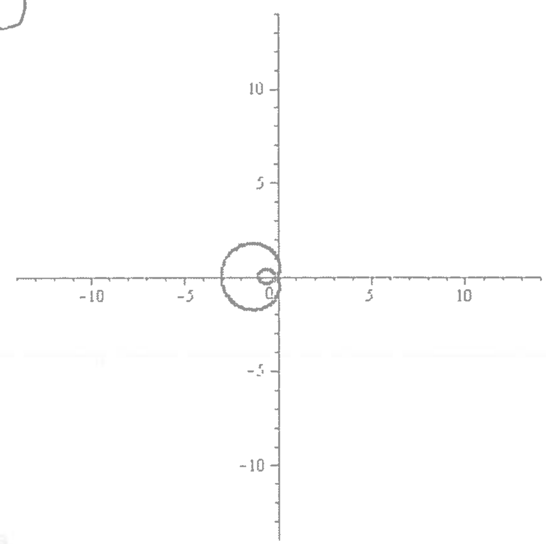
b)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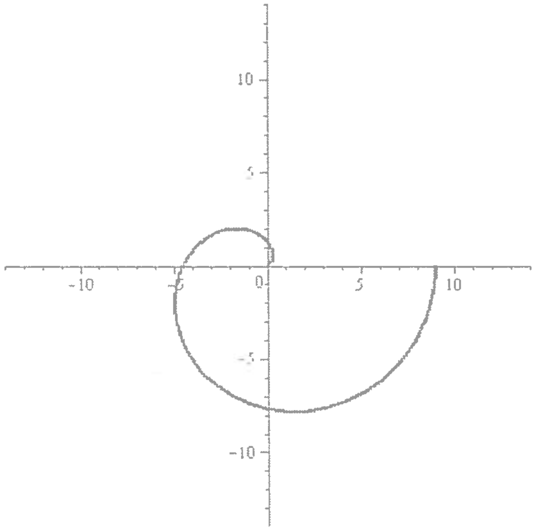
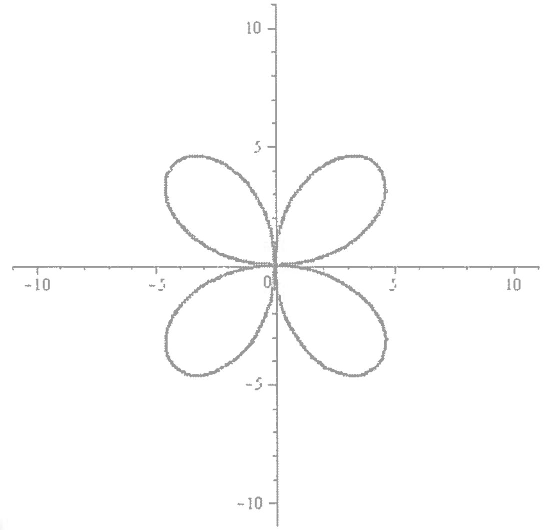
d)



e)



a)

Question 10

E

You did not answer the question.

Which of the following shows the correct sketch of the given polar curve?

$r = 6\cos(2\theta)$

$\Rightarrow$ item 3.② $m = 2$ (even)

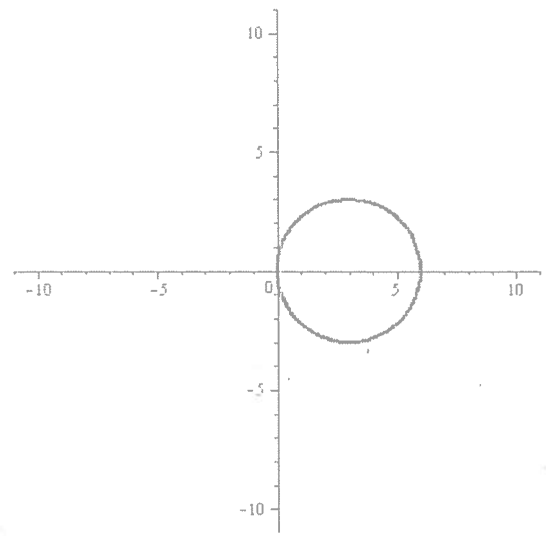
this graph is a flower which has 4 petals

$\Rightarrow$ (a) or (e).

