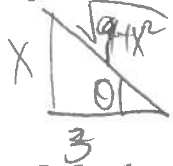
$x\tan\theta = 3$ e, $\frac{2}{3}\int \csc\theta = \frac{2}{3}\ln|\csc\theta - \cot\theta| + C$



$= \frac{2}{3}\ln\left|\frac{\sqrt{9+x^2}}{3} - \frac{3}{x}\right| + C$

as $x=0$.

$0 = A - B + C$

$= \frac{1}{4} - B + \frac{1}{2} \Rightarrow B = \frac{3}{4}$

Math 1432

Exam 3 Review

1. Integrate:

a. $\int \frac{3x^2+3x+3}{x^2+1}\,dx = \int \frac{3x^2+3}{x^2+1}dx + \int \frac{3x}{x^2+1}dx = \int 3dx + \int \frac{3x}{x^2+1} = 3x + \frac{3}{2}\ln(x^2+1) + C$

$\frac{A}{x+1} + \frac{B}{x-1} + \frac{C}{(x-1)^2}$

b. $\int \frac{x^2}{(x+1)(x-1)^2}\,dx = \int \frac{\frac{1}{4}}{x+1} + \frac{\frac{1}{2}}{(x-1)^2} + \frac{\frac{3}{4}}{x-1}dx = \frac{1}{4}\ln|x+1| - \frac{1}{2}(x-1)^{-1} + \frac{3}{4}\ln|x-1| + C$

$\frac{A}{x+1} + \frac{Bx+C}{x^2+1}$

c. $\int \frac{x^2+5x+2}{(x+1)(x^2+1)}\,dx$ $\int \frac{-1}{x+1} + \frac{2x+3}{x^2+1}dx = \int \frac{-dx}{x+1}dx + \int \frac{2x}{x^2+1}dx + \int \frac{3}{x^2+1}dx$

$= -\ln|x+1| + \ln|x^2+1| + 3\arctan x + C$

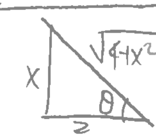
let $x = 3\sin\theta$, $dx = 3\cos\theta d\theta$

d. $\int \frac{2x^2}{\sqrt{9-x^2}}\,dx = \int \frac{18\sin^2\theta}{3\cos\theta}3\cos\theta d\theta = \int 18\left(\frac{1}{2} - \frac{1}{2}\cos 2\theta\right)d\theta = 9\theta - \frac{9}{2}\sin 2\theta + C$

$x = 3\tan\theta$, $dx = 3\sec^2\theta d\theta$

e. $\int \frac{2}{x\sqrt{9+x^2}}\,dx$ $\int \frac{2}{3\tan\theta \cdot 3\sec\theta} 3\sec^2\theta d\theta = \int \frac{2}{3}\frac{\sec\theta}{\tan\theta}d\theta = \frac{2}{3}\int \csc\theta d\theta$ $= 9\arcsin\left(\frac{x}{3}\right) - x\sqrt{9-x^2} + C$

f. $\int \frac{5}{36+(x-1)^2}\,dx = \int \frac{5}{6^2+(x-1)^2}dx = 5\cdot\frac{1}{6}\tan^{-1}\left(\frac{x-1}{6}\right) + C$



let $x = 2\tan\theta$, $dx = 2\sec^2\theta d\theta$

g. $\int \frac{1}{\sqrt{4+x^2}}\,dx = \int \frac{2\sec^2\theta d\theta}{2\sec\theta} = \int \sec\theta d\theta = \ln|\sec\theta + \tan\theta| + C$

$= \ln\left|\frac{\sqrt{4+x^2}}{2} + \frac{x}{2}\right| + C$

h. $\int \frac{5x+14}{(x+1)(x^2-4)}\,dx = \int \frac{-3}{x+1} + \frac{1}{x+2} + \frac{2}{x-2}dx = -3\ln|x+1| + \ln|x+2| + 2\ln|x-2| + C$

$(x+2)(x-2)$

2. Write the equation in polar coordinates:

$x = r\cos\theta$, $y = r\sin\theta$

a. $x^2+y^2=4 \Rightarrow r^2\cos^2\theta + r^2\sin^2\theta = 4 \Rightarrow r^2 = 4 \Rightarrow r = 2$ (or $r=-2$)

b. $x^2+y^2=4x \Rightarrow r^2 = 4r\cos\theta \xRightarrow{r\neq 0} r = 4\cos\theta$.

