

PRINTABLE VERSION

Quiz 5

You scored 0 out of 100

Question 1

You did not answer the question.

E Calculate the integral

- a) $\ln(89)$
- b) $-\ln(89)$
- c) $-\frac{1}{4} \ln\left(\frac{89}{8}\right)$
- d) $\ln\left(\frac{89}{8}\right)$
- e) $\frac{1}{4} \ln\left(\frac{89}{8}\right)$

$$\int \frac{x^3}{8+x^4} dx$$

Let $u = x^4 + 8$, $du = 4x^3 dx \Rightarrow \frac{du}{4} = x^3 dx$

$$\int \frac{x^3}{8+x^4} dx = \frac{1}{4} \int \frac{du}{u} = \frac{1}{4} \ln|u| + C = \frac{1}{4} \ln(x^4 + 8) + C$$

$$= \frac{1}{4} \ln(x^4 + 8) \Big|_0^3 = \frac{1}{4} \ln(89) - \frac{1}{4} \ln(8)$$

$$= \frac{1}{4} \ln\left(\frac{89}{8}\right)$$

Question 2

You did not answer the question.

D Calculate the integral

- a) $-10 \sqrt{2-x^2} + C$
 - b) $10 - 5x^2 + C$
 - c) $\frac{10x}{\sqrt{2-x^2}} + C$
- $$\int \frac{5x}{\sqrt{2-x^2}} dx$$
- Let $u = 2-x^2$, $du = -2x dx \Rightarrow \frac{du}{-2} = x dx$
- $$5 \int \frac{1}{\sqrt{u}} \cdot \frac{du}{-2} = -\frac{5}{2} \int \frac{du}{\sqrt{u}} = -\frac{5}{2} \cdot 2 \sqrt{u} + C$$
- $$= -5 \cdot \sqrt{2-x^2} + C$$

d) $5 \sqrt{2-x^2} + C$

e) $-5 \sqrt{2-x^2} + C$

Question 3

C You did not answer the question.
Calculate the integral:

$$\int_1^2 \frac{e^{\left(\frac{8}{x}\right)}}{x^2} dx$$

Let $u = \frac{8}{x}$, $du = -\frac{8}{x^2} dx \Rightarrow -\frac{du}{8} = \frac{dx}{x^2}$

$$\int \frac{e^{\left(\frac{8}{x}\right)}}{x^2} dx = \int e^u \cdot -\frac{1}{8} du = -\frac{1}{8} \int e^u du = -\frac{1}{8} e^u + C$$

$$= -\frac{1}{8} e^{\frac{8}{x}} + C$$

$$= -\frac{1}{8} e^{\frac{8}{x}} \Big|_1^2 = -\frac{1}{8} (e^8 - e^8)$$

$$= \frac{1}{8} e^8 - \frac{1}{8} e^8$$

Question 4

C You did not answer the question.
Calculate the integral:

$$\int \frac{9e^x}{9+e^{2x}} dx$$

Let $u = e^x$, $du = e^x dx$

$$= \int \frac{9du}{9+u^2} = 9 \cdot \frac{1}{3} \tan^{-1}\left(\frac{u}{3}\right) + C$$

$$= 3 \tan^{-1}\left(\frac{e^x}{3}\right) + C$$

d) $-3 \arctan(v^3) + C$

e) $\frac{2}{3} v^{2/3} + C$

Question 5

D You did not answer the question.
Calculate the integral:

a) $\frac{1}{24} \cosh^4(6x) + C$

b) $\frac{1}{24} \sinh^4(6x) + C$

c) $\frac{1}{36} \cosh^6(6x) + C$

d) $\frac{1}{36} \sinh^6(6x) + C$

e) $\frac{1}{6} \sinh^6(6x) + C$

Question 6

You did not answer the question.

