Math 1432, Section 12869
Spring 2014
Homework Assignment 9
Due Date: 3/21/14 in Lab

Name: Sol.

ID:

Instructions

- Print out this file and complete the problems. You must do all the problems!
- If the problem is from the text, the section number and problem number are in parantheses.
- Use a blue or black pen or a pencil (dark).
- Write your solutions in the spaces provided. You must show work in order receive credit for a problem.
- Remember that your homework must be complete, neatly written and stapled.
- Submit the completed assignment to your Teaching Assistant in lab on the due date.
- If you do not do all of the problems, then your recitation quiz from the previous Friday will automatically become a ZERO.

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1. (Section 10.1, Problem 1)

$(0,2)$

$\text{lub}(0,2) = 2$

$\text{glb}(0,2) = 0.$

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2. (Section 10.1, Problem 2)

$\text{lub}[0,2] = 2$

$\text{glb}[0,2] = 0$

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3. (Section 10.1, Problem 3)

$\text{lub}(0,\infty)$ DNE

$\text{glb}(0,\infty) = 0$

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4. (Section 10.1, Problem 5)

$\{x \mid x^2 < 4\} = \{x \mid -2 < x < 2\}$

$(x^2 < 4 \Rightarrow x^2 - 4 < 0 \Rightarrow (x+2)(x-2) < 0)$

$\overset{+}{}\ {-2}\ \overset{-}{}\ 2\ \overset{+}{} \Rightarrow -2 < x < 2$

$\text{lub}(\{x \mid x^2 < 4\}) = 2$

$\text{glb}(\{x \mid x^2 < 4\}) = -2$

5. (Section 10.1, Problem 6)

$$\{x : |x-1|<2\} = \{x : -2<x-1<2\}$$
$$= \{x : -1<x<3\}$$

$\text{lub}(\{x : |x-1|<2\}) = 3$

$\text{glb}(\{x : |x-1|<2\}) = -1.$

6. (Section 10.1, Problem 10)

$$\{-1, -\frac{1}{2}, -\frac{1}{3}, -\frac{1}{4}, \ldots\} = \{-\frac{1}{n}\}_{n=1}^{\infty}$$

$\text{lub} = 0$

$\text{glb} = -1.$

7. (Section 10.1, Problem 13)

$\{x : \ln x < 1\}$ For the property of "ln" function we have "$x>0$"

and $\ln x < 1$ implies $x < e$

$\Rightarrow \quad 0 < x < e$

$\text{lub} = e$

$\text{glb} = 0$

8. (Section 10.1, Problem 14)

$\{x : \ln x > 0\}$ First. $x>0$

Forthermore, $\ln x > 0 \Rightarrow x > 1$.

$\Rightarrow x > 1$

lub DNE

$\text{glb} = 1.$

9. (Section 10.2, Problem 1)

$$\underbrace{2,5}_{3}\underbrace{,8}_{3}\underbrace{,11}_{3}\underbrace{,14}_{3}$$

$a_n = 2+(n-1)3$

$= 3n-1.$

$a_1 = 2 = 2+0\cdot 3$

$a_2 = 5 = 2+3 = 2+1\cdot 3$

$a_3 = 8 = 2+3+3 = 2+2\cdot 3$

$a_4 = 11 = 2+3+3+3 = 2+3\cdot 3$ ....

10. (Section 10.2, Problem 2)

$2, 0, 2, 0, 2, \ldots$

$\cancel{-2 \; +2 \; -2 \; +2}$

$\cancel{a_1 = 2}$

$\cancel{a_2 = 0 = 2-2}$

$\cancel{a_3 = 2 = 2-2+2}$

$\cancel{a_4 = 0 = 2-2+2-2}$

$a_1 = 2 = 1+1 = 1-(-1)^1$

$a_2 = 0 = 1-1 = 1-(-1)^2$

$a_3 = 2 = 1+1 = 1-(-1)^3$

$a_4 = 0 = 1-1 = 1-(-1)^4$

$\vdots$

$a_n = 1-(-1)^n$

11. (Section 10.2, Problem 3)

$1, -\frac{1}{3}, \frac{1}{5}, -\frac{1}{7}, \frac{1}{9}$ $\Rightarrow$ $a_n = \frac{1}{(-1)^{n-1}(2n-1)}$ or $\frac{(-1)^{n-1}}{2n-1}$