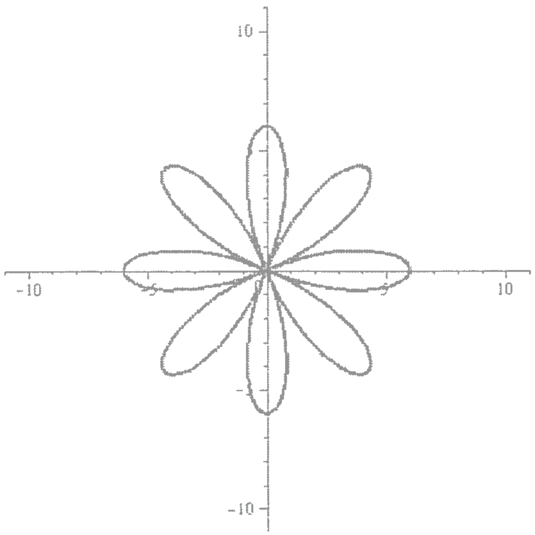
Then check the polar point as $\theta = 0$.

we have $r = 6\cos(2\cdot 0) = 6$

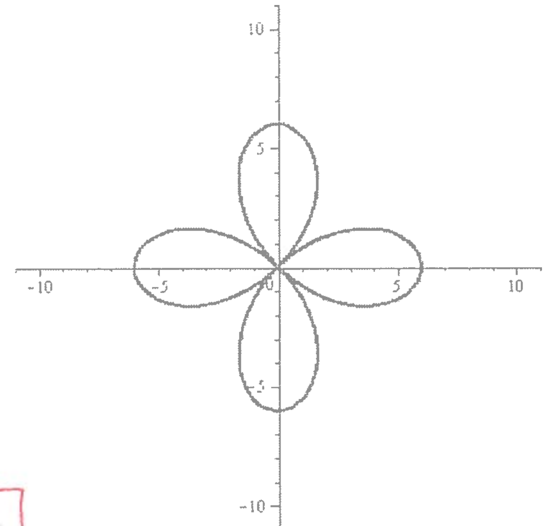
$\Rightarrow$ This graph goes through $[6,0]$ $\Rightarrow$ (e)



b)



c)



e)

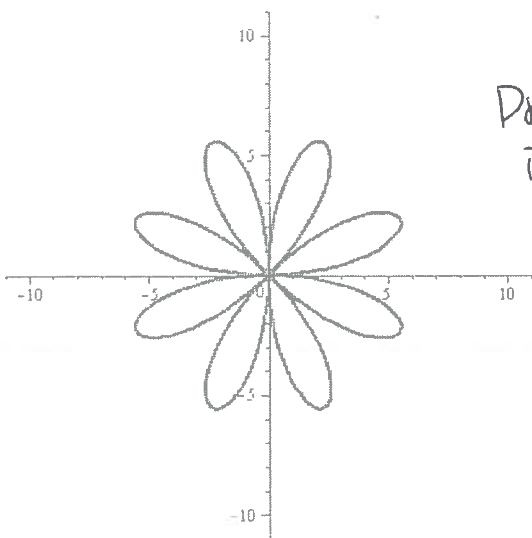
Question 11

B

You did not answer the question.

Which of the following shows the correct sketch of the polar curve given ($0 \le \theta \le \frac{1}{2}\pi$)?

