PRINTABLE VERSION

Quiz 3

You scored 0 out of 100

Question 1

You did not answer the question.

How long does it take for a sum of money to double if compounded continuously at 7 %?

a) approximately 10.904 years

b) approximately 9.404 years

c) approximately 9.904 years

d) approximately 11.404 years

e) approximately 11.904 years

Formula: $A(t) = A_0 e^{rt}$

Initial $\Rightarrow A_0$

final $\Rightarrow 2A_0$ (double)

$r = 0.07$

$\Rightarrow 2A_0 = A_0 e^{0.07t}$

$\Rightarrow 2 = e^{0.07t}$

$\ln 2 = 0.07t$

$t = \frac{100}{7} \cdot \ln 2 = \frac{69}{7} = 9.9$

$(\ln 2 = 0.69)$

Question 2

You did not answer the question.

According to the Bureau of the Census, the population of the United States in 1990 was approximately 249 million and in 2000, 281 million. Use this information to estimate the population in 1960.

a) 169 million

b) 177 million

c) 179 million

d) 173 million

e) 175 million

Formula $P(t) = P_0 e^{rt}$

0	30	40
1960	1990	2000
? $\Rightarrow P_0$	249 million	281

① Find r.

$249 = P(30) = P_0 e^{r \cdot 30}$ —(a)

$281 = P(40) = P_0 e^{r \cdot 40}$ —(b)

$\frac{(a)}{(b)} \Rightarrow \frac{249}{281} = \frac{P_0 e^{r \cdot 30}}{P_0 e^{r \cdot 40}} = \frac{1}{e^{10r}}$

$e^{10r} = \frac{281}{249} \Rightarrow r = \frac{\ln(\frac{281}{249})}{10}$

put it back to (a)

$249 = P_0 \cdot e^{30 \cdot \ln(\frac{281}{249}) \cdot \frac{1}{10}}$

$249 = P_0 e^{3 \ln(\frac{281}{249})} \Rightarrow 249 = P_0 e^{\ln(\frac{281}{249})^3} = P_0 \cdot (\frac{281}{249})^3$

$\Rightarrow P_0 = 249 \cdot (\frac{249}{281})^3 = 173$

Question 3

You did not answer the question.

According to the Bureau of the Census, the population of the United States in 1990 was approximately 249 million and in 2000, 281 million. Use this information to predict the population for 2020.

a) 354 million

b) 362 million

c) 364 million

d) 358 million

e) 360 million

0	10	30
1990	2000	2020
$P_0 = 249$	281	$P(30)$?

① Find r:

$281 = P(10) = P_0 e^{rt} = 249 e^{r \cdot 10}$

$\Rightarrow \frac{281}{249} = e^{r \cdot 10} \Rightarrow r = \ln(\frac{281}{249}) \cdot \frac{1}{10}$

② $P(30) = 249 \cdot e^{[\ln(\frac{281}{249})] \frac{1}{10} \cdot 30}$

$= 249 \cdot e^{\ln(\frac{281}{249})^3}$

$= 249 \cdot (\frac{281}{249})^3 = \frac{281^3}{249^2} = 357.86$

Question 4

You did not answer the question.

The half-life of radium-226 is 1620 years. What percentage of a given amount of the radium will remain after 700 years?

a) 76.0 %

b) 74.0 %

c) 70.0 %

d) 78.0 %

e) 80.0 %

Let initial be A_0,

① Find r: $\frac{A_0}{2} = A_0 e^{r \cdot 1620} \Rightarrow \frac{1}{2} = e^{r \cdot 1620} \Rightarrow r = [\ln(\frac{1}{2})] \frac{1}{1620}$

② $t = 700 \Rightarrow$ we have

$P(700) = A_0 \cdot e^{[\ln(\frac{1}{2})] \cdot \frac{700}{1620}} = A_0 e^{\ln(\frac{1}{2})^{\frac{700}{1620}}}$

$= A_0 \cdot (\frac{1}{2})^{\frac{700}{1620}} \doteq A_0 (\frac{1}{2})^{0.432} = 0.74 A_0$

Question 5

You did not answer the question.