C Calculate the integral:

$$\int_{AE} 6x e^{-9x} dx = -\frac{2}{3} x e^{-9x} - \frac{2}{27} e^{-9x} + C$$

a) $-\frac{2}{3} e^{-9x} + \frac{2}{3} e^{-9x} + C$

b) $-\frac{1}{81} e^{-9x} + C$

c) $-\frac{2}{27} e^{-9x} - \frac{2}{3} x e^{-9x} + C$

STOP \Rightarrow

(dx)u	(g)dv	sign
$6x$	e^{-9x}	+
6	$\frac{e^{-9x}}{-9}$	-
0	$\frac{6}{81} e^{-9x}$	-
	$-\frac{6}{9} x e^{-9x}$	+
	$\frac{6}{81} e^{-9x}$	-

d) $-\frac{2}{3} e^{-9x} - \frac{2}{3} x e^{-9x} + C$

e) $-\frac{2}{27} e^{-9x} + \frac{2}{3} x e^{-9x} + C$

Question 7

C You did not answer the question.

Calculate the integral:

$$\int_{AL} \frac{v \ln(\sqrt{v})}{v} dv$$

a) $\frac{1}{16} + \frac{5}{16} v^5$

b) $\frac{3}{16} + \frac{15}{16} v^6$

c) $\frac{1}{8} + \frac{5}{8} v^6$

d) $\frac{3}{8} + \frac{15}{8} v^6$

e) $\frac{1}{4} + \frac{5}{4} v^6$

Let $u = \ln(\sqrt{v}) \leftarrow dv = x dx$

$\Rightarrow du = \frac{1}{\sqrt{v}} \cdot \frac{1}{2\sqrt{v}} dx \leftarrow v = \frac{x^2}{z} \rightarrow \frac{x}{4}$

$$= \frac{1}{2x} \int \frac{x^2}{z} \ln(\sqrt{v}) |_{e^3} - \int e^3 \frac{x^2}{z} \cdot \frac{1}{2x} dx$$

$$= \frac{e^6}{2} \cancel{\ln(e^3)} - 0 - \left[\frac{x^2}{8} |_{e^3} \right]$$

$$= \frac{3}{2} \cdot \frac{e^6}{2} - \frac{e^6}{8} + \frac{1}{8} = (\frac{3}{4} - \frac{1}{8}) e^6 + \frac{1}{8}$$
$$= \frac{5}{8} e^6 + \frac{1}{8}$$

B Question 8

You did not answer the question.

Calculate the integral:

$$\int \frac{\tan(\ln(6x+4))}{6x+4} dx$$

a) $\frac{1}{6} \sec^2(6x+4) + C$

b) $\frac{1}{6} \ln|\sec(\ln(6x+4))| + C$

c) $-\ln|\sec(\ln(6x+4))| + C$

Let $u = \ln(6x+4), du = \frac{6}{6x+4} dx$

$$= \frac{1}{6} \int \tan(u) du$$

$$= \frac{1}{6} \ln|\sec(u)| + C$$

$$= \frac{1}{6} \ln|\sec(\ln(6x+4))| + C$$

Recall:

$$\int \sec(x) dx$$

$$= \ln |\sec(x) + \tan(x)| + C$$

a) $-\frac{1}{5} \ln |\sec(\ln(5x+4))| + C$

b) $\ln |\sec(\ln(5x+4))| + C$

Question 9

You did not answer the question.

Calculate the integral.

B $\int (\sec(5x) - 1)^2 dx$

$$= \int \sec^2(5x) - 6\sec(5x) + 9 dx$$

a) $\frac{1}{5} \tan(5x) + \frac{6}{5} \ln |\sec(5x)| + \tan(5x) - 9x + C$

b) $\frac{1}{5} \tan(5x) - \frac{6}{5} \ln |\sec(5x) + \tan(5x)| + 9x + C$

c) $\tan(5x) - 9x + C$

d) $\frac{1}{5} \tan(5x) + 9x + C$

e) $-\tan(5x) + 6 \ln |\sec(5x) + \tan(5x)| - 45x + C$

Question 10

You did not answer the question.

B Calculate the integral.

$$\int \frac{x}{6+5x^2} dx$$

Let $u = 6+5x^2$, $du = 10x dx$.

a) $-\frac{1}{10} \ln(|6+5x^2|) + C$

b) $\frac{1}{10} \ln(|6+5x^2|) + C$

c) $\frac{1}{2} \ln(|6+5x^2|) + C$

$$= \frac{1}{10} \ln(6+5x^2) + C$$

d) $-\frac{5x}{(6+5x^2)^2} + C$

e) $-\frac{5}{(6+5x^2)^2} + C$

Question 9

You did not answer the question.

Calculate the integral.