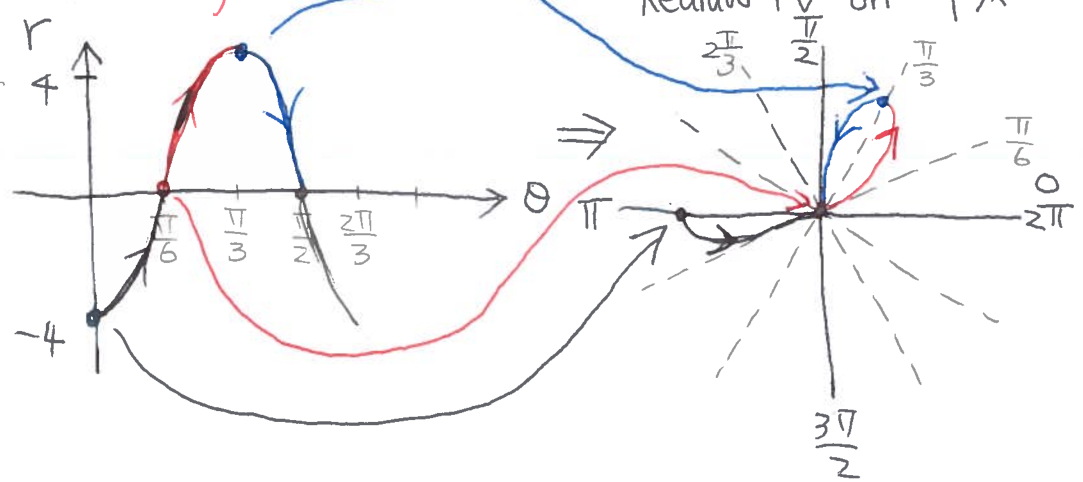
$r = -4\cos(3\theta)$



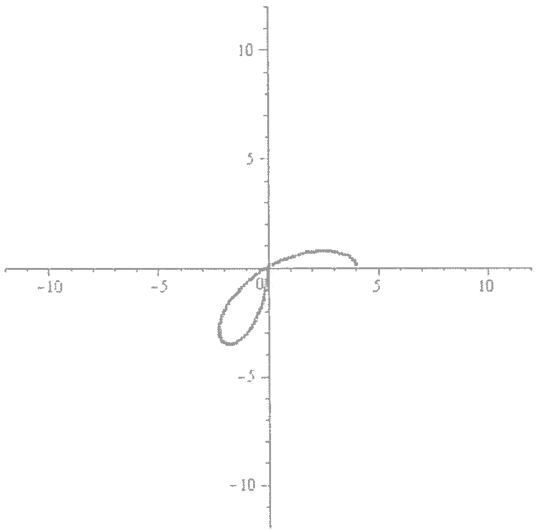
d)

Draw $r = -4\cos(3\theta)$ in this way:

Redraw it on y, x



$\Rightarrow$ (B)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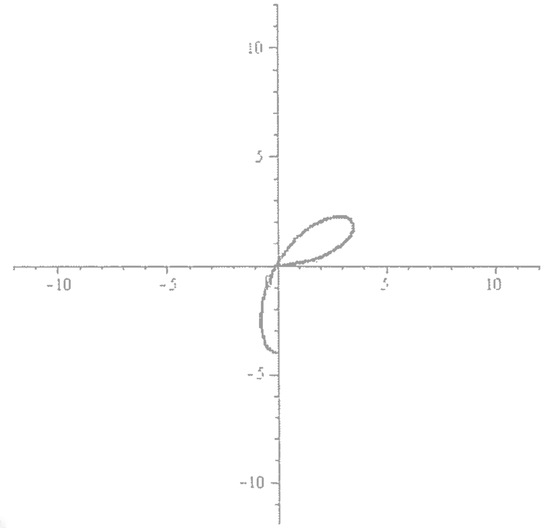
a)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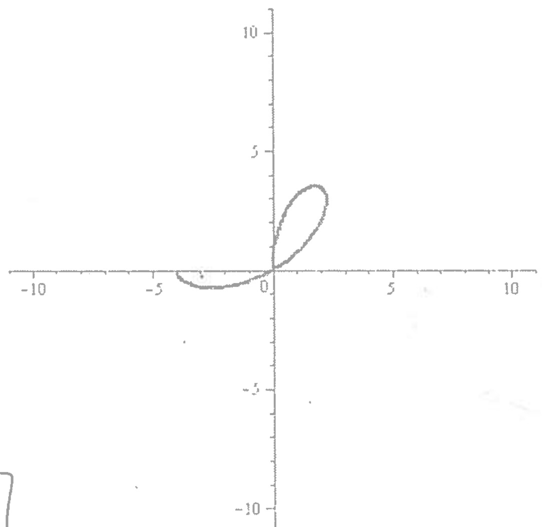
c)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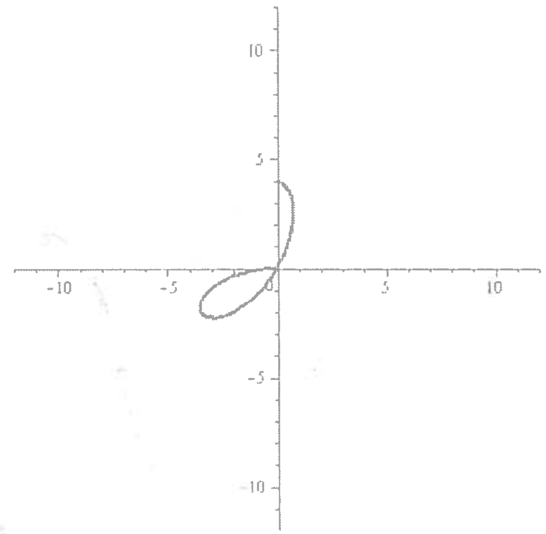
b)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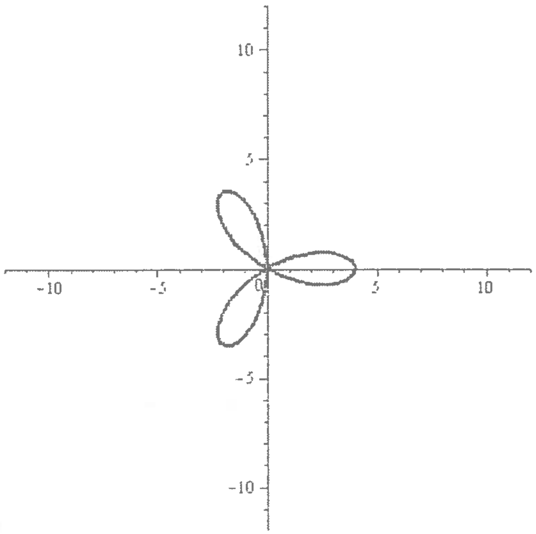
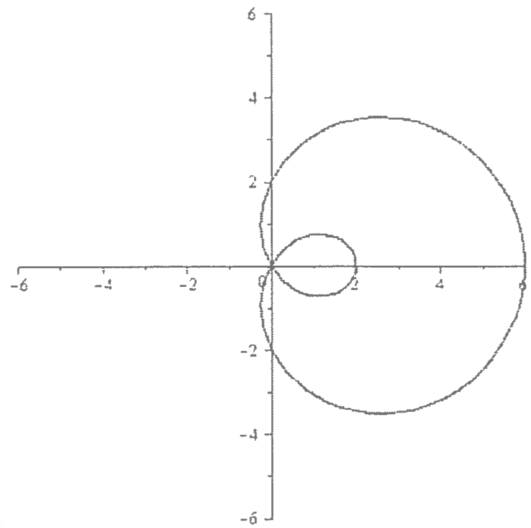
d)



