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Calculus 1432
Quiz 6
February 21, 2014

Give the form of the partial fraction decomposition. Do NOT solve for the "A, B, etc". (1 points)

1. $$\frac{x+1}{\underbrace{(x^2+5x+6)}_{(x+2)(x+3)}(x^2+4)(x-3)^2} = \frac{A}{(x+2)} + \frac{B}{(x+3)} + \frac{Cx+D}{x^2+4} + \frac{E}{x-3} + \frac{F}{(x-3)^2}$$

Find the partial fraction decomposition. DO find the values of "A, B, etc". (2 points)

2. $$\frac{x+1}{\underbrace{x^3-x^2-6x}_{x(x-3)(x+2)}} = \frac{A}{x} + \frac{B}{x-3} + \frac{C}{x+2} = \frac{-\frac{1}{6}}{x} + \frac{\frac{4}{15}}{x-3} + \frac{-\frac{1}{10}}{x+2}.$$

let $f(x) = \frac{x+1}{x(x-3)(x+2)}$

$A = \lim_{x\to 0} x f(x) = -\frac{1}{6}$, $C = \lim_{x\to -2}(x+2)f(x) = \frac{-1}{-2(-5)} = -\frac{1}{10}$

$B = \lim_{x\to 3}(x-3)f(x) = \frac{4}{15}$

or $A(x-3)(x+2) + Bx(x+2) + Cx(x-3) = x+1$

$x=3 \Rightarrow 3B\cdot 5 = 4 \Rightarrow \boxed{B = \frac{4}{15}}$ $\quad x=-2$, $C(-2)(-5) = -1$

$x=0 \Rightarrow A(-3)(2) = 1 \Rightarrow \boxed{A = -\frac{1}{6}}$ $\quad \boxed{C = \frac{-1}{10}}$

Integrate: (3 points each)

3. $$\int \frac{1+x}{x^2+2x}\,dx = \int \frac{\frac{1}{2}}{x} + \frac{\frac{1}{2}}{x+2}\,dx = \frac{1}{2}\ln|x| + \frac{1}{2}\ln|x+2| + C$$

$f(x) = \frac{1+x}{x(x+2)} = \frac{A}{x} + \frac{B}{x+2}$

$A = \lim_{x\to 0} x f(x) = \frac{1}{2}$, $B = \lim_{x\to -2}(x+2)f(x) = \frac{1-2}{-2} = \frac{1}{2}$

or let $u = x^2+2x$, $du = 2x+2\,dx$

$\int \frac{1+x}{x^2+2x}dx = \frac{1}{2}\int \frac{du}{u} = \frac{1}{2}\ln|u| + C$

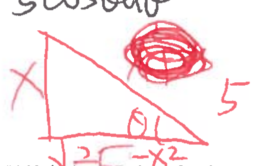
$= \frac{1}{2}\ln|x^2+2x| + C$

4. $$\int \frac{1}{x^2\sqrt{25-x^2}}\,dx = \int \frac{5\cos\theta\,d\theta}{25\sin^2\theta \cdot 5\cos\theta} = \int \frac{d\theta}{25\sin^2\theta} = \frac{1}{25}\int \csc^2\theta\,d\theta = -\frac{1}{25}\cot\theta + C$$

$= -\frac{1}{25}\cdot\frac{\sqrt{25-x^2}}{x} + C$

let $x = 5\sin\theta$

$dx = 5\cos\theta\,d\theta$



$\Rightarrow \cot\theta = \frac{\sqrt{25-x^2}}{x}$

e 5. (1 pt) Which of the following gives the most accurate estimate for definite integrals?

a. Left-endpoint estimate
b. Right-endpoint estimate
c. Midpoint estimate
d. Trapezoid Rule
e. Simpson's Rule

Using "error" to check: