Sol Summer 2014.

PRINTABLE VERSION

Quiz 1

You scored 0 out of 100

Question 1

You did not answer the question.

Determine whether or not the given function is one-to-one and, if so, find the inverse. If f has an inverse, give the domain of f^{-1}.

$$f(x) = 2 - x^2$$

a) $f^{-1}(x) = -\sqrt{x-2}$; domain: $(-\infty, 1)$

b) $f^{-1}(x) = 1 - 2\sqrt{x}$; domain: $(0, \infty)$

c) $f^{-1}(x) = \sqrt{x-2}$; domain: $(2, \infty)$

d) Not one-to-one

e) $f^{-1}(x) = \sqrt{x-2}$; domain: $(-\infty, \infty)$

$f'(x) = -2x \Rightarrow$ NOT MONOTONE

$\Rightarrow$ NOT ONE-TO-ONE

Question 2

You did not answer the question.

Determine whether or not the given function is one-to-one and, if so, find the inverse. If f has an inverse, give the domain of f^{-1}.

$$y = f(x) = x^5 + 2$$

a) $f^{-1}(x) = \sqrt{x-2}$; domain: $(-\infty, 2)$

b) $f^{-1}(x) = (x-2)^5$; domain: $(0, \infty)$

c) Not one-to-one

d) $f^{-1}(x) = (x-2)^{1/5}$; domain: $(-\infty, \infty)$

e) $f^{-1}(x) = (x-2)^{1/5}$; domain: $(2, \infty)$

① $f'(x) = 5x^4 \geq 0$

$\Rightarrow$ always INCREASE.

$\Rightarrow$ ONE-TO-ONE.

② Find inverse function of f.

i. Switch x and y. $\Rightarrow$ $x = y^5 + 2$

ii. Find y $\Rightarrow$ $y = (x-2)^{\frac{1}{5}}$

Domain: $x \in (-\infty, \infty)$

Question 3

You did not answer the question.

Determine whether or not the given function is one-to-one and, if so, find the inverse. If f has an inverse, give the domain of f^{-1}.

$$y = f(x) = 3x^{5/11}$$

a) $f^{-1}(x) = \frac{1}{3}x^{11/5}$; domain: $(-\infty, \infty)$

b) Not one-to-one

c) $f^{-1}(x) = \left(\frac{1}{3}x\right)^{5/11}$; domain: $(0, \infty)$

d) $f^{-1}(x) = \left(\frac{1}{3}x\right)^{11/5}$; domain: $(0, \infty)$

e) $f^{-1}(x) = \left(\frac{1}{3}x\right)^{11/5}$; domain: $(-\infty, \infty)$

① $f'(x) = 3 \cdot \frac{5}{11} x^{-\frac{6}{11}}$

$= \frac{15}{11} \cdot \left(\frac{1}{x^{\frac{1}{11}}}\right)^6 \geq 0$

$\Rightarrow$ INCREASE $\Rightarrow$ ONE-TO-ONE

② f^{-1}: i. Switch x and y

$x = 3y^{\frac{5}{11}}$

ii. Find y

$\left(\frac{x}{3}\right)^{\frac{11}{5}} = y$, $x \in (-\infty, \infty)$

Question 4

You did not answer the question.

Determine whether or not the given function is one-to-one and, if so, find the inverse. If f has an inverse, give the domain of f^{-1}.

$$f(x) = (1 + 2x^2)^5$$

a) $f^{-1}(x) = (1 + 2x^2)^{1/5}$; domain: $(0, \infty)$

b) Not one-to-one

c) $f^{-1}(x) = \sqrt{\frac{1}{2}x^{1/5} - \frac{1}{2}}$; domain: $(0, \infty)$

d) $f^{-1}(x) = (1 + 2x^2)^{1/5}$; domain: $(-\infty, \infty)$

e) $f^{-1}(x) = \sqrt{\frac{1}{2}x^{1/5} - \frac{1}{2}}$; domain: $(-\infty, \infty)$

① $f'(x) = \underbrace{5(1+2x^2)^4}_{\text{all "+"}} \cdot \underbrace{(4x)}_{\text{"+" or "-"}}$

$\Rightarrow$ NOT MONOTONE

$\Rightarrow$ NOT ONE-TO-ONE.

Question 5

You did not answer the question.

