# PRINTABLE VERSION

## Quiz 2

You scored 0 out of 100

**Question 1**

You did not answer the question.

Determine the domain and find the derivative.

$f(x) = \ln(\ln(8x))$ is well-defined

① $\Leftrightarrow \ln(8x) > 0 = \ln 1$

$\Leftrightarrow 8x > 1 \Leftrightarrow x > \frac{1}{8}$ or $x \in (\frac{1}{8}, \infty)$

a) domain: $(-\infty, \infty)$, $f'(x) = \ln(8x)$

b) domain: $(-\infty, \infty)$, $f'(x) = \frac{1}{x\ln(8x)}$

c) domain: $(1/8, \infty)$, $f'(x) = \frac{1}{x\ln(8x)}$

d) domain: $(-\infty, 0)$, $f'(x) = \ln(8x)$

e) domain: $(1, \infty)$, $f'(x) = \frac{1}{\ln(8x)}$

② $f'(x) = \frac{1}{\ln(8x)} \cdot \frac{1}{8x} \cdot 8$

$= \frac{1}{x \cdot \ln(8x)}$

**Question 2**

You did not answer the question.

Determine the domain and find the derivative.

① $f(x) = \cos(\ln(2x))$ is well-defined

$\Leftrightarrow \ln(2x) \in \mathbb{R}$ (can be any real number)

$\Leftrightarrow 2x > 0 \Leftrightarrow x > 0$ or $x \in (0, \infty)$

a) domain: $(0, \infty)$, $f'(x) = \frac{\cos(\ln(2x))}{x}$

b) domain: $(-\infty, 0)$, $f'(x) = \ln(2x)$

c) domain: $(0, \infty)$, $f'(x) = -\frac{1}{2}\frac{\sin(\ln(2x))}{x}$

d) domain: $(0, \infty)$, $f'(x) = -\frac{\sin(\ln(2x))}{x}$

② $f'(x) = -\sin(\ln(2x)) \cdot \frac{1}{2x} \cdot 2$

$= \frac{-\sin(\ln(2x))}{x}$

e) domain: $(-\infty, \infty)$, $f'(x) = -\frac{\sin(\ln(2x))}{x}$

**Question 3**

You did not answer the question.

Calculate the integral.

$$\int \frac{x}{7-4x^2}\,dx$$

Recall U-substitution

a) $\frac{1}{2}\ln(|-7+4x^2|) + C$

b) $-\frac{1}{8}\ln(|-7+4x^2|) + C$

c) $\frac{4x}{(7-4x^2)^2} + C$

d) $\frac{1}{8}\ln(|-7+4x^2|) + C$

e) $\frac{4}{(7-4x^2)^2} + C$

Let $u = 7-4x^2$, $du = -8x\,dx$

$\Rightarrow \frac{du}{-8} = x\,dx$

$\int \frac{x}{7-4x^2}dx = \int \frac{1}{u} \cdot \frac{du}{-8}$

$= -\frac{1}{8}\int \frac{du}{u} = -\frac{1}{8}\ln|u| + C$

$= -\frac{1}{8}\ln(|7-4x^2|) + C$

**Question 4**

You did not answer the question.

Calculate the integral.

$$\int \frac{\ln(5x-9)}{5x-9}\,dx$$

a) $\ln(\ln(5x-9)) + C$

b) $\frac{1}{10}(\ln(5x-9))^2 + C$

c) $\frac{1}{2}(\ln(5x-9))^2 + C$

Let $u = \ln(5x-9)$

$\Rightarrow du = \frac{5}{5x-9}dx \Rightarrow \frac{du}{5} = \frac{dx}{5x-9}$

$\int \frac{\ln(5x-9)}{1} \cdot \frac{dx}{5x-9}$

$= \int u \frac{du}{5} = \frac{1}{5}\int u\,du = \frac{1}{5}\frac{u^2}{2} + C$

$= \frac{1}{5}\frac{[\ln(5x-9)]^2}{2} + C = \frac{1}{10}[\ln(5x-9)]^2 + C$

d) $-\frac{1}{10}(\ln(5x-9))^2+C$

e) $\ln(5x-9)+C$

**Question 5**

You did not answer the question.

