Partial fraction: Let $f(x)=\frac{5}{(x-3)(x+2)}=\frac{A}{(x-3)}+\frac{B}{(x+2)}$

To find $A$, we have

① Times $(x-3)$ on both sides

## PRINTABLE VERSION

Quiz 6

$(x-3)f(x)=A+\frac{(x-3)}{(x+2)}B$

$\frac{5}{x+2}$

② as $x=3$, we have

$\frac{5}{3+2}=A+\frac{\cancel{3-3}}{3+2}B$

$\Rightarrow A=1$

Similarly, $B=-1$

You scored 0 out of 100

**Question 1**

You did not answer the question.

Calculate the integral.

$$\int \frac{5}{(x-3)(x+2)}\,dx$$

$$= \int \left[\frac{1}{x-3}-\frac{1}{x+2}\right]dx$$

$$= \ln|x-3|-\ln|x+2|+C$$

$$= \ln\left|\frac{x-3}{x+2}\right|+C$$

a) $\boxed{\ln\left|\frac{x-3}{x+2}\right|+C}$

b) $\ln|(x-3)(x+2)|+C$

c) $5\ln|(x-3)+(x+2)|+C$

d) $5\ln\left|\frac{x-3}{x+2}\right|+C$

e) $\ln|(x-3)-(x+2)|+C$

**Question 2**

You did not answer the question.

Calculate the integral.

$$\int \frac{5x^4-4x^3+4x^2+2}{x^3-x^2}\,dx$$

Long division.

$$\begin{array}{r|l} & 5+1 \\ 1-1+0+0 & 5-4+4+0+2 \\ & -\;(5-5+0+0) \\ & 1+4+0+2 \\ & -\;(1-1+0+0) \\ & 5+0+2 \end{array}$$

$5x^4-4x^3+4x^2+2$

$=(5x+1)(x^3-x^2)+5x^2+2$

$$=\int \frac{(5x+1)(x^3-x^2)+5x^2+2}{x^3-x^2}\,dx$$

$$=\int \frac{(5x+1)(x^3-x^2)}{(x^3-x^2)}+\frac{5x^2+2}{x^3-x^2}\,dx=\int 5x+1+\frac{-2}{x}+\frac{-2}{x^2}+\frac{7}{x-1}\,dx$$

$x^3-x^2 = x^2(x-1)$

$$=\frac{5}{2}x^2+x-2\ln|x|+7\ln|x-1|+\frac{2}{x}+C$$

a) $\frac{5}{2}x^2+x-2\ln|x|+2\ln|x-1|+C$

b) $-\frac{5}{2}x^2+3\ln|x-1|+\frac{2}{x}+C$

c) $\boxed{\frac{5}{2}x^2+x-2\ln|x|+7\ln|x-1|+\frac{2}{x}+C}$

d) $5x-2\ln|x|+7\ln|x-1|+\frac{2}{x}+C$

e) $\frac{5}{2}x^2+x-2\ln|x|+\frac{2}{x}+C$

**Question 3**

You did not answer the question.

Calculate the integral.

$$\int \frac{3x^2+9}{x(x^2-3)}\,dx$$

$\frac{3x^2+9}{x(x^2-3)}=\frac{A}{x}+\frac{Bx+C}{x^2-3}$

$\Rightarrow A=-3$

as $x=1$, $-6=-3+\frac{B+C}{-2}$ $\Rightarrow$ $6=B+C$

as $x=-1$, $6=+3+\frac{-B+C}{-2}$ $\Rightarrow -B+C=-6$

$\begin{cases}6=B+C\\-6=-B+C\end{cases}\Downarrow\begin{cases}B=6\\C=0\end{cases}$

$$\int \frac{-3}{x}+\frac{6x}{x^2-3}\,dx$$

a) $x\ln|x|-\ln|x^2-3|+C$

b) $-3\ln|x|+3\ln|x^2+3|+C$

c) $\boxed{-3\ln|x|+x\ln|x^2-3|+C}$ $= -3\ln|x|+3\ln|x^2-3|+C.$

d) $-3\ln|x|+3\ln|x^2-3|+C$

e) $\ln|x|+x\ln|x^2+3|+C$

**Question 4**

You did not answer the question.

Calculate the integral.

$$\int \frac{4x+48}{x^2-12x+11}\,dx$$

$x^2-12x+11=(x-1)(x-11)$

$\frac{4x+48}{x^2-12x+11}=\frac{4x+48}{(x-1)(x-11)}=\frac{A}{x-1}+\frac{B}{x-11}$

$\Rightarrow A=\frac{52}{-10}=-\frac{26}{5}$

$B=\frac{92}{10}=\frac{46}{5}$

$$\int \frac{-\frac{26}{5}}{x-1}+\frac{\frac{46}{5}}{x-11}\,dx$$

$$=-\frac{26}{5}\ln|x-1|+\frac{46}{5}\ln|x-11|+C$$

a) $-\frac{46}{5}\ln|x-11|+\frac{26}{5}\ln|x-1|+C$

b) $\frac{92}{15}\ln|x-11|-\frac{52}{15}\ln|x-1|+C$

c) $-\frac{69}{5}\ln|x-11|+\frac{39}{5}\ln|x-1|+C$

d) $\frac{46}{5}\ln|x-11| - \frac{26}{5}\ln|x-1| + C$

e) $\frac{92}{5}\ln|x-11| - \frac{52}{5}\ln|x-1| + C$

$x^2+2x+1-1+2$

$(x^2+2ax+a^2)$

$\Downarrow$

$2x=2ax$

$\Rightarrow a=1$

**Question 5**

You did not answer the question.

Calculate the integral.

$$\int \frac{2}{8x^2+16x+16}\,dx = \int \frac{2}{8}\cdot\frac{dx}{x^2+2x+2}$$

completion of squares

$$= \frac{1}{4}\int \frac{dx}{(x+1)^2+1}$$

$$= \frac{1}{4}\arctan(x+1) + C.$$

a) $-\frac{1}{4}\arcsin(x+1) + C$

b) $\frac{1}{4}\arctan(x+1) + C$

c) $\frac{1}{4}\operatorname{arccot}(x+1) + C$

d) $2(8x^2+16x+16)^{3/2} + C$

e) $\frac{32x}{(8x^2+16x+16)^2} + C$

**Question 6**

You did not answer the question.

Calculate the integral.

$$\int \frac{4x^2}{(x-6)^2(x+6)}\,dx$$

Partial fraction:

$$\frac{4x^2}{(x-6)^2(x+6)} = \frac{A}{x+6} + \frac{B}{(x-6)} + \frac{C}{(x-6)^2}$$

$\Rightarrow \boxed{A = \frac{4\cdot(-6)^2}{(-12)^2} = \frac{4\cdot 6^2}{2^2 6^2} = 1}$

$\boxed{C = \frac{4\cdot 6^2}{12} = 12}$

let $x=0$

$0 = \frac{1}{6} + \frac{B}{-6} + \frac{12}{36}$

$-\frac{1}{2} = -\frac{B}{6} \Rightarrow \boxed{B=3}$

$\Downarrow$

$$\int \frac{1}{x+6} + \frac{3}{x-6} + \frac{12}{(x-6)^2}\,dx$$

$$= \ln|x+6| + 3\ln|x-6| - \frac{12}{(x-6)} + C$$

a) $\ln|x+6| - \frac{12}{x-6} + \ln|x-6| + C$

b) $-\ln|x+6| - \frac{12}{x-6} - 3\ln|x-6| + C$

c) $\ln|x+6| - \frac{12}{x-6} + 3\ln|x-6| + C$

d) $\ln|x+6| - \frac{12}{(x-6)^2} + 3\ln|x-6| + C$

e) $2\ln|x+6| - \frac{12}{x-6} - \ln|x-6| + C$

**Question 7**

You did not answer the question.