Determine whether or not the given function is one-to-one and, if so, find the inverse.

$$f(x) = \frac{4}{3}\cos(x)$$

$$x \in \left[-\frac{1}{2}\pi, \frac{1}{2}\pi\right]$$

$f'(x) = -\frac{4}{3}\sin x \Rightarrow$ NOT MONOTONE

"+" or "−" on $\left[-\frac{1}{2}\pi, \frac{\pi}{2}\right]$

$\Rightarrow$ NOT ONE-TO-ONE.

a) $f^{-1}(x) = \arccos\left(\frac{3}{4}x\right)$

b) $f^{-1}(x) = \sec\left(\frac{3}{4}x\right)$

c) Not one-to-one

d) $f^{-1}(x) = \frac{4}{3}\sin(x)$

e) $f^{-1}(x) = \frac{4}{3}\sec(x)$

Question 6

You did not answer the question.

Determine whether or not the given function is one-to-one and, if so, find the inverse.

$$f(x) = 6x + \frac{7}{x}$$

$f'(x) = 6 - 7\frac{1}{x^2} \Rightarrow$ NOT MONOTONE

$\Rightarrow$ NOT ONE-TO-ONE.

a) Not one-to-one

b) $f^{-1}(x) = -6x - \frac{7}{x}$

c) $f^{-1}(x) = \frac{6}{x} - 7x$

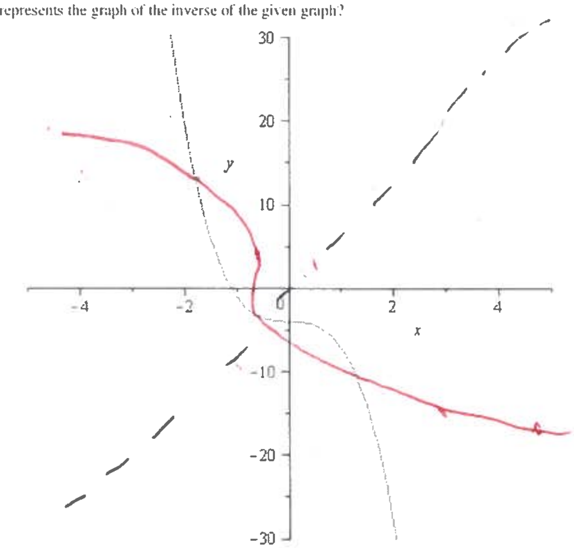
d) $f^{-1}(x) = -\frac{1}{12}x - \frac{1}{12}\sqrt{x^2 - 168}$

e) $f^{-1}(x) = \frac{1}{12}x + \frac{1}{12}\sqrt{x^2 - 168}$

Question 7

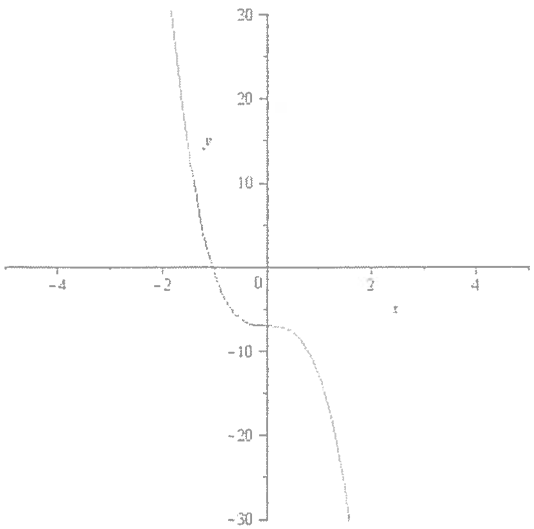
You did not answer the question.

Which of the following represents the graph of the inverse of the given graph?

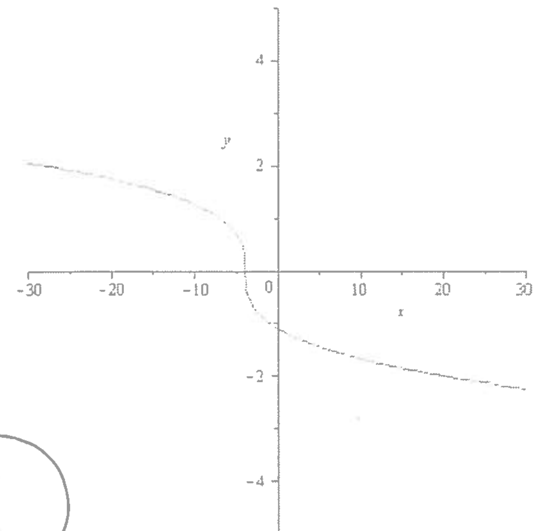


① This graph is ONE-TO-ONE by horizontal line test.