e)



a)

Question 12

You did not answer the question.

Which of the following shows the correct sketch of the given polar curve?

$r = -2 + 4\cos(t)$

$\Rightarrow$ See "Graph".

It is item 4, ③. $a=-2$, $b=4 \Rightarrow |a|<|b|$

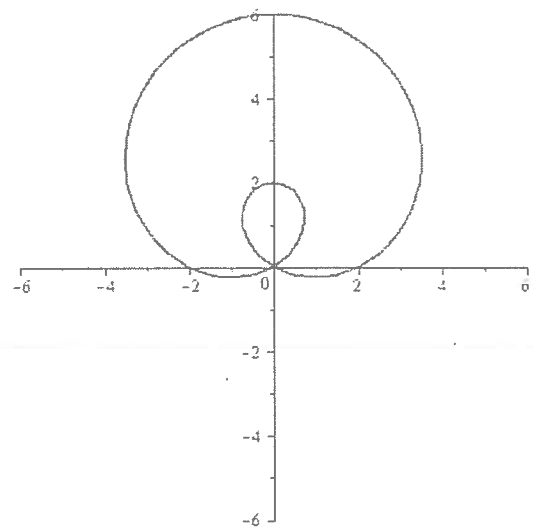
$\Rightarrow$ Limaçon with loop $\Rightarrow$ (a), (b), (c), or (e)

Check point as $\theta = 0$

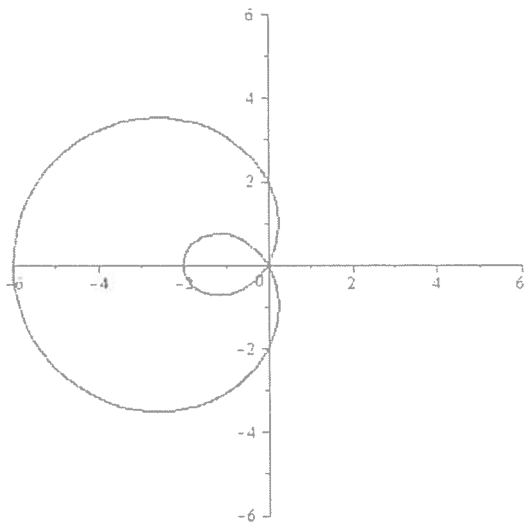
$\Rightarrow r = -2+4\cos(0) = -2+4 = 2.$

$\Rightarrow$ This graph goes through $[r, \theta] = [2, 0] \Rightarrow$ (b) or (e)