Calculate the integral.

$$\int \frac{5}{x^4-16}\,dx$$

Partial fraction

$$\frac{5}{x^4-16} = \frac{5}{(x^2+4)(x-2)(x+2)}$$

$$= \frac{A}{x+2} + \frac{B}{x-2} + \frac{Cx+D}{x^2+4}$$

$\Rightarrow (x^2+4)(x^2-4) = (x^2+4)(x-2)(x+2)$

$\boxed{A = \frac{5}{[(-2)^2+4](-4)} = \frac{5}{-32}}$

$\boxed{B = \frac{5}{8\cdot 4} = \frac{5}{32}}$

as $x=0$, $\frac{5}{-16} = \frac{5}{-32}\cdot\frac{1}{2} + \frac{5}{32}\left(\frac{-1}{2}\right) + \frac{0\cdot C + D}{4}$

$\Rightarrow \frac{5}{32} - \frac{5}{16} = \frac{D}{4}$

$\rightarrow \frac{-5}{32} = \frac{D}{4}$

$\rightarrow \boxed{D = \frac{-5}{8}}$

as $x=1$,

$-\frac{1}{3} = \frac{5}{-32}\cdot\frac{1}{3} + \frac{5}{32}\cdot(-1) + \frac{C-\frac{5}{8}}{5}$

$5\left(\frac{5}{32}\cdot\frac{4}{3} - \frac{1}{3}\right) + \frac{5}{8} = C$

$\boxed{0 = 5\left[\frac{1}{3}\cdot\left(-\frac{3}{8}\right)\right] + \frac{5}{8} = C}$

$$= \int \frac{\frac{-5}{32}}{x+2} + \frac{\frac{5}{32}}{x-2} + \frac{\frac{-5}{8}}{x^2+4}\,dx$$

a) $-\frac{5}{16}\arctan\left(\frac{1}{2}x\right) + \frac{5}{16}\ln\left|\frac{x+2}{x-2}\right| + C$

b) $-\frac{5}{16}\operatorname{arccot}\left(\frac{1}{5}x\right) + \frac{5}{32}\ln\left|\frac{x-2}{x+2}\right| + C$

c) $-\frac{5}{16}\operatorname{arccot}\left(\frac{1}{2}x\right) - \frac{5}{16}\ln\left|\frac{x-2}{x+2}\right| + C$

d) $-\frac{1}{16}\arctan\left(\frac{1}{5}x\right) + \frac{1}{32}\ln\left|\frac{x+2}{x-2}\right| + C$

e) $-\frac{5}{16}\arctan\left(\frac{1}{2}x\right) + \frac{5}{32}\ln\left|\frac{x-2}{x+2}\right| + C$

**Question 8**

You did not answer the question.

Calculate the integral.

$$\int \frac{3x+3}{x^3+x^2}\,dx = \int \frac{3(x+1)}{x^2(x+1)}\,dx = \int \frac{3}{x^2}\,dx$$

$$= -\frac{3}{x} + C$$

a) $3\ln|x+1| - \frac{3}{x} + C.$

b) $2\ln - \frac{1}{x} + C$

c) $C$

d) $-\frac{3}{x} + C$

e) $-\frac{3}{x} + 3\ln|x| + C$

**Question 9**

You did not answer the question.

Evaluate the integral.