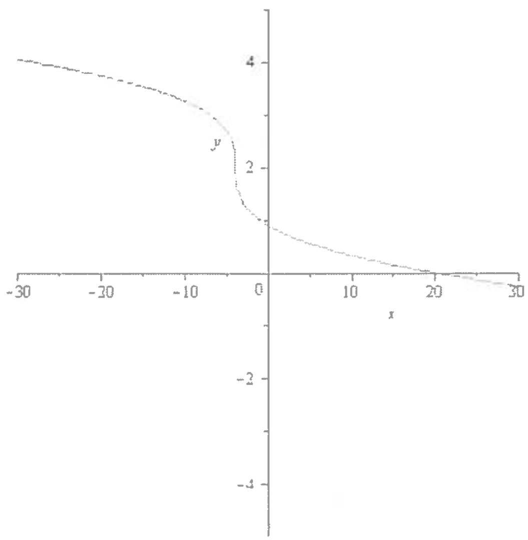
② The graph of the inverse one is the given graph reflected in the line $x = y$.



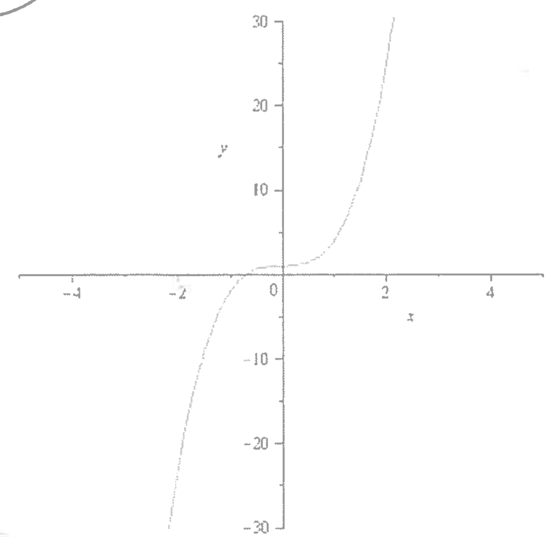
a)



c)



b)



d)

e) *The given function is not one-to-one*

Question 8

You did not answer the question.

Given the following function, with k as a constant, find the values of k for which f is one-to-one.

$$f(x) = \frac{1}{3}x^3 + 8x^2 + kx$$

a) $8 \le k$

b) $64 \le k$

c) $-64 \le k$

d) $k \le \frac{1}{64}$

e) $k \le -\frac{1}{64}$

Find k such that $f'(x)$ is always positive (i.e. f is increasing)

$f'(x) = x^2 + 16x + k$ →

Then "Complete the Square"

$$\Rightarrow x^2+16x+k = (x^2+16x+64)-64+k.$$
$$= (x+8)^2 - 64 + k.$$
$$\Rightarrow -64+k \geq 0 \Rightarrow k \geq 64.$$

Question 9

You did not answer the question.

Suppose that f has an inverse and $f(5) = 6$, $f'(5) = 2/3$. What is $(f^{-1})'(6)$?

a) 3

b) $\frac{3}{2}$

c) $-\frac{2}{3}$

d) $\frac{2}{3}$

e) $\frac{5}{2}$

Formula: $f(a)=b$ and f has an inverse.
Then $(f^{-1})'(b) = \frac{1}{f'(a)}$

Now $a=5$, $b=6$.

$$\Rightarrow (f^{-1})'(6) = \frac{1}{f'(5)} = \frac{1}{\frac{2}{3}} = \frac{3}{2}.$$

Question 10

You did not answer the question.

Suppose that the given function f is differentiable, has an inverse and that $f(1) = -7$. Find $(f^{-1})'(-7)$.

$$f(x) = -4 - 2x - x^3$$

a) $\frac{1}{4}$

b) $-\frac{2}{5}$

c) $-\frac{1}{5}$

d) $\frac{2}{5}$

e) $-\frac{1}{10}$

Now, by Q9, $a=1$, $b=-7$.

and $(f^{-1})'(-7) = \frac{1}{f'(1)} = -\frac{1}{5}$

$f(x) = -4-2x-x^3$
$\Rightarrow f'(x) = -2-3x^2.$
$\Rightarrow f'(1) = -5$

Question 11

You did not answer the question.