The cost of the tuition, fees, room, and board at ABC College is currently \$8000 per year. What would you expect to pay 2 years from now if the costs at ABC are rising at the continuously compounded rate of 8%?

a) \$ 10288

b) \$ 8688

c) \$ 9388

d) \$ 10088

e) \$ 9888

$P_0 = 8000$. $r = 0.08$. $t = 2$

$\Rightarrow P(2) = 8000 \cdot e^{0.08 \cdot 2}$

$= 8000 \, e^{0.16} = 1.17351$

$= 9388$

Question 6

$(\sin^2 x+\cos^2 x=1)$

let $\arcsin\frac{7}{25}=x \Rightarrow \frac{7}{25}=\sin x.$ $\cancel{\Rightarrow \cos x = \frac{\;}{15}}$

You did not answer the question.

Determine the exact value of $\cos\left(2\arcsin\left(\frac{7}{25}\right)\right) = \cos 2x$

a) $\frac{527}{625}$

b) $\frac{21}{25}$

c) $\frac{529}{625}$

d) $\frac{526}{625}$

e) $\frac{106}{125}$

$$= \cos^2 x - \sin^2 x$$
$$= 1-\sin^2 x - \sin^2 x$$
$$= 1-2\sin^2 x$$
$$= 1-2\cdot\left(\frac{7}{25}\right)^2$$
$$= 1-\frac{98}{625} = \frac{527}{625}$$

Question 7

You did not answer the question.

Differentiate

$f(x) = \operatorname{arcsec}(4x^2)$

a) $\frac{4}{x\sqrt{4x^2-1}}$

b) $\frac{2}{x\sqrt{16x^4-1}}$

c) $\frac{2}{x\sqrt{32x^4-1}}$

d) $-\frac{2}{x\sqrt{16x^2-1}}$

e) $-\frac{2}{\sqrt{16x^4-1}}$

$$\Rightarrow f'(x) = \frac{1}{|4x^2|\sqrt{(4x^2)^2-1}}\cdot 8x$$
$$= \frac{8x}{4x^2\sqrt{16x^4-1}}$$
$$= \frac{2}{x\sqrt{16x^4-1}}$$

Question 8

You did not answer the question.

Differentiate

$f(x) = \frac{\arctan(6x)}{x}$ $\cancel{= \arctan(6x)\cdot x^{-1}}$

a) $\frac{6x-\arctan(6x)\,(1+36x^2)}{1+36x^2}$

b) $\frac{x+\arctan(6x)\,(1+36x^2)}{(1+36x^2)\,x^2}$

c) $\frac{6x+\arctan(6x)\,(2+36x^2)}{1+x^2}$

d) $\frac{6x-\arctan(6x)\,(2+36x^2)}{(1-36x^2)\,x^2}$

e) $\frac{6x-\arctan(6x)\,(1+36x^2)}{(1+36x^2)\,x^2}$

$\Rightarrow$ $\cancel{f'(x) = [\arctan(6x)]'\cdot x^{-1} + \arctan(6x)\cdot -x^{-2}}$

By quotient Rule

$$= \frac{x\cdot[\arctan(6x)]' - \arctan(6x)}{x^2}$$
$$= \frac{x\cdot\frac{6}{1+(6x)^2} - \arctan(6x)}{x^2}$$
$$= \frac{6x - \arctan(6x)\,[1+36x^2]}{x^2(1+36x^2)}$$

Question 9

You did not answer the question.