Calculate the integral.

$$\int \frac{\sin(6x)-\cos(6x)}{\sin(6x)+\cos(6x)}\,dx$$

Let $u=\sin(6x)+\cos(6x)$

$\Rightarrow du=[6\cos(6x)-6\sin(6x)]dx$

$\Rightarrow \frac{du}{-6}=[\sin(6x)-\cos(6x)]dx$

$=\int \frac{1}{u}\cdot\frac{du}{-6}=-\frac{1}{6}\int\frac{du}{u}$

$=-\frac{1}{6}\ln|u|+C=-\frac{1}{6}\ln|\sin 6x+\cos 6x|+C$

a) $-\frac{1}{7}\ln|\sin(6x)+\cos(6x)|+C$

b) $\frac{1}{6}\ln|\sin(6x)+\cos(6x)|+C$

c) $-\frac{1}{6}\ln|-\sin(6x)+\cos(6x)|+C$

d) $\frac{1}{7}\ln|-\sin(6x)+\cos(6x)|+C$

e) $\boxed{-\frac{1}{6}\ln|\sin(6x)+\cos(6x)|+C}$

**Question 6**

You did not answer the question.

Calculate the integral.

$$\int \frac{1}{4\sqrt{x}\,(2+\sqrt{x})}\,dx$$

Let $u=2+\sqrt{x}$, $du=\frac{1}{2}\frac{1}{\sqrt{x}}dx$

$\Rightarrow \frac{du}{2}=\frac{dx}{4\sqrt{x}}$

$\int\frac{1}{u}\cdot\frac{du}{2}$

$=\frac{1}{2}\int\frac{du}{u}=\frac{1}{2}\ln|u|+C$

$=\frac{1}{2}\ln|2+\sqrt{x}|+C$

a) $-\frac{1}{2}\ln(4\sqrt{x})+C$

b) $-\frac{1}{2}\ln(2+\sqrt{x})+C$

c) $-4\ln(4\sqrt{x})+C$

d) $\boxed{\frac{1}{2}\ln(2+\sqrt{x})+C}$

e) $-4\ln(1+\sqrt{x})+C$

**Question 7**

You did not answer the question.

Evaluate the definite integral.

$$\int_7^{e^5}\frac{1}{x}\,dx$$

$=\ln|x|\Big|_7^{e^5}$

$=\ln e^5-\ln 7$

$=5\cancel{\ln e}^{1}-\ln 7$

$=5-\ln 7$

a) $\ln(7)-5$

b) $\boxed{-\ln(7)+5}$

c) $-2\ln(7)+10$

d) $-\ln(7)$

e) $-\ln(5)+7$

**Question 8**

You did not answer the question.

Evaluate the definite integral.

$$\int_{\frac{1}{6}\pi}^{\frac{1}{2}\pi}\frac{\cos(x)}{9+\sin(x)}\,dx$$

Let $u=9+\sin x$, $du=\cos x\,dx$.

$\int\frac{\cos(x)}{9+\sin(x)}dx=\int\frac{du}{u}=\ln|u|+C$

$=\ln|9+\sin x|+C$

So. $\int_{\frac{\pi}{6}}^{\frac{1}{2}\pi}\frac{\cos(x)}{9+\sin(x)}dx=\ln|9+\sin x|\Big|_{\frac{\pi}{6}}^{\frac{\pi}{2}}$

$=\ln|9+\sin\frac{\pi}{2}|-\ln|9+\sin\frac{\pi}{6}|$

$=\ln 10-\ln\frac{19}{2}=\ln\left(\frac{10}{\frac{19}{2}}\right)=\ln\left(\frac{20}{19}\right)$

$\left(\ln\left(\frac{a}{b}\right)=\ln a-\ln b\right)$ property of log fn.

a) $\ln\left(\frac{21}{20}\right)$

b) $\ln\left(\frac{18}{17}\right)$

c) $\boxed{\ln\left(\frac{20}{19}\right)}$

Recall Properties of log function

① $\ln ab = \ln a + \ln b$

② $\ln\left(\frac{a}{b}\right) = \ln a - \ln b$

③ $\ln a^c = c \ln a$.

d) $\ln\left(\frac{22}{21}\right)$

e) $\ln\left(\frac{19}{18}\right)$

**Question 9**

You did not answer the question.