Suppose that the given function f is differentiable, has an inverse and that $f(9) = 30$. Find $(f^{-1})'(30)$.

$$f(x) = 2x + 4\sqrt{x}$$
$$x > 0$$

a) $\frac{3}{4}$

b) $\frac{3}{8}$

c) $-\frac{3}{11}$

d) $\frac{3}{16}$

e) $-\frac{3}{4}$

Now $a=9$, $b=30$, Then.

$$(f^{-1})'(30) = \frac{1}{f'(9)} = \frac{1}{\frac{8}{3}} = \frac{3}{8}$$

$f(x) = 2x + 4\sqrt{x}.$
$f'(x) = 2 + \frac{1}{2}\frac{4}{\sqrt{x}} = 2 + \frac{2}{\sqrt{x}}$
$f'(9) = 2 + \frac{2}{\sqrt{9}} = 2 + \frac{2}{3} = \frac{8}{3}$

Question 12

You did not answer the question.

Suppose that the given function f is differentiable, has an inverse and that $f(\frac{3}{2}\pi) = \frac{1}{2}\pi$. Find $(f^{-1})'(\frac{1}{2}\pi)$.

$f(x) = x - \pi + \cos(x)$

$0 < x < 2\pi$

Now $a = \frac{3}{2}\pi$, $b = \frac{\pi}{2}$. Then

$$(f^{-1})'\left(\frac{\pi}{2}\right) = \frac{1}{f'\left(\frac{3}{2}\pi\right)} = \frac{1}{2}$$

$$\boxed{\begin{aligned} f(x) &= x - \pi + \cos(x) \\ f'(x) &= 1 - \sin(x) \\ f'\left(\tfrac{3}{2}\pi\right) &= 1 - \sin\left(\tfrac{3\pi}{2}\right) = 1 - (-1) = 2 \end{aligned}}$$

a) $\frac{1}{4}$

b) $-\frac{1}{2}$

c) 1

d) -1

e) $\frac{1}{2}$

Question 13

You did not answer the question.

Use the properties of logarithms and the table given below to estimate ln(56).

n	$\ln n$	n	$\ln n$
1	0.00	6	1.79
2	0.69	7	1.95
3	1.10	8	2.08
4	1.39	9	2.20
5	1.61	10	2.30

$$\boxed{\begin{aligned} &① \ \log ab = \log a + \log b \\ &② \ \log \frac{a}{b} = \log a - \log b \\ &③ \ \log a^c = c \log a \end{aligned}}$$

$$\ln 56 = \ln 7 \cdot 8 = \ln 7 \cdot 2^3$$

$$\overset{①}{=} \ln 7 + \ln 2^3$$

$$\overset{②}{=} \ln 7 + 3 \ln 2$$

$$= 1.95 + 3 \cdot 0.69 = 4.02$$

a) 3.83

b) 4.03

c) 4.06

d) 3.63

e) 4.43

the closest one

Question 14

You did not answer the question.

Use the properties of logarithms and the table given below to estimate $\ln(4\sqrt{5})$.

n	$\ln n$	n	$\ln n$
1	0.00	6	1.79
2	0.69	7	1.95
3	1.10	8	2.08
4	1.39	9	2.20
5	1.61	10	2.30

$$\ln 4\sqrt{5} = \ln 4 \cdot 5^{\frac{1}{2}} = \ln 2^2 \cdot 5^{\frac{1}{2}}$$

$$\overset{①}{=} \ln 2^2 + \ln 5^{\frac{1}{2}}$$

$$\overset{③}{=} 2 \ln 2 + \frac{1}{2} \ln 5$$

$$= 2 \cdot 0.69 + \frac{1}{2} \cdot 1.61 = 1.38 + 0.805 = 2.185$$

a) 2.39

b) 2.19

c) 2.59

d) 1.79

e) 2.74

Question 15

You did not answer the question.

Estimate:

$$\ln(2.1) = \int_1^{2.1} \frac{1}{t}\,dt$$

$f(t) = \frac{1}{t}$

Lower sum, upper sum

Using the approximation $1/2[L_f(P) + U_f(P)]$ with $P = \{1 = 10/10, 11/10, 12/10, 13/10, 14/10, 15/10, 16/10, 17/10, 18/10, 19/10, 20/10, 21/10 = 2.1\}$.