B $\int (\sec(5x) - 1)^2 dx$

$$= \int \sec^2(5x) - 6\sec(5x) + 9 dx$$

a) $\frac{1}{5} \tan(5x) + \frac{6}{5} \ln |\sec(5x)| + \tan(5x) - 9x + C$

b) $\frac{1}{5} \tan(5x) - \frac{6}{5} \ln |\sec(5x) + \tan(5x)| + 9x + C$

c) $\tan(5x) - 9x + C$

d) $\frac{1}{5} \tan(5x) + 9x + C$

e) $-\tan(5x) + 6 \ln |\sec(5x) + \tan(5x)| - 45x + C$

Question 11

E You did not answer the question.

Calculate the given integral:

$$\int \frac{4x}{\sqrt{8-x^2}} dx$$

let $u = 8-x^2$, $du = -2x dx$.
 $\Rightarrow \frac{du}{-2} = x dx$

$$+ \int \frac{1}{\sqrt{u}} \cdot \frac{du}{-2} = \frac{1}{2} \int \frac{du}{\sqrt{u}}$$

$$= -2 \cdot 2 \sqrt{u} + C$$

$$= -4 \sqrt{8-x^2} + C$$

e) $-4 \sqrt{8-x^2} + C$

Question 12

B You did not answer the question.

Calculate the given integral:

$$\int \frac{3\sqrt{4-x^2}}{(4-x^2)^{3/2}} dx$$

a) $-\frac{3}{4} \sqrt{4-x^2} + 3 \arcsin\left(\frac{1}{2}x\right) + C$

b) $-\frac{3}{2} \sqrt{4-x^2} + 6 \arcsin\left(\frac{1}{2}x\right) + C$

c) $\frac{3}{2} \sqrt{4-x^2} - 6 \arcsin\left(\frac{1}{2}x\right) + C$

d) $\frac{3(2\sin u)^2}{2\cos u} \cdot 2\cos u du$

e) $\int z \sin^2 u du = \int z \left(\frac{1}{2} - \frac{1}{2} \cos 2u\right) du$

$$= 6 \int 1 - \cos 2u du = 6 \left(u - \frac{\sin 2u}{2}\right) + C$$

$$= 6u - 3 \cdot 2 \sin u \cos u + C = 6 \cdot \sin^{-1}\left(\frac{x}{2}\right) - 6 \cdot \frac{x}{2} \cdot \frac{\sqrt{4-x^2}}{2} + C$$

$$\sqrt{4-x^2} = \sqrt{4-4\sin^2 u} = \sqrt{4\cos^2 u}$$

$$\Rightarrow \sqrt{4\cos^2 u} = 2\cos u$$

$$\text{let } x = z \sin u, dx = z \cos u du$$

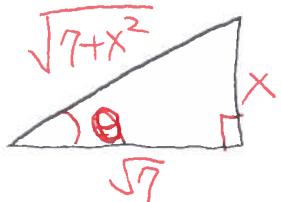
$$\int \frac{3(2\sin u)^2}{2\cos u} \cdot z \cos u du$$

$$= \int z \sin^2 u du = \int z \left(\frac{1}{2} - \frac{1}{2} \cos 2u\right) du$$

$$= 6 \int 1 - \cos 2u du = 6 \left(u - \frac{\sin 2u}{2}\right) + C$$

$$= 6u - 3 \cdot 2 \sin u \cos u + C = 6 \cdot \sin^{-1}\left(\frac{x}{2}\right) - 6 \cdot \frac{x}{2} \cdot \frac{\sqrt{4-x^2}}{2} + C$$

$$\begin{aligned}
 & \text{(*)} \int \sec^3 \theta d\theta = \int \underline{\sec^2 \theta} \cdot \underline{\sec \theta} d\theta = \sec \theta - \int \tan \theta \sec \theta d\theta \\
 & \text{IBP} \quad u = \sec \theta \quad dv = \sec^2 \theta d\theta \\
 & du = \sec \theta \tan \theta \quad v = \tan \theta \\
 & \text{(b)} \quad \frac{11}{3} \sqrt{3} + \frac{11}{6} \pi \\
 & \text{(c)} \quad 33 \sqrt{3} - \frac{11}{3} \pi \\
 & \Rightarrow 2 \int \sec^3 \theta d\theta = \sec \theta + \ln |\sec \theta + \tan \theta| \\
 & \Rightarrow \int \sec^3 \theta d\theta = \frac{1}{2} \sec \theta + \frac{1}{2} \ln |\sec \theta + \tan \theta|
 \end{aligned}$$



d) $= 3x \sqrt{4-x^2} + 12 \arcsin\left(\frac{1}{2}x\right) + C$

e) $\frac{3}{2} \frac{1}{(4-x^2)^{1/2}} + \frac{3}{2} \arctan\left(\frac{x}{\sqrt{4-x^2}}\right) + C$

Question 13

You did not answer the question.