($1, +3, 5, +7, 9$ $\Rightarrow$ odd numbers $\Rightarrow 2n-1$)

($1, -3, 5, -7, 9$ $\Rightarrow$ alternative odd number $\Rightarrow (-1)^{n-1}(2n-1)$)

12. (Section 10.2, Problem 4)

$\frac{1}{2}, \frac{3}{4}, \frac{7}{8}, \frac{15}{16}, \frac{31}{32}$

$= 1-\frac{1}{2}, 1-\frac{1}{4}, 1-\frac{1}{8}, 1-\frac{1}{16}, 1-\frac{1}{32}$ $\Rightarrow$ $a_n = 1-\frac{1}{2^n}$

($2, 4, 8, 16, 32 \Rightarrow 2^n$)

13. (Section 10.2, Problem 5)

$2, \frac{5}{2}, \frac{10}{3}, \frac{17}{4}, \frac{26}{5}$ $\Rightarrow$ $a_n = \frac{1+n^2}{n}$

$1+\sum_{k=1}^{n}[1+2(k-1)]$

$2, 5, 10, 17, 26$
differences $3, 5, 7, 9$ $\Rightarrow 1+2(n-1)$
differences $2, 2, 2$ $\Rightarrow 2$

backward

$1+\sum_{k=1}^{n}(2k-1) = 1+\frac{(1+2n-1)n}{2} = 1+n^2$

or $2, 5, 10, 17, 26$
$= 1+1, 4+1, 9+1, 16+1, 25+1 \Rightarrow n^2+1$

14. (Section 10.2, Problem 6)

$-\frac{1}{4}, \frac{2}{9}, -\frac{3}{16}, \frac{4}{25}, -\frac{5}{36}$ $\Rightarrow \frac{(-1)^n n}{(\ )^2}$

$\Rightarrow \frac{(-1)^n n}{(n+1)^2}$

15. (Section 10.2, Problem 9)

$a_n = \frac{2}{n}$ $\quad a_{n+1} = \frac{2}{n+1}$ $\Rightarrow a_n > a_{n+1}$

decreasing

$a_1 = 2$
$a_2 = 1$
$a_3 = \frac{2}{3}$
$a_4 = \frac{2}{4}$ ... 0

bounded below by 0
bounded above by 2

16. (Section 10.2, Problem 11)

$a_n = \frac{n+(-1)^n}{n}$

$a_1 = \frac{1+(-1)}{1} = 0$ increasing
$a_2 = \frac{2+(-1)^2}{2} = \frac{3}{2}$ decreasing
$a_3 = \frac{3+(-1)^3}{3} = \frac{2}{3}$ increasing
$a_4 = \frac{4+(-1)^4}{4} = \frac{5}{4}$

not monotone

$a_n = \frac{n+(-1)^n}{n} = \frac{n}{n}+\frac{(-1)^n}{n} = 1+\frac{(-1)^n}{n}$

$0 \quad \frac{2}{3} \quad 1 \quad \frac{5}{4} \quad \frac{3}{2}$

bdd below by 0 above by $\frac{3}{2}$

$$\left(\frac{n+1}{n}\right)^2 = \left(1+\frac{1}{n}\right)^2$$

$a_1 = 2^2 = 4 \leftarrow$ above

$a_2 = (1+\frac{1}{2})^2 = \frac{9}{4}$ decreasing

$1 \leftarrow$ below

17. (Section 10.2, Problem 13)

$a_n = (0.9)^n$

$$\frac{a_{n+1}}{a_n} = \frac{(0.9)^{n+1}}{(0.9)^n} = 0.9 < 1$$

$a_1 = 0.9$

$a_2 = 0.81$

$a_3 = 0.729$

$\vdots$

0

$\Rightarrow a_{n+1} < a_n \Rightarrow$ decreasing

bdd below by 0

above by 0.9

20. (Section 10.2, Problem 23)