Calculate the derivative by logarithmic differentiation.

$$g(x) = (x^2+1)^3 (x-1)^6 x^4$$

First, Take "ln" ⇒

$\ln(g(x)) = \ln\left[(x^2+1)^3 (x-1)^6 x^4\right]$

① & ③ ⇒ $= 3\ln(x^2+1) + 6\ln(x-1) + 4\ln x$

Then find derivative on both sides

$$\frac{g'(x)}{g(x)} = \frac{3\cdot(2x)}{x^2+1} + \frac{6}{x-1} + \frac{4}{x}$$

$$\Rightarrow g'(x) = g(x)\left[\frac{6x}{x^2+1} + \frac{6}{x-1} + \frac{4}{x}\right]$$

a) $g'(x) = \frac{6x}{x^2+1} - \frac{6}{x-1} - \frac{4}{x}$

b) $g'(x) = (x^2+1)^3 (x-1)^6 x^4 \left(\frac{3x}{x^2+1} + \frac{6}{x-1} + \frac{4}{x}\right)$

c) $g'(x) = (x^2+1)^3 (x-1)^6 x^4 \left(\frac{6x}{x^2+1} - \frac{6}{x-1} - \frac{4}{x}\right)$

d) $g'(x) = \frac{6x}{x^2+1} + \frac{6}{x-1} + \frac{4}{x}$

e) $g'(x) = (x^2+1)^3 (x-1)^6 x^4 \left(\frac{6x}{x^2+1} + \frac{6}{x-1} + \frac{4}{x}\right)$

**Question 10**

You did not answer the question.

Find the points of inflection for the function

$$f(x) = 3x^2 \ln\left(\frac{1}{2}x\right)$$

$(x, y)$ s.t. $f''(x) = 0$

$f'(x) = 6x \ln\left(\frac{x}{2}\right) + 3x^2 \cdot \frac{2}{x} \cdot \frac{1}{2}$

product rule

$= 6x\ln\left(\frac{x}{2}\right) + 3x$.

$f''(x) = 6\ln\left(\frac{x}{2}\right) + 6x \cdot \frac{2}{x} \cdot \frac{1}{2} + 3$

$= 6\ln\left(\frac{x}{2}\right) + 6 + 3 = 6\ln\left(\frac{x}{2}\right) + 9 = 0$

$\Rightarrow 6\ln\left(\frac{x}{2}\right) = -9 \Rightarrow \ln\left(\frac{x}{2}\right) = -\frac{9}{6} = -\frac{3}{2} \Rightarrow \frac{x}{2} = e^{-\frac{3}{2}} \Rightarrow x = 2e^{-\frac{3}{2}}$

Take "e" on both sides

$x = 2e^{-\frac{3}{2}}$.

$y = f(2e^{-\frac{3}{2}})$

$= 3(2e^{-\frac{3}{2}})^2 \ln\left(\frac{1}{2}\cdot 2e^{-\frac{3}{2}}\right)$

$= 12e^{-3} \cdot \ln(e^{-\frac{3}{2}})$

$= -\frac{3}{2}\cdot 12e^{-3}$

$= -18e^{-3}$

a) $\left(-18e^{-3}, 2e^{-\frac{3}{2}}\right)$

b) $\left(-2e^{-\frac{3}{2}}, 18e^{-3}\right)$

c) $\left(4e^{-\frac{3}{2}}, -36e^{-3}\right)$

d) $\left(2e^{-\frac{3}{2}}, 0\right)$

e) $\boxed{\left(2e^{-\frac{3}{2}}, -18e^{-3}\right)}$

**Question 11**

You did not answer the question.

Differentiate

$$y = \frac{e^{-3x}}{x^3} = e^{-3x}\cdot x^{-3}$$

By Product Rule

$y' = -3e^{-3x}\cdot x^{-3} + e^{-3x}\cdot(-3x^{-4})$

$= -\frac{3e^{-3x}}{x^3} - \frac{3e^{-3x}}{x^4}$

a) $y' = \frac{e^{-3x}}{x^4} + \frac{9e^{-3x}}{x^3}$

b) $y' = -\frac{e^{-3x}}{x^4} - \frac{e^{-3x}}{x^3}$

c) $y' = -\frac{3e^{-3x}}{x^4} - \frac{3e^{-3x}}{x^3}$

d) $y' = -\frac{4e^{-3x}}{x^3}$

e) $y' = \frac{3e^{-3x}}{x^4} + \frac{3e^{-3x}}{x^3}$

**Question 12**

You did not answer the question.