Differentiate

$f(x) = \arcsin\left(\sqrt{9-3x^2}\right)$

a) $\frac{3x}{\sqrt{(-8+3x^2)\,(9-3x^2)}}$

b) $-\frac{3x}{\sqrt{(-8+3x^2)\,(9-3x^2)}}$

c) $-\frac{x}{\sqrt{(-8+3x^2)\,(9-3x^2)}}$

$$f'(x) = \frac{1}{\sqrt{1-(\sqrt{9-3x^2})^2}}\cdot(\sqrt{9-3x^2})'$$
$$= \frac{1}{\sqrt{1-9+3x^2}}\cdot\frac{1}{2}\cdot\frac{1}{\sqrt{9-3x^2}}\cdot(-6x)$$
$$= \frac{-3x}{\sqrt{3x^2-8}\,\sqrt{9-3x^2}}$$

d) $-\frac{3x}{\sqrt{9-3x^2}}$

e) $\frac{x}{\sqrt{(-8+3x^2)(9-3x^2)}}$

Question 10

You did not answer the question.

Evaluate the given integral.

$$\int_0^{\frac{1}{2}\sqrt{2}} \frac{4}{\sqrt{1-x^2}}\,dx = 4\int_0^{\frac{\sqrt{2}}{2}} \frac{1}{\sqrt{1-x^2}}dx$$

$$= 4\cdot \arcsin(x)\Big|_0^{\frac{\sqrt{2}}{2}}$$

$$= 4\arcsin\left(\frac{\sqrt{2}}{2}\right) - 4\arcsin 0$$

$$= 4\cdot\frac{\pi}{4} - 4\cdot 0 = \pi$$

a) -2π

b) $\frac{1}{3}\pi$

c) 2π

d) $\frac{1}{2}\pi$

e) $\boxed{\pi}$

Question 11

You did not answer the question.

Evaluate the given integral.

$$\int_0^{\frac{2}{3}} \frac{1}{16+36x^2}\,dx = \int_0^{\frac{2}{3}} \frac{dx}{16(1+\frac{36}{16}x^2)}$$

$$= \frac{1}{16}\int_0^{\frac{2}{3}} \frac{dx}{1+(\frac{6}{4}x)^2}$$

$$= \frac{1}{16}\,\frac{4}{6}\arctan\left(\frac{6}{4}x\right)\Big|_0^{\frac{2}{3}} = \frac{4}{96}\left[\cancel{\arctan 1}^{\frac{\pi}{4}} - \cancel{\arctan 0}^{0}\right] = \frac{\pi}{96}$$

a) $\boxed{\frac{1}{96}\pi}$

b) $-\frac{1}{48}\pi$

c) $\frac{1}{288}\pi$

d) $\frac{1}{48}\pi$

e) $\frac{1}{192}\pi$

Question 12

You did not answer the question.

Evaluate the given integral.

$$\int \frac{e^x}{1+e^{2x}}dx$$

Let $u=e^x$. $du=e^x dx$

$$= \int \frac{du}{1+u^2} = \tan^{-1}(u)+C$$

$$= \tan^{-1}(e^x)+C$$

$$\int_0^{\ln(7)} \frac{e^x}{1+e^{2x}}\,dx = \arctan(e^x)\Big|_0^{\ln 7}$$

$$= \arctan(e^{\ln 7}) - \arctan(e^0)$$

$$= \arctan(7) - \arctan(1)$$

$$= \arctan(7) - \frac{\pi}{4}$$

a) $-\frac{1}{8}\pi + \frac{1}{2}\arctan(7)$

b) $-\frac{1}{2}\pi + 2\arctan(7)$

c) $\boxed{-\frac{1}{4}\pi + \arctan(7)}$

d) $\frac{1}{2}\pi - 2\arctan(7)$

e) $-\frac{1}{12}\pi + \frac{1}{3}\arctan(7)$

Question 13

You did not answer the question.