c. $(x^2+y^2)^2 = 4xy$ $(r^2)^2 = 4r^2\cos\theta\sin\theta \xRightarrow{r\neq 0} r^2 = 4\cos\theta\sin\theta$

d. $x = 4y$ $r\cos\theta = 4r\sin\theta \Rightarrow \tan\theta = \frac{1}{4}$

3. Write the given equations in rectangular coordinates:

$x^2+y^2=r^2$, $\cos\theta = \frac{x}{r}$, $\sin\theta = \frac{y}{r}$

a. $r = -2\sin\theta$ $\Rightarrow r = -2\cdot\frac{y}{r} \Rightarrow r^2 = -2y \Rightarrow x^2+y^2 = -2y$

b. $r\cos\theta = 5$ $\Rightarrow r\cdot\frac{x}{r} = 5 \Rightarrow x = 5$

4. Recognize all types of polar graphs.

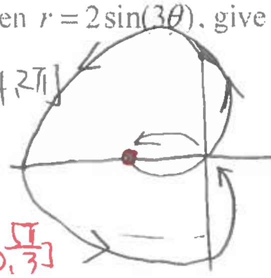
$r = \cos|m\theta|$: m petals, m is odd; 2m petals, m is even.

$r = a + b\cos\theta$: $|a|=|b|$, $|a|<|b|$, $|a|>|b|$

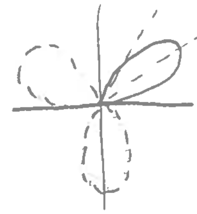
5. Given $r = 4 - 8\cos\theta$, give the formula (only) for the area inside the inner loop.

6. Given $r = 2\sin(3\theta)$, give the formula (only) for the area of one petal.

$[r,\theta] = [4,0]$ $[4,2\pi]$ $[0,\frac{\pi}{3}]$ $[4,\frac{\pi}{2}]$ $[12,\pi]$ $[4,\frac{3\pi}{2}]$ $[0,\frac{5\pi}{3}]$



$A = \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \frac{1}{2}(4-8\cos\theta)^2 d\theta$



$[r,\theta] = [0,0]$, $[2,\frac{\pi}{6}]$, $[0,\frac{\pi}{3}]$

$\int_0^{\frac{\pi}{3}} \frac{1}{2}(2\sin 3\theta)^2 d\theta$

7. Find the arc length for the following:

$\int_a^b \sqrt{1+(f'(x))^2}\,dx$ a. $f(x)=\frac{2}{3}(x-1)^{3/2} \quad x\in[1,2]$ $\Rightarrow \int_1^2 \sqrt{1+x-1}\,dx = \frac{2}{3}x^{\frac{3}{2}}\Big|_1^2 = \frac{2}{3}(2)^{\frac{3}{2}} - \frac{2}{3}$

$\int_a^b \sqrt{[x'(t)]^2+[y'(t)]^2}\,dt$ b. $x(t)=\sin(2t),\ y(t)=\cos(2t), \quad t\in\left[0,\frac{\pi}{2}\right]$ $\int_0^{\frac{\pi}{2}} \sqrt{(2\cos 2t)^2+(-2\sin 2t)^2}\,dt = \int_0^{\frac{\pi}{2}} \sqrt{4}\,dt = 2\cdot\frac{\pi}{2} = \pi$

$\int_a^b \sqrt{[r(\theta)]^2+(r'(\theta))^2}\,d\theta$ c. $r=2\sec(\theta), \quad t\in\left[0,\frac{\pi}{4}\right]$ $\Rightarrow \int_0^{\frac{\pi}{4}} \sqrt{4\sec^2\theta + 4\sec^2\theta\tan^2\theta}\,d\theta = \int_0^{\frac{\pi}{4}} 2\sec^2\theta\,d\theta = 2\tan\theta\Big|_0^{\frac{\pi}{4}}$ $= 2$

8. Find the equation of the tangent and the normal lines to the parametric curves at the given points:

a. $x(t)=-2\cos 2t,\ y(t)=4+2t,\ (-2,4)$ $m_{tangent} = \frac{y'}{x'} = \frac{2}{4\sin 2t}\Big|_{t=0} \Rightarrow x'=0 \Rightarrow x=-2$

b. $x(t)=3\cos(3t)+2t,\ y(t)=1+5t,\ (3,1)$ normal $\Rightarrow y=4$

9. Find the points (x, y) at which the curve $x(t)=3-4\sin(t),\quad y(t)=4+3\cos(t)$ has: (a) a horizontal tangent; (b) a vertical tangent. (a) $y'=0 \Rightarrow -3\sin t=0$ $t=0,\pi$. $(x,y)=(3,7)$ $(3,1)$