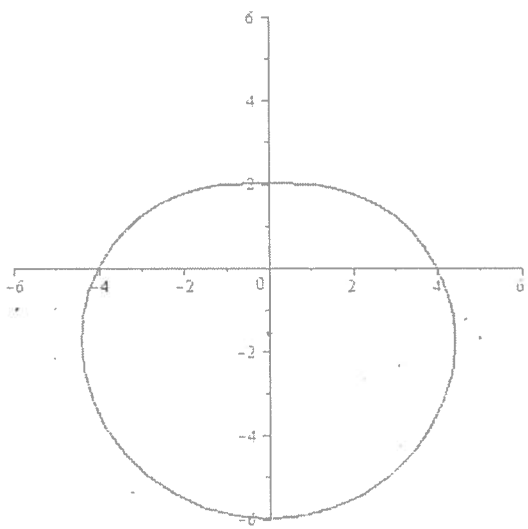
check one more point as $\theta = \frac{\pi}{2} \Rightarrow r = -2+4\cos\frac{\pi}{2} = -2. \Rightarrow [-2, \frac{\pi}{2}] \Rightarrow$ (E)



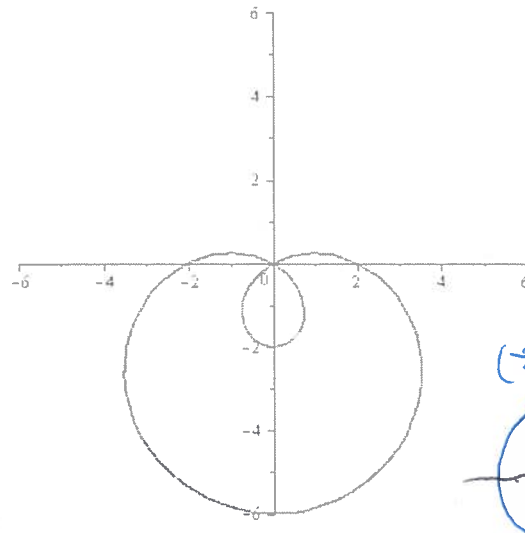
b)



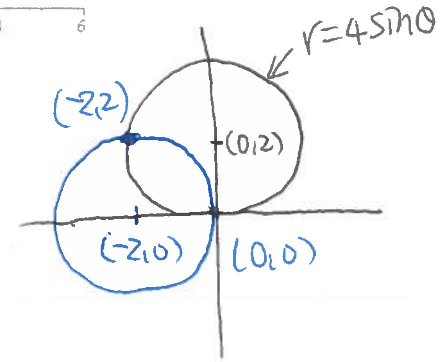
c)



d)



e)



Question 13

You did not answer the question.

Find the rectangular coordinates of the point(s) of intersection of the following polar curves.

$$r = 4\sin(\theta)$$
$$r = -4\cos(\theta)$$

By Graph ⇒ see item 1

a) $(-2, 2)$

b) $[(0, 0), (-2, 2)]$

c) $[(1, 1), (-2, 2)]$

d) $(0, 0)$

e) $[(0, 0), (-4, 4)]$

$r = 4\sin\theta$ ⇒ a circle with center $(0,2)$ radius 2

$a=2$

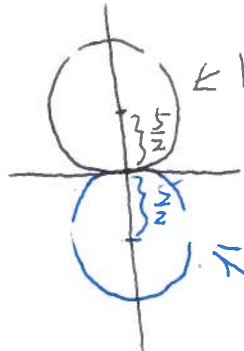
$r = -4\cos\theta$ ⇒ a circle with center $(-2,0)$ radius 2.

$a=-2$

Question 14

You did not answer the question.

Calculate the area enclosed by $r^2 = 25\sin^2(\theta)$ ⇒ $r = \pm 5\sin\theta$



$\leftarrow r=5\sin\theta$

$\nwarrow r=-5\sin\theta$

a) $\boxed{\frac{25}{2}\pi}$

b) $\frac{25}{3}\pi$

c) 25π

d) $\frac{75}{2}\pi$

e) $\frac{75}{4}\pi$

$\left(\frac{5}{2}\right)^2\pi \times 2 = \frac{25}{2}\pi$

or By formula $A=\frac{1}{2}\int r^2\,d\theta$.

$\Rightarrow \frac{1}{2}\int_0^{2\pi} 25\sin^2\theta\,d\theta = \frac{25}{2}\int_0^{2\pi}\frac{1}{2}-\frac{1}{2}\cos 2\theta\,d\theta = \frac{25}{2}\pi$

Question 15

You did not answer the question.

Calculate the area of the given region:

$r=3\cos(\theta)$
$r=3\sin(\theta)$

and the rays: $\theta=0$ and $\theta=\frac{1}{4}\pi$

?

a) $\frac{9}{2}$

b) $\frac{9}{4}$

c) $\frac{27}{8}$

d) $\frac{27}{4}$

e) $\frac{3}{2}$

Question 16

You did not answer the question.