Differentiate

$$y = \left(e^{x^4} + 2\right)^2$$

$y=(e^{x^4}+2)^2$

$y'=2(e^{x^4}+2)\cdot 4x^3\cdot e^{x^4}$

$=8x^3e^{x^4}(e^{x^4}+2)$

a) $y'=\frac{(e^{x^4}+2)e^{x^4}}{x}$

b) $y'=8(e^{x^4}+2)^2x^3e^{x^4}$

c) $y'=2(e^{x^4}+2)x^3e^{x^4}$

d) $y'=8(e^{x^4}+2)x^3e^{x^4}$

e) $y'=4(e^{x^4}+2)x^3e^{x^4}$

**Question 13**

You did not answer the question.

Calculate the given integral.

$$\int 4e^{-2x}\,dx$$

$$= \frac{4}{-2}e^{-2x}+C = -2e^{-2x}+C.$$

a) $2e^{-2x}+C$

b) $-2e^{-2x}+C$

c) $-4e^{-2x}+C$

d) $\frac{1}{2}e^{-2x}+C$

e) $-\frac{1}{2}e^{-2x}+C$

**Question 14**

You did not answer the question.

Calculate the given integral.

$$\int 5e^{\ln(3x)}\,dx$$

$e^x$ is the inverse of $\ln x$,

$\Rightarrow e^{\ln(3x)}=3x.$

$\int 5\cdot 3x\,dx=\int 15x\,dx$

$=\frac{15}{2}x^2+C$

a) $30x^2+C$

b) $\frac{15}{2}x^2+C$

c) $-\frac{5}{2}x^2+C$

d) $\frac{5}{2}\ln(3)+\frac{5}{2}\ln(x)+C$

e) $\frac{3}{2}e^{3x}+C$

**Question 15**

You did not answer the question.

Calculate the given integral.

$$\int \frac{\sin(7e^{-6x})}{e^{6x}}\,dx$$

Let $u=7e^{-6x} \Rightarrow du=-42e^{-6x}dx$

$=-\frac{42}{e^{6x}}dx.$

$\Rightarrow \frac{du}{-42}=\frac{dx}{e^{6x}}$

$\int \sin(u)\frac{du}{-42}=-\frac{1}{42}\int \sin(u)du$

$=-\frac{1}{42}\cdot -\cos(u)+C.$

$=\frac{1}{42}\cos(7e^{-6x})+C.$

a) $7\cos(7e^{6x})+C$

b) $-\frac{1}{6}\cos(7e^{-6x})+C$

c) $-\frac{1}{7}\cos(7e^{6x})+C$

d) $\frac{6}{7}\cos(7e^{-6x})+C$

e) $\frac{1}{42}\cos(7e^{-6x})+C$

**Question 16**

You did not answer the question.

Find the 4th derivative of $f(x)=e^{4x}$

$f'(x)=4\cdot e^{4x}$

$f''(x)=4\cdot 4e^{4x}=16e^{4x}$

$f'''(x)=4\cdot 16e^{4x}=64e^{4x}$

$f^{(4)}(x)=4\cdot 64e^{4x}=256e^{4x}$

a) $-256e^{4x}$

b) $1024e^{4x}$

Changing Base: $\log_p x = \dfrac{\ln x}{\ln p}$

$(\ln y = \log_e y)$

c) $-64\, e^{4x}$

d) $64\, e^{4x}$

e) $\boxed{256\, e^{4x}}$

**Question 17**

You did not answer the question.

Differentiate the given function.

$$f(x) = \frac{\log_9(x)}{x^3} = \frac{\ln x}{\ln 9} \cdot \frac{1}{x^3} = \frac{\ln x}{\ln 9} \cdot x^{-3}$$

constant

product rule ⇒ $\dfrac{1}{\ln 9} \cdot \dfrac{1}{x} \cdot x^{-3} + \dfrac{\ln x}{\ln 9} \cdot (-3x^{-4})$