Calculate the given indefinite integral.

$$\int \frac{3x}{\sqrt{1-x^4}}\,dx$$

Let $u=x^2$. $du=2xdx$

$$\int \frac{4}{\sqrt{1-u^2}}du = 4\arcsin(u)+C$$

$$= 4\arcsin(x^2)+C$$

a) $4\arccos(x^2)+C$

b) $8\arcsin(x^2)+C$

c) $4\arcsin(x^2)+C$

d) $8\arccos(x^2)+C$

e) $4\arcsin(x^4)+C$

Question 14

You did not answer the question.

Calculate the given indefinite integral.

$$\int \frac{11}{x(1+[\ln(2x)]^2)}\,dx$$

Let $u=\ln(2x)$, $du=\frac{2}{2x}dx$

$\Rightarrow du=\frac{dx}{x}$

$$\int \frac{11\cdot du}{1+u^2} = 11\cdot\arctan(u)+C$$

$$=11\arctan(\ln(2x))+C$$

a) $-11\,\mathrm{arcsec}(\ln(2x))+C$

b) $-11\arctan(\ln(2x))+C$

c) $\frac{1}{11}\mathrm{arcsec}(\ln(2x))+C$

d) $\arctan(\ln(2x))+C$

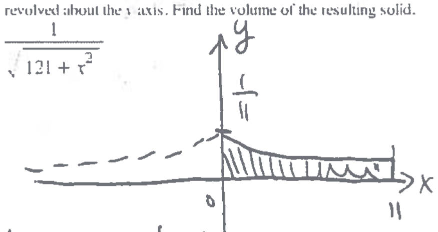
e) $11\arctan(\ln(2x))+C$

Question 15

You did not answer the question.

The region bounded by the graph of f between $x=0$ and $x=11$ is revolved about the y axis. Find the volume of the resulting solid.

$$f(x)=\frac{1}{\sqrt{121+x^2}}$$



a) $-11\pi+11\sqrt{2}\pi$

b) $-\frac{22}{3}\pi+\frac{22}{3}\sqrt{2}\pi$

c) $-22\pi+22\sqrt{2}\pi$

By Shell method

$$\int_0^{11} 2\pi x\frac{1}{\sqrt{121+x^2}}\,dx$$

$$=2\pi\frac{2}{2}\cdot\sqrt{121+x^2}\Big|_0^{11} = 22\sqrt{2}\pi-22\pi.$$

d) $-44\pi+44\sqrt{2}\pi$

e) $-66\pi+66\sqrt{2}\pi$

$\frac{d}{dx}(\sinh(10x))=10\cosh(10x)$

$\frac{d}{dx}(\cosh(10x))=10\sinh(10x)$

Question 16

You did not answer the question.

Differentiate the given function.

$$f(x)=\sinh(10x)\cosh(10x)$$

$=10\cosh(10x)\cosh(10x)+\sinh(10x)\cdot 10\sinh(10x)$

$=10[\cosh(10x)]^2+10[\sinh(10x)]^2$

a) $10[\cosh(10x)]^2-10[\sinh(10x)]^2$

b) $10[\cosh(10x)]^2$

c) $10[\cosh(10x)]^2+10[\sinh(10x)]^2$

d) $[\cosh(10x)]^2+[\sinh(10x)]^2$

e) $10[\sinh(10x)]^2$

Question 17

You did not answer the question.

Differentiate the given function.

$$y=\arctan(\sinh(11x))$$

$$y'=\frac{1}{1+(\sinh(11x))^2}\cdot[\cosh(11x)]\cdot 11$$

$$=\frac{11\cosh(11x)}{1+(\sinh(11x))^2}$$

a) $\frac{11\sinh(11x)}{1+(\cosh(11x))^2}$

b) $\frac{11}{\cosh(11x)}$

c) $\frac{1}{\cosh(11x)}$

d) $-\frac{11}{\cosh(11x)}$

e) $\frac{\cosh(11x)}{1-(\sinh(11x))^2}$?

Question 18

You did not answer the question.

Calculate the indefinite integral.

$$\int \sinh(10x)\,(\cosh(10x))^4\,dx$$

let $u = \cosh(10x)$. $du = 10\sinh(10x)dx$

$\frac{1}{10}\int u^4 du = \frac{1}{10}\frac{u^5}{5} + C$

$= \frac{1}{50}(\cosh(10x))^5 + C$.

a) $-\frac{1}{50}(\cosh(10x))^5 + C$

b) $\frac{1}{5}(\cosh(10x))^5 + C$

c) $\frac{1}{10}(\cosh(10x))^5 + C$

d) $\frac{1}{50}(\cosh(10x))^5 + C$

e) $\frac{1}{4}(\cosh(10x))^4 + C$

Question 19

You did not answer the question.

Calculate the indefinite integral.

$$\int \frac{\cosh(4x)}{\sinh(4x)}\,dx = \frac{1}{4}\ln|\sinh(4x)| + C.$$

(let $u = \sinh(4x)$, $du = 4\cosh(4x)dx$)

a) $\frac{1}{4}\ln(\cosh(4x)) + C$

b) $\frac{1}{4}\ln(\sinh(4x)) + C$

c) $\frac{1}{4}\ln(\coth(4x)) + C$

d) $-\frac{1}{4}\ln(\tanh(4x)) + C$

e) $-\frac{1}{4}\ln(\coth(4x)) + C$

Question 20

You did not answer the question.

Find the average value of $f(x)$ on the interval $[-2,2]$.

$$f(x) = 6\cosh(x)$$

a) $\frac{3}{2}\cosh(2)$

b) $\frac{3}{2}\sinh(2)$

c) $3\cosh(2)$

d) $6\sinh(2)$

e) $3\sinh(2)$

Average of $f(x)$ on $[-2,2]$

$= A.V. = \frac{\int_{-2}^{2} f(x)dx}{2-(-2)}$

$= \frac{1}{4}\int_{-2}^{2} 6\cosh(x)\,dx$

$= \frac{1}{4} 6\sinh(x)\Big|_{-2}^{2} = \frac{6}{4}[\sinh(2) - \sinh(-2)]$

$= 3\sinh(2)$

$[\sinh(-2) = -\sinh(2)]$

Def:

$\sinh(x) = \frac{e^x - e^{-x}}{2}$

$\cosh(x) = \frac{e^x + e^{-x}}{2}$.

$$\frac{d}{dx}(\sin^{-1}x) = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\cos^{-1}x) = -\frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2}$$

$$\frac{d}{dx}(\sec^{-1}x) = \frac{1}{|x|\sqrt{x^2-1}}$$

$$\frac{d}{dx}(\csc^{-1}x) = -\frac{1}{|x|\sqrt{x^2-1}}$$

$$\frac{d}{dx}(\cot^{-1}x) = -\frac{1}{1+x^2}$$

$$\int \frac{dx}{\sqrt{1-x^2}} = \arcsin(x) + C$$

$$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\left(\frac{x}{a}\right) + C$$

$$\int \frac{dx}{1+x^2} = \arctan(x) + C$$

$$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\left(\frac{x}{a}\right) + C$$

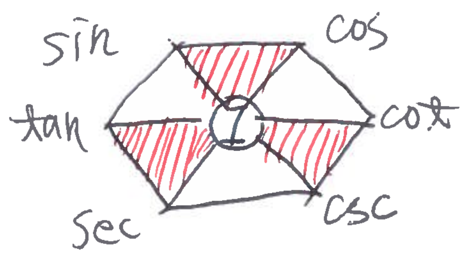
Review.

Sum and Difference Formulas

$$\sin(x \pm y) = \sin x \cos y \pm \sin y \cos x$$

$$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$$

Pythagorean Identities



$\Rightarrow$

$$\sin^2 x + \cos^2 x = 1$$

$$\tan^2 x + 1^2 = \sec^2 x$$

$$1^2 + \cot^2 x = \csc^2 x$$