$a_n = \frac{(n+1)^2}{n^2}$

$$\cancel{\frac{a_{n+1}}{a_n} = \frac{(n+2)^2}{(n+1)^2} \cdot \frac{n^2}{(n+1)^2} =}$$

$$a_{n+1} - a_n = \frac{(n+2)^2}{(n+1)^2} - \frac{(n+1)^2}{n^2} = \frac{n^2(n+2)^2 - (n+1)^4}{(n+1)^2 n^2}$$

$$= \frac{n^4+4n^3+4n^2 - n^4 - 4n^3 - 6n^2 - 4n - 1}{(n+1)^2 n^2} = \frac{-4n^3 - 2n^2 - 4n + 3}{(n+1)^2 n^2} < 0$$

$\Rightarrow$ decreasing

18. (Section 10.2, Problem 15)

$a_n = \frac{n^2}{n+1}$

$$\cancel{\frac{a_{n+1}}{a_n} = \frac{(n+1)^2}{n+2} \cdot \frac{n+1}{n^2}}$$

$a_1 = \frac{1}{2}$ increasing

$a_2 = \frac{4}{3}$ increasing

$a_3 = \frac{9}{4}$

$$a_{n+1} - a_n = \frac{(n+1)^2}{n+2} - \frac{n^2}{n+1} = \frac{(n+1)^3 - n^2(n+2)}{(n+2)(n+1)}$$

$$= \frac{n^3+3n^2+3n+1 - n^3 - 2n^2}{(n+1)(n+2)} = \frac{n^2+3n+1}{(n+1)(n+2)} > 0$$

$\Rightarrow a_{n+1} > a_n$

below $\frac{1}{2}$

~~not above~~

21. (Section 10.2, Problem 24)

$a_n = (-1)^n \sqrt{n}$

alternating

$-\sqrt{3}$ -1 0 $\sqrt{2}$ $\sqrt{4}$

$\Rightarrow$ not monotone

~~above~~

~~below~~

19. (Section 10.2, Problem 20)

$$a_n = \frac{n^2}{\sqrt{n^2+1}} = \sqrt{\frac{n^4}{n^2+1}} = \sqrt{\frac{n^4+n-n}{n^3+1}}$$

$$= \sqrt{\frac{n(n^3+1)}{n^3+1} - \frac{n}{n^3+1}} = \sqrt{n - \frac{n}{n^3+1}}$$

$a_1 = \frac{1}{\sqrt{2}}$ increasing

$a_2 = \frac{4}{\sqrt{9}} = \frac{4}{3}$ increasing

$a_3 = \frac{9}{\sqrt{28}}$

$\Rightarrow$ increasing

below $\frac{1}{\sqrt{2}}$

~~above~~

22. (Section 10.2, Problem 26)

$a_n = \ln\left(\frac{n+1}{n}\right) = \ln(n+1) - \ln n$

$$a_{n+1} - a_n = \ln\left(\frac{n+2}{n+1}\right) - \ln\left(\frac{n+1}{n}\right)$$

$$= \ln\left(\frac{n+2}{n+1} \cdot \frac{n}{n+1}\right)$$

$$= \ln\left(\frac{n^2+2n}{n^2+2n+1}\right) < 0 \Rightarrow \text{decreasing}$$

$\because \frac{n^2+2n}{n^2+2n+1} < 1$

above

$a_1 = \ln(2)$

$a_2 = \ln(\frac{3}{2})$ decreasing

$\vdots$

$\ln 1 = 0 \rightarrow$ below

29. (Section 10.3, Problem 3)

$$a_n = \frac{(-1)^n}{n}$$

$$\left|\frac{(-1)^n}{n} - 0\right| = \left|\frac{1}{n}\right| < \varepsilon \text{ as } n \geq K.$$

limit 0

30. (Section 10.3, Problem 4)

$$a_n = \sqrt{n}. \Rightarrow \text{ limit DNE.}$$

31. (Section 10.3, Problem 5)

$$a_n = \frac{n-1}{n}$$

$$|a_n - 1| = \left|\frac{n-1}{n} - 1\right| = \left|\frac{1}{n}\right| < \varepsilon \text{ as } n \geq K.$$

$$\Rightarrow a_n \to 1 \text{ as } n \to \infty$$

32. (Section 10.3, Problem 6)

$$a_n = \frac{n+(-1)^n}{n}$$

$$|a_n - 1| = \left|\frac{n+(-1)^n}{n} - 1\right| = \left|\frac{(-1)^n}{n}\right| = \left|\frac{1}{n}\right| < \varepsilon \text{ as } n \geq K$$