B

Calculate the given integral:

$$\begin{aligned}
 & \int \frac{3x^2}{\sqrt{7+x^2}} dx \Rightarrow dx = \sqrt{7} \sec^2 \theta d\theta \\
 & \text{and } \sqrt{7+x^2} = \sqrt{7+\tan^2 \theta} = \sqrt{7} \sec \theta
 \end{aligned}$$

a) $\frac{3}{2} \frac{1}{(7+x^2)^{1/2}} + C$

$$\int \frac{3 \cdot 7 \tan^2 \theta}{\sqrt{7} \sec \theta} \cdot \sqrt{7} \sec^2 \theta d\theta = 21 \int \tan^2 \theta \sec \theta d\theta$$

b) $\frac{3}{2} x \sqrt{7+x^2} - \frac{21}{2} \ln |x + \sqrt{7+x^2}| + C$

c) $\frac{3}{2} x \sqrt{7+x^2} + \frac{3}{2} \ln |x + \sqrt{7+x^2}| + C$

d) $\frac{3}{2} \frac{1}{(7+x^2)^{1/2}} - \frac{21}{2} \ln |\sqrt{7+x^2}| + C$

e) $\frac{3}{2} \sqrt{7+x^2} + 21 \ln |x + (7+x^2)^{1/2}| + C$

$$= 21 \left(\frac{1}{2} \sec \theta + \frac{1}{2} \ln |\sec \theta + \tan \theta| \right) + C$$

$$= 21 \left(\frac{1}{2} \sec \theta + \frac{1}{2} \ln |\sec \theta + \tan \theta| \right) + C$$

Question 14

B You did not answer the question.

Calculate the given integral:

$$= \frac{21}{2} \cdot \left(\frac{\sqrt{7+x^2}}{\sqrt{7}} \cdot \frac{x}{\sqrt{7}} - \ln \left| \frac{\sqrt{7+x^2} + x}{\sqrt{7}} \right| \right) + C$$



a) $\frac{11}{3} \sqrt{3} + \frac{11}{3} \pi$

Let $x = \sin \theta$. $dx = \cos \theta d\theta$

$$\int \frac{11x^2}{(1-x^2)^{3/2}} dx = \int \frac{11 \sin^2 \theta}{\cos^2 \theta} \cdot \cos \theta d\theta$$

$$= \int 11 \frac{\sin^2 \theta}{\cos^2 \theta} d\theta = 11 \int \tan^2 \theta d\theta = 11 \int (\sec^2 \theta - 1) d\theta$$

$$\begin{aligned}
 & = 11 \cdot \tan \theta - 11\theta + C = 11 \cdot \frac{x}{\sqrt{1-x^2}} - 11 \ln |\sec \theta + \tan \theta| + C
 \end{aligned}$$

$\theta = \arcsin x$ and $\tan \theta = \frac{x}{\sqrt{1-x^2}}$

$$\int \frac{11x^2}{(1-x^2)^{3/2}} dx = \frac{11}{2} \frac{x}{\sqrt{1-x^2}} - 11 \ln |\sec \theta + \tan \theta| \Big|_0^{\frac{1}{2}}$$

$$= \frac{11}{2} \frac{1}{\sqrt{7}} - 11 \ln \left| \sec \frac{\pi}{4} + \tan \frac{\pi}{4} \right| + 11 \ln 1$$

$$= \frac{11}{2} \frac{1}{\sqrt{7}} - 11 \cdot \frac{\sqrt{2}}{2} + 0$$

$$= \frac{11}{2} \frac{1}{\sqrt{7}} - \frac{11\sqrt{2}}{2}$$

$$= \frac{11}{2} \frac{1}{\sqrt{7}} - \frac{11\sqrt{2}}{2}$$

Question 16

You did not answer the question.