$$\int_0^2 \frac{x}{x^2+8x+7}\,dx$$

a) $-\frac{7}{4}\ln(7) + \frac{13}{4}\ln(3)$

b) $-\frac{7}{2}\ln(7) + \frac{13}{2}\ln(3)$

c) $-\frac{7}{6}\ln(7) + \frac{13}{6}\ln(3)$

d) $-\frac{7}{3}\ln(7) + \frac{13}{3}\ln(3)$

e) $-\frac{7}{9}\ln(7) + \frac{13}{9}\ln(3)$

Q9. Partial fraction:

$$\text{let } f(x) = \frac{x}{x^2+8x+7} = \frac{x}{(x+1)(x+7)} = \frac{A}{x+1} + \frac{B}{x+7}$$

$$A = \frac{-1}{6}, \quad B = \frac{7}{6}$$

$$\int_0^2 \frac{x}{x^2+8x+7}\,dx = \int_0^2 \left(\frac{-\frac{1}{6}}{x+1} + \frac{\frac{7}{6}}{x+7}\right)dx$$

$$= -\frac{1}{6}\ln|x+1| + \frac{7}{6}\ln|x+7|\Big|_0^2$$

$$= -\frac{1}{6}\ln 3 + \cancel{\frac{1}{6}\ln 1}^{\,0} + \frac{7}{6}\ln 9^{\to 3^2} - \frac{7}{6}\ln 7$$

$$= -\frac{1}{6}\ln 3 + \frac{7}{6}\cdot 2\cdot\ln 3 - \frac{7}{6}\ln 7$$

$$= \frac{13}{6}\ln 3 - \frac{7}{6}\ln 7$$

**Question 10**

You did not answer the question.

Evaluate the integral.

$$\int_1^3 \frac{1}{x^3+6x}\,dx$$

a) $\frac{1}{8}\ln(7) - \frac{1}{8}\ln(5) + \frac{1}{8}\ln(3)$

b) $\frac{1}{12}\ln(7) - \frac{1}{12}\ln(5) + \frac{1}{12}\ln(3)$

c) $\frac{1}{4}\ln(7) - \frac{1}{4}\ln(5) + \frac{1}{4}\ln(3)$

d) $\frac{1}{18}\ln(7) - \frac{1}{18}\ln(5) + \frac{1}{18}\ln(3)$

e) $\frac{1}{6}\ln(7) - \frac{1}{6}\ln(5) + \frac{1}{6}\ln(3)$

Q10. Let $f(x) = \frac{1}{x^3+6x} = \frac{A}{x} + \frac{Bx+C}{x^2+6}$

$\boxed{A = \frac{1}{6}}$

$f(1) = \frac{1}{7} = \frac{1}{6} + \frac{B+C}{7} \Rightarrow 7\cdot\left(\frac{1}{7}-\frac{1}{6}\right) = B+C$

$f(-1) = -\frac{1}{7} = -\frac{1}{6} + \frac{-B+C}{7} \Rightarrow 7\cdot\left(\frac{1}{6}-\frac{1}{7}\right) = -B+C$

$\Rightarrow \begin{cases} B+C = -\frac{1}{6} \\ -B+C = \frac{1}{6} \end{cases} \Rightarrow \boxed{\begin{cases} B = -\frac{1}{6} \\ C = 0 \end{cases}}$

$$= \int_1^3 \frac{\frac{1}{6}}{x} + \frac{-\frac{1}{6}x}{x^2+6}\,dx$$

let $u = x^2+6$, $du = 2x\,dx$, $\Rightarrow \frac{du}{2} = x\,dx$

$$= \frac{1}{6}\ln|x| + \frac{1}{2}\cdot\left(-\frac{1}{6}\right)\cdot\ln|x^2+6|\Big|_1^3$$

$$= \frac{1}{6}\ln 3 - \cancel{\frac{1}{6}\ln 1}^{\,0} - \frac{1}{12}\underbrace{\ln 15}_{\ln 3+\ln 5} + \frac{1}{12}\ln 7 = \left(\frac{1}{6}-\frac{1}{12}\right)\ln 3 - \frac{1}{12}\ln 5 + \frac{1}{12}\ln 7$$

**Question 11**

You did not answer the question.