Calculate the area of the given region:

$r=22\cos(\theta)$

?

$r=11\cos(\theta)$

and the rays: $\theta=0$ and $\theta=\frac{1}{4}\pi$

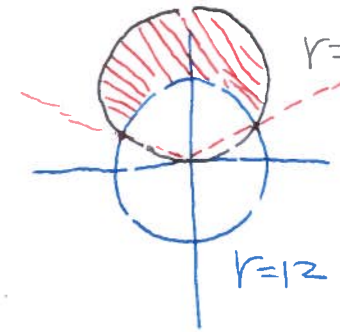
a) $\frac{1089}{16}+\frac{1089}{32}\pi$

b) $\frac{363}{8}+\frac{363}{16}\pi$

c) $\frac{1089}{8}+\frac{1089}{16}\pi$

d) $\frac{363}{4}+\frac{363}{3}\pi$

e) $\frac{121}{4}+\frac{121}{8}\pi$



$r=24\sin\theta$

$r=12$

$\parallel$

Question 17

You did not answer the question.

Which of the following represents the area outside $r=12$, but inside $r=24\sin(\theta)$?

a) $\int_{\frac{1}{6}\pi}^{\frac{5}{6}\pi}\frac{1}{2}\left((12)^2-(24\sin(\theta))^2\right)d\theta$

b) $\int_{\frac{1}{6}\pi}^{\frac{5}{6}\pi}\frac{1}{2}\left((24\sin(\theta))^2-(12)^2\right)d\theta$

c) $\int_{\frac{1}{4}\pi}^{\frac{3}{4}\pi}\frac{1}{2}\left((24\sin(\theta))^2-(12)^2\right)d\theta$

$=\frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}}(24\sin\theta)^2\,d\theta-\frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}}(12)^2\,d\theta$

$12=24\sin\theta$

$\Rightarrow \sin\theta=\frac{1}{2}$

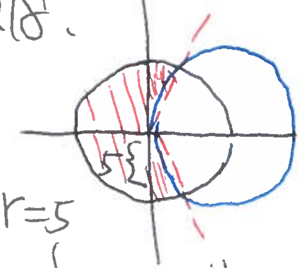
$\Rightarrow \theta=\frac{\pi}{6}$ or $\frac{5\pi}{6}$

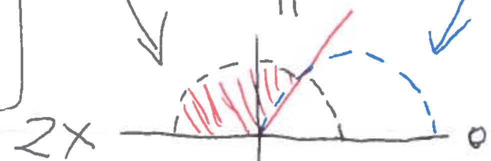
$=\frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}}(24\sin\theta)^2-(12)^2\,d\theta$

d) $\displaystyle\int_{\frac{1}{3}\pi}^{\frac{2}{3}\pi} \frac{1}{2}\left((24\sin(\theta))^2 - (12)^2\right) d\theta$

e) $\displaystyle\int_{\frac{1}{3}\pi}^{\frac{2}{3}\pi} \frac{1}{2}\left((12)^2 - (24\sin(\theta))^2\right) d\theta$

Q18.



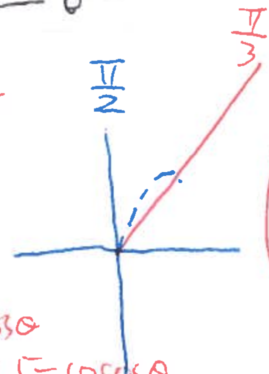


Question 18

You did not answer the question.

Which of the following represents the area inside $r = 5$, but outside $r = 10\cos\theta$?

$-$



$\begin{cases} r=5 \\ r=10\cos\theta \end{cases} \Rightarrow 5 = 10\cos\theta$

$\cos\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$

a) $\displaystyle\int_{\frac{1}{3}\pi}^{\frac{1}{2}\pi} \frac{1}{2}\left((5)^2 - (10\cos(\theta))^2\right) d\theta$

b) $\displaystyle 2\left(\int_{\frac{1}{3}\pi}^{\frac{1}{2}\pi} \frac{1}{2}\left((5)^2 - (10\cos(\theta))^2\right) d\theta\right) + 2\left(\int_{\frac{1}{2}\pi}^{\pi} \frac{1}{2}\left((5)^2\right) d\theta\right)$