Calculate the given integral:

$$\int \frac{2x^2}{(x^2+7)^{3/2}} dx = \int \frac{2x^2}{(x^2+7)^{3/2}} \cdot \frac{2x}{2x} dx$$

$$= \int \frac{4x^3}{(x^2+7)^{3/2}} dx = \int \frac{4x^3}{2(x^2+7)^{3/2}} \cdot 2x dx$$

$$= 32 \int \sin^3 \theta d\theta = 32 \int (1-\cos^2 \theta) \sin \theta d\theta$$

$$= 32 \left[\cos \theta + \frac{\cos^3 \theta}{3} \right] + C$$

$$= +\frac{32}{3} \cos^3 \theta - 32 \cos \theta + C = +\frac{32}{3} \left(\frac{\sqrt{4-x^2}}{2} \right)^3 - 32 \left(\frac{\sqrt{4-x^2}}{2} \right) + C$$

Let $x = \sqrt{7} \tan \theta$. $dx = \sqrt{7} \sec^2 \theta d\theta$

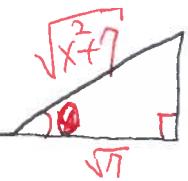
$$\int \frac{2x^2}{(x^2+7)^{3/2}} dx = \int \frac{2x^2}{(7+7\tan^2 \theta)^{3/2}} \cdot \frac{x}{\sqrt{7}} \sec^2 \theta d\theta$$

$$= \int \frac{2 \cdot 7 \tan^2 \theta}{7 \sqrt{7} \sec^3 \theta} \cdot \sqrt{7} \sec^2 \theta d\theta = \int 2 \cdot \frac{\tan^2 \theta}{\sec \theta} d\theta = \int 2 \frac{\sec \theta}{\tan \theta} d\theta = \int 2 \frac{\sec \theta}{\sec \theta - \tan \theta} d\theta$$

$$= \int 2 \frac{\sec \theta}{\sec \theta - \tan \theta} d\theta = 2 \int \frac{\sec^2 \theta - \sec \theta \tan \theta}{\sec^2 \theta - \sec \theta \tan \theta} d\theta = 2 \int \frac{\sec \theta + \tan \theta}{\sec \theta - \tan \theta} d\theta$$

$$= 2 \ln |\sec \theta + \tan \theta| - 2 \ln |\sec \theta - \tan \theta| + C$$

$$= 2 \ln |\sec \theta + \tan \theta| - 2 \ln |\sec \theta - \tan \theta| + C$$



$$\sec \theta = \frac{\sqrt{x^2 + 7}}{x}, \sin \theta = \frac{x}{\sqrt{x^2 + 7}}$$

$$a) -\frac{x}{\sqrt{x^2 + 7}} - 2 \ln |x + \sqrt{x^2 + 7}| + C$$

$$b) -\frac{2x}{(x^2 + 7)^{3/2}} - 2 \ln |x + (x^2 + 7)^{1/2}| + C$$

$$c) -\frac{2x}{\sqrt{x^2 + 7}} + 2 \ln |x + \sqrt{x^2 + 7}| + C$$

$$d) \frac{2x}{(x^2 + 7)^{3/2}} + 2 \ln |x + \sqrt{x^2 + 7}| + C$$

$$e) \frac{x}{\sqrt{x^2 + 7}} - \ln |2x + \sqrt{x^2 + 7}| + C$$

Question 17

You did not answer the question.

Calculate the given integral:

$$= 49 \cdot \sin^2(\frac{x}{7}) + \frac{49}{2} \cdot 2 \frac{x}{7} \cdot \frac{\sqrt{49-x^2}}{7} + C$$

$$a) \frac{147}{2} \pi$$

$$b) \frac{147}{4} \pi$$

$$c) \frac{49}{2} \pi$$

$$d) \frac{49}{4} \pi$$

$$e) \frac{49}{3} \pi$$

$$\rightarrow 49 \sin^2(\frac{x}{7}) + x \sqrt{49-x^2} \Big|_0^7$$

$$= 49 \sin^2(1) - 49 \sin^2 0 + 7 \cdot 0 - 0 \cdot 7 \\ = 49 \frac{\pi}{2} = \frac{49}{2} \pi$$

Question 18

You did not answer the question.

