Name: Sol

PSID:

Calculus 1432
Quiz 13
April 18, 2014

1 point each

For problems 1-6, determine whether each series converges absolutely, converges conditionally or diverges.

1. $\sum \frac{(-1)^k}{\sqrt{2k^5-1}}$ abs. convergent by Limit Comparison test (or P-series test) and compare it with $\sum \frac{1}{k^{\frac{5}{2}}}$

2. $\sum (-1)^k \left(\frac{5}{6}\right)^k$ abs. convergent by Root test (let $a_k = (\frac{5}{6})^k$, $\sqrt[k]{a_k} = \frac{5}{6} < 1$)

3. $\sum \frac{(-1)^k}{k-1}$ Conditional convergent by Alternating Series Test ($\frac{1}{k-1} \to 0$ as $k \to \infty$) (For $\sum \left|\frac{(-1)^k}{k}\right| = \sum \frac{1}{k}$, it diverges by p-series test)

4. $\sum (-1)^k \left(\frac{k}{k^3-2k}\right)$ abs. convergent by Limit Comparison test and compare it with $\sum \frac{k}{k^3} = \sum \frac{1}{k^2}$

5. $\sum \left(\frac{2k-1}{3k+5}\right)^k$ abs. convergent by Root test (let $a_k = (\frac{2k-1}{3k+5})^k$ and $\sqrt[k]{a_k} = (\frac{2k-1}{3k+5}) \to \frac{2}{3} < 1$ as $k \to \infty$)

6. $\sum \frac{(-1)^k k!}{(k+3)!}$ abs. convergent by Limit Comparison Test

" $\sum (-1)^k \cdot \frac{1}{(k+3)(k+2)(k+1)}$ and compare it with $\sum \frac{1}{k^3}$

7. Give the 3rd degree Taylor polynomial approximation and the Taylor series for $f(x) = e^x$ centered at $x = 0$.

$f(x) = e^x$, $f(0) = 1$

$f'(x) = e^x$, $f'(0) = 1$

$f''(x) = e^x$, $f''(0) = 1$

$f'''(x) = e^x$, $f'''(0) = 1$

Taylor polynomial in x:

$\Rightarrow f(0) + \frac{f'(0)}{1!}x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!}$

8. Give the 5th degree Taylor polynomial approximation and the Taylor series for $f(x) = \sin(x)$ centered at $x = 0$.

$f(x) = \sin x$, $f(0) = 0$

$f'(x) = \cos x$, $f'(0) = 1$

$f''(x) = -\sin x = -1 \cdot \sin x$, $f''(0) = 0$

$f'''(x) = -1 \cdot \cos x$, $f'''(0) = -1$

$f^{(4)}(x) = (-1)^2 \cdot \sin x$, $f^{(4)}(0) = 0$

$f^{(5)}(x) = (-1)^2 \cdot \cos x$, $f^{(5)}(0) = (-1)^2$

$\Rightarrow$ Taylor series of $f(x)$ in x

$0 + \frac{1}{1!}x + \frac{0}{2!}x^2 + \frac{-1}{3!}x^3 + \frac{0}{4!}x^4 + \frac{(-1)^2}{5!}x^5 = x + \frac{-1}{3!}x^3 + \frac{(-1)^2}{5!}x^5$

9. Give the 4th degree Taylor polynomial approximation and the Taylor series for $f(x) = \cos(x)$ centered at $x = 0$.

$f(x) = \cos x$, $f(0) = 1$

$f'(x) = (-1) \cdot \sin x$, $f'(0) = 0$

$f''(x) = (-1) \cdot \cos x$, $f''(0) = -1$

$f'''(x) = (-1)^2 \cdot \sin x$, $f'''(0) = 0$

$f^{(4)}(x) = (-1)^2 \cos x$, $f^{(4)}(0) = (-1)^2$

$\Rightarrow$ Taylor series of $f(x)$ in x

$\Rightarrow 1 + \frac{0}{1!}x + \frac{-1}{2!}x^2 + \frac{0}{3!}x^3 + \frac{(-1)^2}{4!}x^4 = 1 + \frac{-1}{2!}x^2 + \frac{(-1)^2}{4!}x^4$

10. Give the coefficient of x^6 for the 10th degree Taylor polynomial approximation to $\sin(x)$ centered at $x = 0$.

See Q8. We have $f^{(5)}(x) = (-1)^2 \cos x$, so $f^{(6)}(x) = (-1)^3 \sin x$.

and $f^{(6)}(0) = (-1)^3 \cdot \sin 0 = 0$ $\Rightarrow$ The coefficient of x^6 of Taylor series of $f(x)$ in x is $\frac{0}{6!} = 0$