

Name: Sol

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2 points each

1. At what rate r of continuous compounding does a sum of money triple in 5 years?

Initial: A_0 , $A(5) = 3A_0$. $A(t) = A_0 \cdot e^{rt}$. Find r .

$$3A_0 = A(5) = A_0 e^{r \cdot 5} \Rightarrow 3 = e^{r \cdot 5} \xrightarrow{\ln} \ln 3 = r \cdot 5 \Rightarrow r = \frac{\ln 3}{5}$$

2. $\int \sec 2x dx = \frac{1}{2} \int \sec u du = \frac{1}{2} |\sec u + \tan u| + C$

Let $u = 2x$,
 $du = 2dx$
 $\frac{du}{2} = dx$

$$= \frac{1}{2} |\sec 2x + \tan 2x| + C$$

$$\frac{12x^2}{5(4x^3-1)}$$

3. Find $f'(x)$ if $f(x) = \ln(\sqrt[5]{4x^3-1})$

or

$$f(x) = \ln((4x^3-1)^{\frac{1}{5}}) = \frac{1}{5} \ln(4x^3-1)$$

$$f'(x) = \frac{1}{5} \cdot \frac{12x^2}{4x^3-1} \cdot \frac{1}{(4x^3-1)^{\frac{4}{5}}} \cdot \frac{1}{(4x^3-1)^{\frac{1}{5}}}$$

$$f(x) = \frac{1}{5} \cdot \frac{12x^2}{4x^3-1} \cdot \frac{(\ln x)^3}{x} dx = \int \ln e^2 u^3 du = \int_0^2 u^3 du = \frac{u^4}{4} \Big|_0^2 = \frac{16}{4} = 4$$

Let $u = \ln x$
 $du = \frac{dx}{x}$

5. Use logarithmic differentiation to find y' given: $y = (x^2+1)^x$

① Take "ln", $\ln y = \ln(x^2+1)^x = x \ln(x^2+1)$

② Take " $\frac{d}{dx}$ ", $\frac{d}{dx}(\ln y) = \frac{d}{dx}(x \ln(x^2+1))$

↓ product rule

$$\frac{y'}{y} = \ln(x^2+1) + \frac{x}{x^2+1} \cdot 2x$$

③ $y' = (x^2+1)^x \left(\ln(x^2+1) + \frac{2x^2}{x^2+1} \right)$