(b) $x'=0 \Rightarrow -4\cos t=0 \Rightarrow t=\frac{\pi}{2},\frac{3\pi}{2}$. $(x,y)=(-1,4)$ $(7,4)$

10. Give an equation relating x and y for the curve given parametrically by

$\cos^2 t+\sin^2 t=1$ a. $x(t)=-1+3\cos t \quad y(t)=1+2\sin t$

$\cosh^2 t-\sinh^2 t=1$ b. $x(t)=-1+3\cosh t \quad y(t)=1+2\sinh t$

c. $x(t)=-1+4e^t \quad y(t)=2+3e^{-t}$ $\frac{x+1}{4}=e^t,\ \frac{y-2}{3}=e^{-t} \Rightarrow \frac{x+1}{4}=\frac{3}{y-2}$

11. Find a parameterization for:

$x(t)=x_0+t(x_1-x_0)$ a. Line segment from (-1, 3) to (5, 4) (x_0 y_0 x_1 y_1)

$y(t)=y_0+t(y_1-y_0)$ b. Circle with radius 2 and center (2, -1) $2\cos t-2$, $2\sin t+1$

12. Write an expression for the nth term of the sequence:

a. 1, 4, 7, 10, ... $3n-2$

b. $2, -1, \frac{1}{2}, -\frac{1}{4}, \frac{1}{8}, \ldots$ $2^1, -2^0, 2^{-1}, -2^{-2}, 2^{-3}, \cdots$ $(-1)^{n+1}(2)^{2-n}$

13. Determine if the following sequences are monotonic. Also indicate if the sequence is bounded and if it is give the least upper bound and/or greatest lower bound.

$a_1=1$, $a_2=\frac{4}{3}$ increasing, $a_3=\frac{6}{4}=\frac{3}{2}$ increasing

a. $a_n=\frac{2n}{1+n}$ $\rightarrow 2$. increasing from 1 and finally tends to 2 $\Rightarrow$ bounded.

b. $a_n=\frac{\cos n}{n}$ $a_1=\cos 1$ $a_2=\frac{\cos 2}{2}$ $a_3=\frac{\cos 3}{3}$ $\Rightarrow$ not monotone but lub $=\cos 1$ glb $=\frac{\cos 3}{3}$

14. Determine if the following sequences converge or diverge. If they converge, give the limit.

a. $\left\{(-1)^n\left(\frac{n}{n+1}\right)\right\}$ Diverge

b. $\left\{\frac{6n^2-2n+1}{4n^2-1}\right\}$ leading coefficient $\Rightarrow \frac{3}{2}$.

c. $\left\{\frac{(n+2)!}{n!}\right\}$ $\frac{(n+2)!}{n!} = \frac{(n+2)(n+1)n!}{n!} = (n+2)(n+1) \Rightarrow$ Diverges.

d. $\left\{\frac{3}{e^n}\right\}$ $e^n \to \infty$ as $n \to \infty$ and $\frac{3}{x}$ is continuous $\Rightarrow \frac{3}{e^n} \to 0$

e. $\left\{\frac{4n+1}{n^2-3n}\right\}$ $\frac{P}{Q}$ and $\deg(P) < \deg Q \Rightarrow 0$

f. $\left\{\frac{e^n}{n^3}\right\}$ $\to$ diverge since e^n is faster than n^3

15. Determine the values of n which guarantee a theoretical error less than ε if the integral is estimated by the trapezoidal rule and then by Simpson's rule if $\varepsilon = 0.01$.

a. $\int_1^3 \left(\frac{1}{4}x^2 + 3x - 2\right) dx$

b. $\int_1^3 \cos(5x)dx$

Error for trapezoidal :

$$E_n^T = -\frac{(b-a)^3}{12n^2} f''(c)$$

Error for Simpson

$$E_n^S = -\frac{(b-a)^5}{2880n^4} f^{(4)}(c)$$

$2x \quad \frac{x}{\sqrt{9-x^2}}$

$2 \quad -\sqrt{9-x^2}$

0