Calculate the integral.

$$\int \frac{\cos(x)}{(\sin(x))^2 - 3\sin(x) - 10}\,dx$$

a) $-\frac{1}{7}\ln\left|\frac{\sin(x)}{\sin(x)+2}\right| + C$

b) $\frac{1}{7}\ln\left|\frac{\sin(x)-2}{\sin(x)+5}\right| + C$

c) $\frac{1}{7}\ln\left|\frac{\sin(x)-5}{\sin(x)+2}\right| + C$

d) $\frac{1}{7}\ln\left|\frac{\sin(x)+2}{\sin(x)-5}\right| + C$

e) $\frac{2}{7}\ln\left|\frac{\sin(x)-5}{\sin(x)+2}\right| + C$

Q.11. let $u = \sin(x)$, $du = \cos(x)\,dx$

$$\int \frac{\cos(x)}{\sin^2(x) - 3\sin(x) - 10}\,dx = \int \frac{du}{u^2-3u-10}$$

let $f(u) = \frac{1}{u^2-3u-10} = \frac{1}{(u-5)(u+2)} = \frac{A}{u-5} + \frac{B}{u+2}$

$\Rightarrow A = \frac{1}{7}, \quad B = -\frac{1}{7}$

$$= \int \frac{\frac{1}{7}}{u-5} + \frac{-\frac{1}{7}}{u+2}\,du$$

$$= \frac{1}{7}\ln|u-5| - \frac{1}{7}\ln|u+2| + C$$

$$= \frac{1}{7}\ln|\sin(x)-5| - \frac{1}{7}\ln|\sin(x)+2| + C$$

$$= \frac{1}{7}\ln\frac{|\sin(x)-5|}{|\sin(x)+2|} + C$$

**Question 12**

You did not answer the question.

Calculate the integral.

$$\int \frac{e^x}{e^{2x} + 9e^x + 14}\,dx$$

Q12. let $u = e^x$, $du = e^x\,dx$

$$\int \frac{e^x}{e^{2x}+9e^x+14}\,dx = \int \frac{du}{u^2+9u+14}$$

Let $f(u) = \frac{1}{u^2+9u+14} = \frac{1}{(u+2)(u+7)}$

$= \frac{A}{u+2} + \frac{B}{u+7}$

$\Rightarrow A = +\frac{1}{5}, \quad B = -\frac{1}{5}$

$$= \int \frac{\frac{1}{5}}{u+2} + \frac{-\frac{1}{5}}{u+7}\,du$$

$$= \frac{1}{5}\ln|u+2| - \frac{1}{5}\ln|u+7| + C$$

$$= \frac{1}{5}\ln|e^x+2| - \frac{1}{5}\ln|e^x+7| + C$$

$$= \frac{1}{5}\ln\left|\frac{e^x+2}{e^x+7}\right| + C$$

Q13. $n=12$, $x\in[0,6]$, $f(x)=2x^2$

| $x_0$ | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | $x_6$ | $x_7$ | $x_8$ | $x_9$ | $x_{10}$ | $x_{11}$ | $x_{12}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | $\frac{1}{2}$ | 1 | $\frac{3}{2}$ | 2 | $\frac{5}{2}$ | 3 | $\frac{7}{2}$ | 4 | $\frac{9}{2}$ | 5 | $\frac{11}{2}$ | 6 |

Mid. pt. ; $\frac{x_n+x_{n+1}}{2}$ → $\frac{1}{4}$ $\frac{3}{4}$ $\frac{5}{4}$ $\frac{7}{4}$ $\frac{9}{4}$ $\frac{11}{4}$ $\frac{13}{4}$ $\frac{15}{4}$ $\frac{17}{4}$ $\frac{19}{4}$ $\frac{21}{4}$ $\frac{23}{4}$

$f\left(\frac{x_n+x_{n+1}}{2}\right)$ $\frac{2}{16}$ $\frac{18}{16}$ $\frac{50}{16}$ $\frac{98}{16}$ $\frac{}{16}$ . . . . . . . .

a) $\frac{2}{5}\ln\left|\frac{e^x+2}{e^x+7}\right|+C$

b) $\frac{1}{5}\ln\left|\frac{e^x+2}{e^x+7}\right|+C$

c) $\frac{1}{5}\ln\left|\frac{e^x+7}{e^x+2}\right|+C$

d) $\frac{2}{5}\ln|(e^x-2)(e^x-7)|+C$

e) $-\frac{1}{5}\ln|(e^x+7)(e^x+2)|+C$

$M_{12}$ of $\int_0^6 2x^2dx$ is $\frac{6-0}{12}\left[f(\tfrac14)+f(\tfrac34)+\cdots+f(\tfrac{23}{4})\right]$

$=\frac{1}{2}\cdot 2\left[(\tfrac14)^2+(\tfrac34)^2+(\tfrac54)^2+(\tfrac74)^2+\cdots+(\tfrac{23}{4})^2\right]$

$=\frac{1}{16}\left[1^2+3^2+5^2+7^2+\cdots+23^2\right]$

$=\frac{1}{16}[23\cdot 100]=143\frac{12}{16}\fallingdotseq 144$

Question 15

You did not answer the question.

Estimate the given integral by the trapezoidal rule, $n=5$.

$$\int_0^1 \sin^2\left(\frac{1}{5}\pi x\right)dx$$

a) 303

b) 260

c) 318

d) 289

e) 275

Q15. $n=5$. $x\in[0,1]$, $f(x)=\sin^2(\frac{1}{5}\pi x)$

| | $x_0$ | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ |
|---|---|---|---|---|---|---|
| $x$ | 0 | $\frac{1}{5}$ | $\frac{2}{5}$ | $\frac{3}{5}$ | $\frac{4}{5}$ | 1 |
| $f(x)$ | 0 | $\sin^2(\frac{\pi}{25})$ | $\sin^2(\frac{2\pi}{25})$ | $\sin^2(\frac{3\pi}{25})$ | $\sin^2(\frac{4\pi}{25})$ | $\sin^2(\frac{5\pi}{25})$ |
| calculator | | 0.015 | 0.06 | 0.13 | 0.23 | 0.34 |

$T_5$ of $\int_0^1 \sin^2(\frac{\pi}{5}x)dx$ is

$\frac{1-0}{5}\left[\frac12(f(x_0)+f(x_5))+f(x_1)+f(x_2)+\cdots+f(x_4)\right]$

$=\frac15\left[\frac12(0+0.34)+0.44575\right]$

$=\frac15[0.61515]=0.10303$

Question 13

You did not answer the question.

Estimate the given integral by the midpoint estimate, $n=12$.

$$\int_0^6 2x^2dx$$

a) 158

b) 151

c) 144

d) 130

e) 137

Q.14. $n=12$, $x\in[0,6]$, $f(x)=4x^2$.

| | $x_0$ | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | $x_6$ | $x_7$ | $x_8$ | $x_9$ | $x_{10}$ | $x_{11}$ | $x_{12}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 0 | $\frac12$ | 1 | $\frac32$ | 2 | $\frac52$ | 3 | $\frac72$ | 4 | $\frac92$ | 5 | $\frac{11}{2}$ | 6 |
| $f(x)$ | 0 | $4\cdot(\frac12)^2$ | $4\cdot(1)^2$ | $4\cdot(\frac32)^2$ | $4\cdot(2)^2$ | $4(\frac52)^2$ | . . . | | | | | | $4(6)^2$ |

$T_{12}$ of $\int_0^6 4x^2dx$ is $\frac{6-0}{12}\left[\frac12(f(x_0)+f(x_{12}))+f(x_1)+f(x_2)+\cdots+f(x_{11})\right]$

Question 14

You did not answer the question.