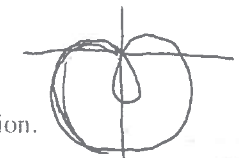
$\displaystyle 2\times\left[\int_{\frac{\pi}{2}}^{\pi} \frac{1}{2}(5)^2 d\theta + \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{1}{2}(5)^2 d\theta\right] - 2\left[\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{1}{2}(10\cos\theta)^2 d\theta\right]$

c) $\displaystyle\int_{\frac{1}{6}\pi}^{\frac{1}{2}\pi} \frac{1}{2}\left((5)^2 - (10\cos(\theta))^2\right) d\theta$

d) $\displaystyle 2\left(\int_{\frac{1}{3}\pi}^{\frac{1}{2}\pi} \frac{1}{2}\left((5)^2 - (10\cos(\theta))^2\right) d\theta\right) - \left(\int_{\frac{1}{2}\pi}^{\pi} \frac{1}{2}\left((5)^2\right) d\theta\right)$

e) $\displaystyle\int_{\frac{1}{3}\pi}^{\frac{1}{2}\pi} \frac{1}{2}\left((5)^2 - (10\cos(\theta))^2\right) d\theta + \int_{\frac{1}{2}\pi}^{\pi} \frac{1}{2}\left((5)^2\right) d\theta$

Question 19



You did not answer the question.

Which of the following represents the area inside the inner loop of $r = 8 - 16\sin(\theta)$?

$8 - 16\sin\theta = 0$

$\Rightarrow \sin\theta = \frac{1}{2}$

$\theta = \frac{\pi}{6}, \frac{5\pi}{6}$

$[r, \theta]$
$[8, 0]$
$[-8, \frac{\pi}{2}]$
$[8, \pi]$
$[32, \frac{3\pi}{2}]$
$[8, 2\pi]$

$[0, \frac{\pi}{6}]$

$[0, \frac{5\pi}{6}]$

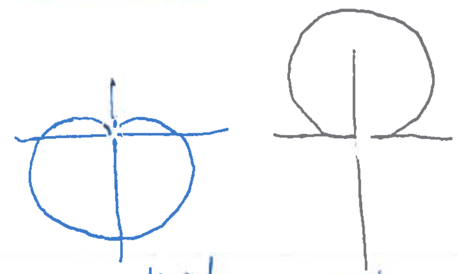
a) $\displaystyle\int_{\frac{1}{4}\pi}^{\frac{3}{4}\pi} \frac{1}{2}(8 - 16\sin(\theta))^2 d\theta$

b) $\displaystyle\int_{\frac{1}{3}\pi}^{\frac{2}{3}\pi} \frac{1}{2}(8 - 16\sin(\theta))^2 d\theta$

c) $\displaystyle\int_{\frac{1}{6}\pi}^{\frac{5}{6}\pi} \frac{1}{2}(8 - 16\sin(\theta))^2 d\theta$

d) $\displaystyle\int_{\frac{1}{3}\pi}^{\frac{4}{3}\pi} \frac{1}{2}(8 - 16\sin(\theta))^2 d\theta$

e) $\displaystyle\int_{\frac{1}{6}\pi}^{\frac{7}{6}\pi} \frac{1}{2}(8 - 16\sin(\theta))^2 d\theta$



Question 20

You did not answer the question.

cardiod

circle

Which of the following represents the area interior to both $r = 8 - 8\sin(\theta)$ and $r = 8\sin(\theta)$?

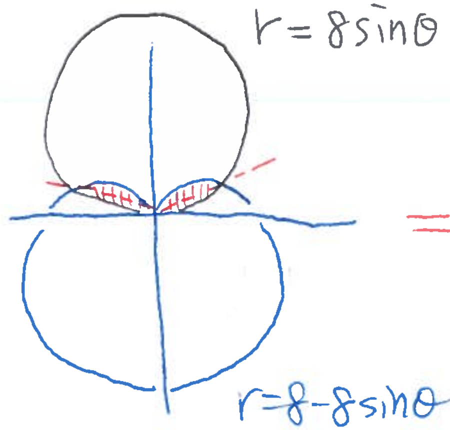
a) $\displaystyle 2\left(\int_{0}^{\frac{1}{3}\pi} \frac{1}{2}(8\sin(\theta))^2 d\theta + \int_{\frac{1}{3}\pi}^{\frac{1}{2}\pi} \frac{1}{2}(8 - 8\sin(\theta))^2 d\theta\right)$