Calculate the given integral:

$$\text{Let } x = 7\tan \theta, \Rightarrow \tan \theta = \frac{x}{7}$$

$$dx = 7\sec^2 \theta d\theta$$

$$\int \frac{2}{x^2 \sqrt{49+x^2}} dx$$

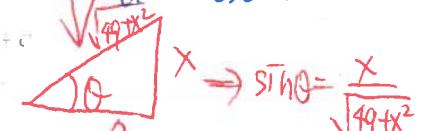
$$= \int \frac{2}{49 \tan^2 \theta \cdot 7 \sec^2 \theta} \cdot 7 \sec^2 \theta d\theta$$

$$= \frac{2}{49} \int \frac{\sec \theta}{\tan^2 \theta} d\theta = \frac{2}{49} \int \frac{1}{\cos^2 \theta} \cdot \frac{\cos^2 \theta}{\sin^2 \theta} du$$

$$= \frac{2}{49} \int \frac{\cos \theta}{\sin^2 \theta} d\theta = \frac{2}{49} \int \frac{du}{u^2} = \frac{2}{49} \cdot (-\frac{1}{u}) + C$$

$$u = \sin \theta \\ du = \cos \theta d\theta \\ = -\frac{2}{49} \cdot \frac{1}{\sin \theta} + C$$

$$= -\frac{2}{49} \cdot \frac{\sqrt{49+x^2}}{x} + C$$



$$\rightarrow \sin \theta = \frac{x}{\sqrt{49+x^2}}$$

Question 19

You did not answer the question.

Calculate the given integral:

$$\int \frac{2}{e^x \sqrt{6+e^{2x}}} dx = \int \frac{2}{e^x \sqrt{6+(e^x)^2}} dx$$

$$\text{Let } e^x = \sqrt{6} \tan \theta \Rightarrow e^x dx = \sqrt{6} \sec^2 \theta d\theta$$

$$\Rightarrow dx = \frac{\sqrt{6} \sec^2 \theta}{e^x} d\theta$$

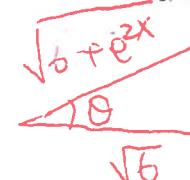
$$= \frac{\sqrt{6} \sec^2 \theta}{\sqrt{6} \tan \theta} d\theta = \frac{\sec^2 \theta}{\tan \theta} d\theta$$

$$= \frac{\sec^2 \theta}{\tan \theta} d\theta = \frac{1}{\sin^2 \theta} d\theta$$

$$= \frac{1}{3} \frac{\sqrt{6-e^{2x}}}{e^x} + C$$

$$(see Q8) = \frac{1}{3} \cdot \frac{1}{\sin \theta} + C = -\frac{1}{3} \cdot \frac{\sqrt{6+e^{2x}}}{e^x} + C$$

$$\rightarrow \sin \theta = \frac{e^x}{\sqrt{6+e^{2x}}}$$



$$\text{e) } \frac{2 \left(b + e^{x^2} \right)^{1/2}}{e^x} + C$$

Question 20

You did not answer the question.

Calculate the given integral:

$$\begin{aligned}
 & \int \frac{x+5}{\sqrt{x^2+10x+10}} dx \\
 & \text{let } u = x^2 + 10x + 10 \\
 & du = 2x + 10 dx \Rightarrow \frac{du}{2} = (x+5)dx \\
 & \text{complete the square} \\
 & \cancel{x^2 + 10x + 10} \\
 & = x^2 + 10x + 25 - 25 + 10 \\
 & = (x+5)^2 - 15 \\
 & \int \frac{du}{2} \cdot \frac{1}{\sqrt{u}} = \frac{1}{2} \cdot 2\sqrt{u} + C \\
 & = \sqrt{u} + C \\
 & = \sqrt{x^2 + 10x + 10} + C
 \end{aligned}$$

a) $-\left(x^2 + 10x + 10\right)^{3/2} + C$

b) $\sqrt{x^2 + 10x + 10} + C$

c) $2x\sqrt{x^2 + 10x + 10} + C$

d) $-x\sqrt{x^2 + 10x + 10} + C$

e) $\left(x^2 + 10x + 10\right)^{3/2} + C$