P	Max	length	min
$[1, \frac{11}{10}]$	1	$\frac{1}{10}$	$\frac{10}{11}$
$[\frac{11}{10}, \frac{12}{10}]$	$\frac{10}{11}$	$\frac{1}{10}$	$\frac{10}{12}$
$[1.2, 1.3]$	$\frac{10}{12}$	$\frac{1}{10}$	$\frac{10}{13}$
$[1.3, 1.4]$	$\frac{10}{13}$	$\frac{1}{10}$	$\frac{10}{14}$
$[1.4, 1.5]$	$\frac{10}{14}$	$\frac{1}{10}$	$\frac{10}{15}$
$[1.5, 1.6]$	$\frac{10}{15}$	$\frac{1}{10}$	$\frac{10}{16}$
$[1.6, 1.7]$	$\frac{10}{16}$	$\frac{1}{10}$	$\frac{10}{17}$
$[1.7, 1.8]$	$10/17$	$\frac{1}{10}$	$\frac{10}{18}$
$[1.8, 1.9]$	$10/18$	$\frac{1}{10}$	$\frac{10}{19}$
$[1.9, 2.0]$	$10/19$	$\frac{1}{10}$	$\frac{10}{20}$
$[2.0, 2.1]$	$10/20$	$\frac{1}{10}$	$\frac{10}{21}$

$U_f = 0.768$

$L_f = 0.715$

$\frac{1}{2}(U_f + L_f) = 0.74$

a) 0.769

b) 1.43

c) 1.49

d) 0.743

e) 0.716

Question 16

Differentials estemate.

$f(x+h) \doteq f(x) + hf'(x)$.

You did not answer the question.

Taking ln(5) is approximately 1.61, use differentials to estimate ln(5.1).

Given $\ln 5 = 1.61$ and $f(x) = \ln(x) \Rightarrow f'(x) = \frac{1}{x}$

Find $\ln(5.1) = f(5.1) \doteq f(5) + 0.1 \cdot f'(5)$

$x=5,\ h=0.1$

$= 1.61 + 0.1 \cdot \frac{1}{5}$

$= 1.61 + 0.02$

$= 1.63$

a) 1.71

b) 1.63

c) 1.75

d) 1.58

e) 1.55

Question 17

You did not answer the question.

Taking ln(5) is approximately 1.61, use differentials to estimate ln(5.3).

Given $\ln 5 = 1.61$, $f(x) = \ln x$, $f'(x) = \frac{1}{x}$.

To Find $\ln 5.3 = f(5.3) \doteq f(5) + 0.3 \cdot f'(5)$

$\boxed{\cancel{x}=5,\ h=0.3}$

$= 1.61 + 0.3 \cdot \frac{1}{5} = 1.61 + 0.06 = 1.67$

a) 1.75

b) 1.67

c) 1.79

d) 1.72

e) 1.59

Question 18

You did not answer the question.

Solve the equation for x.

$\ln(x) = 1$

a) e^3

b) e^2

c) $\frac{1}{e}$

e^x is the inverse of $\ln x$.

$\Rightarrow e^{\ln x} = x$.

Take "e" on both sides.

we have $e^{\ln x} = e^1 \Rightarrow x = e$.

d) 1

e) e

Question 19

You did not answer the question.

Solve the equation for x.

$\frac{1}{2}\ln(x) = \ln(2x-10)$

property ③ $\Rightarrow \ln(x)^{\frac{1}{2}} = \ln(2x-10)$.

Take "e" on both sides

$\Rightarrow e^{\ln(x)^{\frac{1}{2}}} = e^{\ln(2x-10)}$

$\Rightarrow x^{\frac{1}{2}} = 2x - 10$

$\Rightarrow x = (2x-10)^2$

$\Rightarrow x = 4x^2 - 40x + 100$

$\Rightarrow 4x^2 - 41x + 100 = 0$

$\Rightarrow (x-4)(4x-25) = 0$

$\Rightarrow x = \cancel{4}$ or $\frac{25}{4}$

a) 3

b) $\frac{25}{4}$

c) 4 or $\frac{25}{4}$

d) 4

e) -4 or $-\frac{25}{4}$

Question 20

You did not answer the question.

Solve the equation for x.

$\ln((2x+3)(x+10)) = 2\ln(x+10)$

Take "e"

$\Rightarrow (2x+3)(x+10) = (x+10)^2$.

$\Rightarrow (2x+3)(x+10) - (x+10)^2 = 0$

$\Rightarrow (x+10)[(2x+3) - (x+10)] = 0$

$\Rightarrow (x+10)(x-7) = 0$

$\Rightarrow x = \cancel{-10}$ or 7

a) 0

b) 7

c) 5

d) 9

e) 10