$$\Rightarrow a_n \to 1. \text{ as } n \to \infty$$

33. (Section 10.3, Problem 7)

$$a_n = \frac{n+1}{n^2}$$

$$|a_n - 0| = \left|\frac{n+1}{n^2}\right| = \left|\frac{1}{n} + \frac{1}{n^2}\right| < \left|\frac{1}{n}\right| + \left|\frac{1}{n^2}\right| < \varepsilon \text{ as } n \geq K$$

$$a_n \to 0 \text{ as } n \to \infty$$

34. (Section 10.3, Problem 8)

$$a_n = \sin\frac{\pi}{2n} \to 0 \text{ as } n \to \infty$$

since $\frac{\pi}{2n} \to 0$ and $\sin x$ is continuous

23. (Section 10.2, Problem 27)

$a_n = (-1)^{2n+1}\sqrt{n}$

$a_1 = (-1)^3\sqrt{1} = -\sqrt{1} = -1 \leftarrow$ above

$a_2 = (-1)^5\sqrt{2} = -\sqrt{2}$

$a_3 = (-1)^7\sqrt{3} = -\sqrt{3}$ $\downarrow$ decreasing

$\vdots$

$a_n = -\sqrt{n}$

~~below~~

24. (Section 10.2, Problem 31)

$a_n = \sin\frac{\pi}{n+1}$

$a_1 = \sin\frac{\pi}{2} = 1 \rightarrow$ above

$a_2 = \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

$a_3 = \sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$ $\downarrow$ decreasing.

$a_4 = \sin\frac{\pi}{5}$

$\rightarrow 0 =$ below

25. (Section 10.2, Problem 36)

$a_n = \cos n\pi$

above: 1, below $= -1$.

$a_1 = \cos\pi = -1$

$a_2 = \cos 2\pi = 1$ alternating

$a_3 = \cos 3\pi = -1$ $\Downarrow$

$a_4 = \cos 4\pi = 1$ not monotone

26. (Section 10.2, Problem 45)

$a_1 = 1 \quad a_{n+1} = \frac{1}{n+1}a_n$

$a_2 = \frac{1}{1+1}a_1 = \frac{1}{2}$

$a_3 = \frac{1}{2+1}a_2 = \frac{1}{3}\cdot\frac{1}{2} = \frac{1}{6} = \frac{1}{3!}$

$a_4 = \frac{1}{3+1}a_3 = \frac{1}{4}\cdot\frac{1}{6} = \frac{1}{24} = \frac{1}{4!}$

$a_5 = \frac{1}{5!} \quad a_6 = \frac{1}{6!}$

$\Rightarrow a_n = \frac{1}{n!}$

27. (Section 10.2, Problem 48)

see below

$a_1 = 1. \quad a_{n+1} = a_n + 2$

$n=1$: $a_2 = a_1 + 2 = 1+2 = 3$

$n=2$: $a_3 = a_2 + 2 = 3+2 = 5$

$n=3$: $a_4 = a_3 + 2 = 7$

$n=4$: $a_5 = 9$

$n=5$: $a_6 = 11$

1, 3, 5, 7, 9, 11

$\Rightarrow a_n = 2n-1.$

$(n = 1, 2, 3 \ldots)$

28. (Section 10.2, Problem 55)

$a_1 = 1, \ a_2 = 3 \quad a_{n+1} = 2a_n - a_{n-1} \quad n \geq 2$

$n=2$: $a_3 = 2a_2 - a_1 = 2\cdot 3 - 1 = 5.$

$n=3$: $a_4 = 2a_3 - a_2 = 10 - 3 = 7.$

$n=4$: $a_5 = 2a_4 - a_3 = 14 - 5 = 9$

$n=5$: $a_6 = 2a_5 - a_4 = 18 - 7 = 11.$

1, 3, 5, 7, 9, 11

$\Rightarrow \boxed{a_n = 2n-1.}$

140

41. (Section 10.3, Problem 21)

$a_n = \cos n\pi$.

~~$\because \cos x$ is conti. and~~

$a_1 = \cos\pi = -1$.
$a_2 = \cos 2\pi = 1$.
$a_3 = \cos 3\pi = -1$.

$\Rightarrow -1, 1, -1, 1, -1, \cdots$

$\Rightarrow$ limit DNE.

42. (Section 10.3, Problem 23)

$a_n = e^{\frac{1}{\sqrt{n}}}$.

$\because e^x$ is continuous and $\frac{1}{\sqrt{n}} \to 0$ as $n \to \infty$

$\Rightarrow a_n \to e^0 = 1$ as $n \to \infty$

43. (Section 10.3, Problem 25)

$a_n = \ln n - \ln(n+1) = \ln\left(\frac{n}{n+1}\right)$

$\because \ln x$ is conti. and $\frac{n}{n+1} \to 1$ as $n \to \infty$

$\therefore a_n \to \ln 1 = 0$ as $n \to \infty$

44. (Section 10.3, Problem 28)

$a_n = \frac{1}{n} - \frac{1}{n+1} = \frac{n+1-n}{n(n+1)} = \frac{1}{n(n+1)} = \frac{1}{n} \cdot \frac{1}{n+1}$

$0 < a_n < \frac{1}{n \cdot n}$

$\boxed{\frac{1}{n+1} < \frac{1}{n}}$

$0 \qquad 0$

$\Rightarrow a_n \to 0$ as $n \to \infty$.

45. (Section 10.3, Problem 29)

$(1+\frac{1}{n})^{2n} = \left[(1+\frac{1}{n})^n\right]^2$

$\because \boxed{(1+\frac{1}{n})^n \to e}$ as $n \to \infty$ as x^2 is conti.

$\therefore (1+\frac{1}{n})^{2n} \to e^2$ as $n \to \infty$

46. (Section 10.3, Problem 32)

$a_n = 2\ln 3n - \ln(n^2+1)$

$= \ln(3n)^2 - \ln(n^2+1) = \ln\left(\frac{(3n)^2}{n^2+1}\right) = \ln\left(\frac{9n^2}{n^2+1}\right)$

$\because \ln x$ is conti. and $\frac{9n^2}{n^2+1} \to 9$ as $n \to \infty$

$\therefore a_n \to \ln 9$ as $n \to \infty$

35. (Section 10.3, Problem 9)

$a_n = \frac{2^n}{4^n+1}$

$0 < a_n = \frac{2^n}{4^n+1} < \frac{2^n}{4^n} \quad \forall n$

as $n\to\infty$ $\downarrow$ 0 $\qquad \frac{2^n}{4^n} = (\frac{1}{2})^n \to 0 \ (\because \frac{1}{2}<1)$

$\Longrightarrow a_n \to 0$ as $n\to\infty$

36. (Section 10.3, Problem 12)

$a_n = \frac{4n}{\sqrt{n^2+1}} = \frac{\sqrt{16n^2}}{\sqrt{n^2+1}} = \sqrt{\frac{16n^2 \cdot \frac{1}{n^2}}{(n^2+1)\frac{1}{n^2}}} = \frac{\sqrt{16}}{\sqrt{1+\frac{1}{n^2}}} \to 4$ as $n\to\infty$.

$\because 1+\frac{1}{n^2}\to 1$ and $\sqrt{x}$ is continuous

$\therefore \sqrt{1+\frac{1}{n^2}} \to \sqrt{1}=1$ as $n\to\infty$

37. (Section 10.3, Problem 15)

$a_n = \tan\frac{n\pi}{4n+1}$

since $\tan x$ is ~~###~~ continuous

and $\frac{n\pi}{4n+1} \to \frac{\pi}{4}$ as $n\to\infty$

$\Rightarrow a_n \to \tan\frac{\pi}{4} = 1$ as $n\to\infty$

38. (Section 10.3, Problem 17)

$a_n = \frac{(2n+1)^2}{(3n-1)^2} = \left(\frac{2n+1}{3n-1}\right)^2$

$\because x^2$ is conti. and $\frac{2n+1}{3n-1} \to \frac{2}{3}$ as $n\to\infty$

$\Rightarrow a_n \to \left(\frac{2}{3}\right)^2$ as $n\to\infty$.