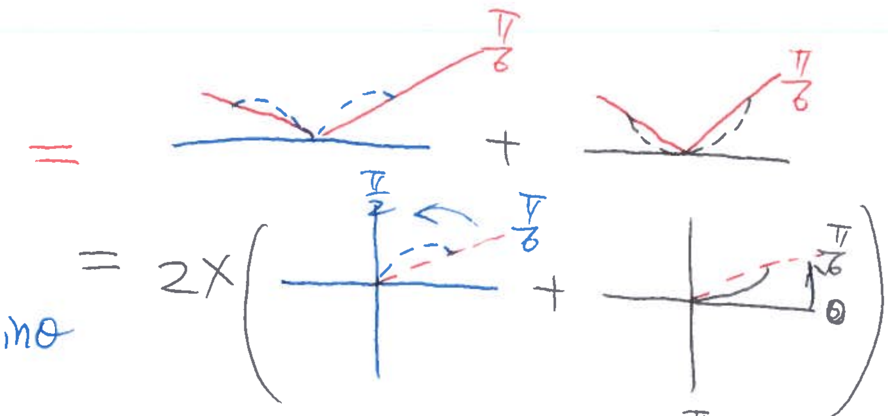
b) $\int_0^{\frac{1}{6}\pi} \frac{1}{2}(8\sin(\theta))^2\, d\theta + \int_{\frac{1}{6}\pi}^{\frac{1}{2}\pi} \frac{1}{2}(8-8\sin(\theta))^2\, d\theta$

c) $2\left[\int_0^{\frac{1}{6}\pi} \frac{1}{2}(8\sin(\theta))^2\, d\theta + \int_{\frac{1}{6}\pi}^{\frac{1}{2}\pi} \frac{1}{2}(8-8\sin(\theta))^2\, d\theta\right]$

d) $2\left[\int_0^{\frac{1}{4}\pi} \frac{1}{2}(8-8\sin(\theta))^2\, d\theta + \int_{\frac{1}{4}\pi}^{\frac{1}{2}\pi} \frac{1}{2}(8\sin(\theta))^2\, d\theta\right]$

e) $2\left[\int_0^{\frac{1}{6}\pi} \frac{1}{2}(8-8\sin(\theta))^2\, d\theta + \int_{\frac{1}{6}\pi}^{\frac{1}{2}\pi} \frac{1}{2}(8\sin(\theta))^2\, d\theta\right]$





$$8-8\sin\theta = 8\sin\theta$$

$$\Rightarrow 8 = 16\sin\theta$$

$$\sin\theta = \frac{8}{16} = \frac{1}{2}$$

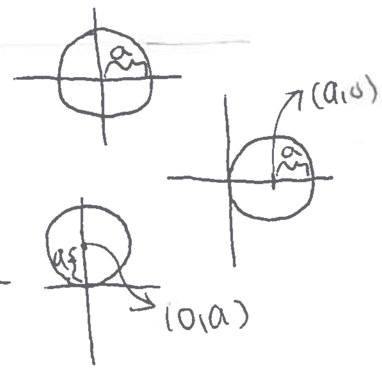
$$\theta = \frac{\pi}{6}$$

$$= 2\times\left(\frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} (8-8\sin\theta)^2\, d\theta + \frac{1}{2}\int_0^{\frac{\pi}{6}} (8\sin\theta)^2\, d\theta\right)$$

Graph

1. Circle

polar	rectangular
$r=a$	$x^2+y^2=a^2$
$r=2a\cos\theta$	$(x-a)^2+y^2=a^2$
$r=2a\sin\theta$	$x^2+(y-a)^2=a^2$



2. Line

$r=a\csc\theta$ $\quad y=a \quad$ Horizontal lines

$r=a\sec\theta$ $\quad x=a \quad$ Vertical lines

$\theta=\arctan a$ $\quad y=ax \quad$ line though the origin

$r=\dfrac{c}{a\cos\theta+b\sin\theta}$ $\quad ax+by=c \quad$ arbitrary lines

3. Flowers

$r=a\cos(m\theta)$ or $r=a\sin(m\theta)$ $\quad m\neq -1,\ a>0$

① m is odd $\Rightarrow$ # petals $=m$

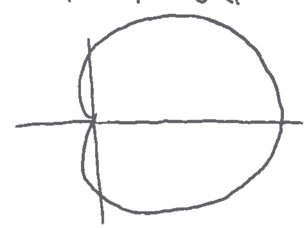
② m is even $\Rightarrow$ # petals $=2m$.

4. Polar curves of the form

$r=a+b\cos\theta$ or $r=a+b\sin\theta$

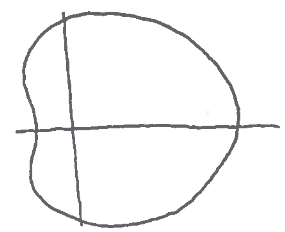
① $|a|=|b|$

Cardioid



② $|a|>|b|$

Limaçon with dimple



③ $|a|<|b|$

Limaçon with loop