$$= \frac{1 - 3\ln x}{x^4 \cdot \ln 9} \quad 9 \to 3^2$$

$$= \frac{1 - 3\ln x}{2x^4 \cdot \ln 3}$$

a) $f'(x) = -\dfrac{1}{2}\, \dfrac{-x^3 + 3\ln(x)}{x^4 \ln(3)}$

b) $f'(x) = -\dfrac{1}{2}\, \dfrac{-x + 3\ln(x)}{x^4 \ln(3)}$

c) $f'(x) = -\dfrac{-1 + 3\ln(x)}{x^4 \ln(3)}$

d) $\boxed{f'(x) = -\dfrac{1}{2}\, \dfrac{-1 + 3\ln(x)}{x^4 \ln(3)}}$

e) $f'(x) = \dfrac{1}{2}\, \dfrac{1 + 6\ln(x)\ln(3)}{x^4 \ln(3)}$

**Question 18**

You did not answer the question.

Calculate the given integral.

$$\int (x^4 + 5^{-x})\, dx$$

$$= \frac{x^5}{5} - \frac{1}{\ln 5} \cdot 5^{-x} + C$$

$$\frac{d}{dx} 5^{-x} = \frac{d}{dx}\left(e^{\ln 5^{-x}}\right) = \frac{d}{dx}\left(e^{-x \ln 5}\right) = e^{-x\ln 5} \cdot (-\ln 5) = -\ln 5 \cdot 5^{-x}$$

$$5^{-x} = \int \left(\frac{d}{dx} 5^{-x}\right) dx = \int -\ln 5 \cdot 5^{-x}\, dx$$

a) $\boxed{\dfrac{1}{5} x^5 - \dfrac{5^{-x}}{\ln(5)} + C}$

b) $\dfrac{1}{5} x^5 + \dfrac{5^{-x}}{\ln(5)} + C$

c) $\dfrac{1}{4} x^4 - \dfrac{5^{-x}}{\ln(5)} + C$

d) $-\dfrac{1}{5} x^5 - \dfrac{5^{-x}}{\ln(6)} + C$

e) $\dfrac{1}{4} x^4 - \dfrac{5^{-x}}{\ln(6)} + C$

✓

**Question 19**

You did not answer the question.

Find the derivative by logarithmic differentiation.

$$\frac{d}{dx}(3x+2)^x = \frac{d}{dx}\left(e^{\ln(3x+2)^x}\right) = \frac{d}{dx}\left(e^{x\ln(3x+2)}\right) = e^{x\ln(3x+2)} \cdot [x\ln(3x+2)]' = (3x+2)^x \cdot \left[\ln(3x+2) + \frac{3x}{3x+2}\right]$$

a) $-(3x+2)^x(\ln(3x) + 3x)$

b) $(3x+2)^x(\ln(3x) + 1)$

c) $-(3x+2)^x(\ln(3x+2) + x(3x+2))$

d) $(3x+2)^x\left(\ln(3x+2) + \dfrac{3}{3x+2}\right)$

e) $\boxed{(3x+2)^x\left(\ln(3x+2) + \dfrac{3x}{3x+2}\right)}$

**Question 20**

You did not answer the question.

Evaluate the given integral.

$$\int_3^4 2^{-x}\, dx = \frac{1}{-\ln 2} \cdot 2^{-x}\Big|_3^4 = \frac{1}{-\ln 2}\left(2^{-4} - 2^{-3}\right) = \frac{1}{-\ln 2}\left(\frac{1}{16} - \frac{1}{8}\right) = \frac{1}{16\ln 2}$$

a) $-\dfrac{8}{\ln(2)}$

Changing Base

$$\log_p x = \frac{\ln x}{\ln p}.$$

b) $-\frac{8}{\ln(3)}$

c) $\boxed{\frac{1}{16\ln(2)}}$

d) $\frac{3}{16\ln(3)}$

e) $\frac{1}{32\ln(2)}$

$$\begin{cases} \int x^r dx = \frac{x^{r+1}}{r+1} + c & \text{for } r \neq -1. \\ \int x^{-1} dx = \ln|x| + c \end{cases}$$

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Base p, (p is a real number).

$$\frac{d}{dx}(p^x) = \ln p \cdot p^x.$$

$$\frac{d}{dx}(p^{f(x)}) = \ln p \cdot f'(x) \cdot p^x.$$

$$\int p^x dx = \frac{1}{\ln p} \cdot p^x + C, \quad \underline{p > 0. \quad p \neq 1}$$