39. (Section 10.3, Problem 18)

$\ln\left(\frac{2n}{n+1}\right)$

$\because \ln x$ is conti and $\frac{2n}{n+1} \to 2$ as $n\to\infty$

$\Rightarrow \ln\left(\frac{2n}{n+1}\right) \to \ln 2$ as $n\to\infty$

40. (Section 10.3, Problem 19)

$a_n = \frac{n^2}{\sqrt{2n^4+1}} = \sqrt{\frac{n^4}{2n^4+1}} = \sqrt{\frac{n^4\cdot\frac{1}{n^4}}{(2n^4+1)\cdot\frac{1}{n^4}}} = \frac{1}{\sqrt{2+\frac{1}{n^4}}}$

since $\sqrt{x}$ is conti. and $2+\frac{1}{n^4} \to 2$ as $n\to\infty$

$\Rightarrow \sqrt{2+\frac{1}{n^4}} \to \sqrt{2}$

$\Rightarrow a_n \to \frac{1}{\sqrt{2}}$ as $n\to\infty$

47. (Section 10.3, Problem 55)

$a_1=1. \quad a_{n+1}=\frac{1}{n+1}a_n$

$\boxed{n=1}$ $a_2=\frac{1}{1+1}a_1=\frac{1}{2}=\frac{1}{2!}$

$a_3=\frac{1}{2+1}a_2=\frac{1}{3}\cdot\frac{1}{2}=\frac{1}{3!}$

$a_4=\frac{1}{3+1}a_3=\frac{1}{4}\cdot\frac{1}{3}\cdot\frac{1}{2}=\frac{1}{4!}$

$\Rightarrow a_n=\frac{1}{n!}\to 0$ as $n\to\infty$

(Say $0\le a_n\le\frac{1}{n}$. and by Pinching Thm)

48. (Section 10.3, Problem 57)

$a_1=1$

$a_{n+1}=1-a_n$

$a_1=1$

$a_2=1-a_1=0$

$a_3=1-a_2=1-0=1$

$a_4=1-a_3=1-1=0$

$\Rightarrow 1,0,1,0,1,0$

No limit!

27. $a_1=1, \quad a_{n+1}=\frac{1}{2}a_n+1$

$n=1$ $a_2=\frac{1}{2}a_1+1=\frac{1}{2}+1=\frac{1}{2^1}+\frac{1}{2^0}$

$n=2$ $a_3=\frac{1}{2}a_2+1=\frac{1}{2}(\frac{1}{2}+1)+1=\frac{1}{2^2}+\frac{1}{2}+\frac{1}{2^0}$

$n=3$ $a_4=\frac{1}{2}a_3+1=\frac{1}{2}(\frac{1}{2^2}+\frac{1}{2^1}+\frac{1}{2^0})+1$

$=\frac{1}{2^3}+\frac{1}{2^2}+\frac{1}{2^1}+\frac{1}{2^0}$

$\vdots$

$a_n=\frac{1}{2^{n-1}}+\frac{1}{2^{n-2}}+\cdots+\frac{1}{2^2}+\frac{1}{2^1}+\frac{1}{2^0}$

a_n is a series with first term $\frac{1}{2^0}=1$ and common ratio "$\frac{1}{2}$". Thus

$a_n=\frac{1\cdot(1-(\frac{1}{2})^n)}{1-\frac{1}{2}}=\frac{1\cdot(1-(\frac{1}{2})^n)}{\frac{1}{2}}=2\cdot(1-(\frac{1}{2})^n)$

Formula of a sum of a common ratio sequence.

(or call it "geometric sequence")

If a sequence has first term "a" and common ratio "r"

we have $\{a, ar, ar^2, ar^3, \ldots\}$

Then $a+ar+ar^2+\cdots+ar^n$

$$=\begin{cases}\frac{a(r^{n+1}-1)}{r-1}, & \text{If } |r|>1\\ \frac{a(1-r^{n+1})}{1-r}, & \text{If } |r|<1\end{cases}$$

The proof of this formula: (let $|r|<1$).

Let $S = a + ar + ar^2 + \cdots + ar^n$, — (1)

Then $rS = ar + ar^2 + ar^3 + \cdots + ar^{n+1}$ — (2)

Cancel out

Do subtraction,

(1) − (2) $\Rightarrow$

$$S - rS = a - ar^{n+1}$$

$$\Rightarrow (1-r)S = a(1-r^{n+1})$$

$$\Rightarrow S = \frac{a(1-r^{n+1})}{1-r}$$

(When $|r|>1$, we can do $rS - S$.)