Estimate the given integral by the trapezoidal rule, $n=12$.

$$\int_0^6 4x^2dx$$

$=\frac12\left[\frac12(0+4\cdot36)+4\cdot(\tfrac12)^2+4(\tfrac22)^2+4(\tfrac32)^2+\cdots+4(\tfrac{11}{2})^2\right]$

$=\frac12\left[\frac12(4\cdot36)+\frac44(1^2+2^2+3^2+\cdots+11^2)\right]=\frac12[72+506]=\frac{578}{2}$

$=289$

a) 0.0738

b) 0.0984

c) 0.123

d) 0.1722

e) 0.1476

Q14. $n=4$, $x\in[0,2]$, $f(x)=\frac{1}{\sqrt{4+x^3}}$

| | $x_0$ | $x_1$ | $x_2$ | $x_3$ | $x_4$ |
|---|---|---|---|---|---|
| $x$ | 0 | $\frac12$ | 1 | $\frac32$ | 2 |
| $f(x)$ | $\frac{1}{\sqrt4}$ | $\frac{\sqrt8}{\sqrt{33}}$ | $\frac{1}{\sqrt5}$ | $\sqrt{\frac{8}{59}}$ | $\frac{1}{\sqrt{12}}$ |
| calculator | $\frac12$ | 0.49 | 0.44 | 0.36 | 0.28 |

1.3

Question 16

You did not answer the question.

Estimate the given integral by the trapezoidal rule, $n=4$.

$$\int_0^2 \frac{1}{\sqrt{4+x^3}}dx$$

a) 0.6808

b) 0.851

c) 1.0212

$T_4$ of $\int_0^2\frac{1}{\sqrt{4+x^3}}dx$ is

$\frac{2-0}{4}\cdot\left[\frac12(f(x_0)+f(x_4))+f(x_1)+f(x_2)+f(x_3)\right]$

$=\frac12\left[\frac12(\tfrac12+0.28)+1.307809\right]=0.851073$

$$\star\ S_n = \frac{1}{3}T_n + \frac{2}{3}M_n$$

Simpson's — Trapezoidal — Midpoint

d) 0.7659

e) 0.9361

**Question 17**

You did not answer the question.

Estimate the given integral by Simpson's rule, n = 4.

$$\int_0^1 \frac{1}{\sqrt{2+x^3}}\,dx$$

Let $f(x)=\frac{1}{\sqrt{2+x^3}}$

$n=4,\ x\in[0,1]$

| | $x_0$ | $x_1$ | $x_2$ | $x_3$ | $x_4$ |
|---|---|---|---|---|---|
| $x$ | $0$ | $\frac{1}{4}$ | $\frac{1}{2}$ | $\frac{3}{4}$ | $1$ |
| $f(x)$ | $\frac{1}{\sqrt{2}}$ | $\sqrt{\frac{64}{129}}$ | $\sqrt{\frac{8}{17}}$ | $\sqrt{\frac{64}{155}}$ | $\frac{1}{\sqrt{3}}$ |
| calculator | 0.707 | 0.704 | 0.68 | 0.64 | 0.57 |
| $\frac{x_n+x_{n+1}}{2}$ | | $\frac{1}{8}$ | $\frac{3}{8}$ | $\frac{5}{8}$ | $\frac{7}{8}$ |
| $f\left(\frac{x_n+x_{n+1}}{2}\right)$ | | $\sqrt{\frac{512}{1025}}$ | $\sqrt{\frac{512}{1051}}$ | $\sqrt{\frac{512}{1149}}$ | $\sqrt{\frac{512}{1367}}$ |
| | | 0.706 | 0.69 | 0.66 | 0.611 |

a) 0.8028

b) 0.669

c) 0.5352

d) 0.6021

e) 0.7359

$$T_n = \frac{1-0}{4}\left[\tfrac{1}{2}f(x_0)+\tfrac{1}{2}f(x_4)+f(x_1)+f(x_2)+f(x_3)\right] = 1.0943.$$

$$M_n = \frac{1-0}{4}\left[f\left(\tfrac{x_0+x_1}{2}\right)+f\left(\tfrac{x_1+x_2}{2}\right)+f\left(\tfrac{x_2+x_3}{2}\right)+f\left(\tfrac{x_3+x_4}{2}\right)\right]$$

$$= 0.67 \qquad S_n = 0.811$$

**Question 18**

You did not answer the question.

Determine the values of $n$ which guarantee a theoretical error less than $\varepsilon = 0.001$ if the integral is estimated by the trapezoidal rule.

$$\int_1^4 \sqrt{x}\,dx$$

$$E_n^T = -\frac{(b-a)^3}{12n^2}\cdot f''(c) \qquad c\in(a,b)$$

$\rightarrow f(x)=\sqrt{x},\ x\in[1,4]$ ($a$, $b$), $n=?$

a) $n\geq 25$

b) $n\geq 22$

c) $n\geq 24$

d) $n\geq 26$

e) $n\geq 27$

$$|E_n^T| = \left|-\frac{(4-1)^3}{12n^2}\cdot f''(c)\right| \leq \frac{3^3}{12n^2}\cdot\frac{1}{4} \leq 0.001$$

$$f'(x)=\frac{1}{2}x^{-\frac{1}{2}},\ f''(x)=-\frac{1}{4}x^{-\frac{3}{2}}\quad x\in[1,4]$$

$\Rightarrow$ Maximum of $|f''(x)|$ as $x\in[1,4]$ is $|f''(1)|=\frac{1}{4}$.

$$\frac{\cancel{27}^{\,9}}{\cancel{48}_{\,16}\,n^2} \leq \frac{1}{1000}$$

$$\Rightarrow \frac{2250}{4} \leq n^2$$

$$\Rightarrow \frac{15}{2}\sqrt{10} \leq n$$

$$\Rightarrow n \geq 24$$

**Question 19**

You did not answer the question.

Determine the values of $n$ which guarantee a theoretical error less than $\varepsilon = 0.1$ if the integral is estimated by the trapezoidal rule.

$$\int_1^3 e^x\,dx$$

$f(x)=e^x,\ x\in[1,3]$ ($a$, $b$)

$n=?$

$$f''(x)=e^x \xRightarrow[x\in[1,3]]{\text{Max}} e^3$$

a) $n\geq 15$

b) $n\geq 10$

c) $n\geq 11$

d) $n\geq 14$

e) $n\geq 12$

$$|E_n^T| = \left|-\frac{(3-1)^3}{12n^2}\cdot f''(c)\right| \leq \frac{2^3}{12n^2}\cdot e^3 \leq 0.1$$

$$\Rightarrow \frac{2^3}{12n^2}e^3 \leq \frac{1}{10} \Rightarrow n^2 \geq \frac{80e^3}{12} \Rightarrow n^2 \geq 133.90$$

$$\Rightarrow n \geq 12$$

**Question 20**

You did not answer the question.

Determine the values of $n$ which guarantee a theoretical error less than $\varepsilon = 0.01$ if the integral is estimated by the trapezoidal rule.

$$\int_3^{e^2} \ln(x)\,dx$$

$f(x)=\ln(x).\ x\in[3,e^2]$ ($a$, $b$)

$$f'(x)=\frac{1}{x},\ f''(x)=-\frac{1}{x^2}$$

$$\Rightarrow \text{max. of } |f''(x)| = \frac{1}{9} \quad x=3$$

a) $n\geq 10$

b) $n\geq 11$

c) $n\geq 9$

d) $n\geq 12$

e) $n\geq 7$

$$|E_n^T| = \left|-\frac{(e^2-3)^3}{12n^2}\cdot f''(c)\right| \leq \frac{(e^2-3)^3}{12n^2}\cdot\frac{1}{9} \leq \frac{1}{100}$$

$$\Rightarrow \frac{100(e^2-3)^3}{108} \leq n^2$$

$$\Rightarrow n^2 \geq 78.28$$

$$\Rightarrow n \geq 9$$

Reviews:

$$1+2+3+\cdots+n = \frac{n(n+1)}{2}.$$

$$1^2+2^2+3^2+\cdots+n^2 = \frac{n(n+1)(2n+1)}{6}$$