Math 1432, Section 12869
Spring 2014
Homework Assignment 8
Due Date: 3/17/14 in Lab

Name: Sol

ID:

Instructions

- Print out this file and complete the problems. You must do all the problems!
- If the problem is from the text, the section number and problem number are in parantheses.
- Use a blue or black pen or a pencil (dark).
- Write your solutions in the spaces provided. You must show work in order receive credit for a problem.
- Remember that your homework must be complete, neatly written and stapled.
- Submit the completed assignment to your Teaching Assistant in lab on the due date.
- If you do not do all of the problems, then your recitation quiz from the previous Friday will automatically become a ZERO.

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1. (Section 9.6, Problem 1)

$x(t)=t^2,\ y(t)=2t+1 \Rightarrow \frac{y-1}{2}=t$

$\Rightarrow x=\left(\frac{y-1}{2}\right)^2$

$\left(\text{or } x=\frac{(y-1)^2}{4} \Rightarrow (y-1)^2-4x=0\right)$

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2. (Section 9.6, Problem 3)

$x(t)=t^2,\ y(t)=4t^4+1 = 4(t^2)^2+1$

$\Rightarrow y=4x^2+1$

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3. (Section 9.6, Problem 5)

$x(t)=2\cos t \quad y(t)=3\sin t$

(* Use $\boxed{\cos^2 t+\sin^2 t=1}$ *)

$\Rightarrow \frac{x}{2}=\cos t,\quad \frac{y}{3}=\sin t$

$\Rightarrow \left(\frac{x}{2}\right)^2+\left(\frac{y}{3}\right)^2=1$

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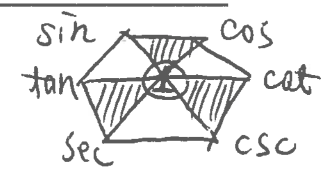
4. (Section 9.6, Problem 6)

$x(t)=\sec^2 t,\ y(t)=2+\tan t$

(* Use $\boxed{\tan^2 t+1=\sec^2 t}$)



$\Rightarrow y-2=\tan t$

$\Rightarrow (y-2)^2+1=x$

5. (Section 9.6, Problem 8)

$x = 2 - \sin t$, $y(t) = \cos t$

$x - 2 = \sin t$

$\Rightarrow (x-2)^2 + y^2 = 1$

(or $(2-x)^2 + y^2 = 1$)

6. (Section 9.6, Problem 9)

$x(t) = \sin t$, $y(t) = 1 + \cos^2 t$

$\Rightarrow y - 1 = \cos^2 t$

$\Rightarrow x^2 + (y-1) = 1$

7. (Section 9.6, Problem 10)

$x(t) = e^t$, $y(t) = 4 - e^{2t}$

$\Rightarrow -y + 4 = +e^{2t} = (e^t)^2$

$\Rightarrow -y + 4 = x^2$

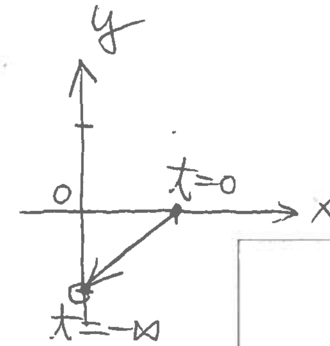
8. (Section 9.6, Problem 13)

$x(t) = e^{2t}$, $y(t) = e^{2t} - 1$

$\Rightarrow y = x - 1$

$t \le 0$

t	$x(t)$	$y(t)$
$t=0$	1	0
$t=-1$	$e^{-2} = \frac{1}{e^2}$	$\frac{1}{e^2} - 1$
$-\infty$	0	-1



9. (Section 9.6, Problem 17)

$x(t) = 3 + 2t$, $y(t) = 5 - 4t$

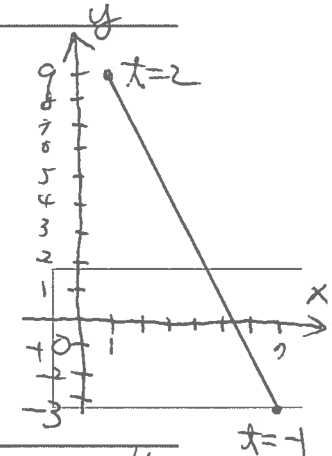
$\Rightarrow x - 3 = 2t$, $y - 5 = -4t = -2(2t)$

$y - 5 = -2(x-3)$ $-1 \le t \le 2$

$\Rightarrow \boxed{2x + y = 1}$

$\Rightarrow$ (a line)

t	$x(t)$	$y(t)$
2	7	-3
-1	1	9



10. (Section 9.6, Problem 18)

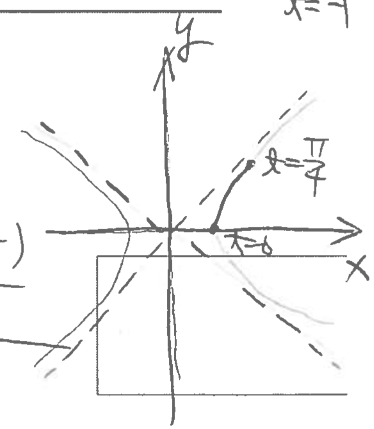
$x(t) = \sec t = \frac{1}{\cos t}$, $y(t) = \tan t$

$\Rightarrow y^2 + 1 = x^2$ $0 \le t \le \frac{\pi}{4}$

$\Rightarrow x^2 - y^2 = 1$

(hyperbolic curve)

t	$x(t)$	$y(t)$
0	1	0
$\frac{\pi}{4}$	$\frac{2}{\sqrt{2}} = \sqrt{2}$	1



11. (Section 9.6, Problem 20)

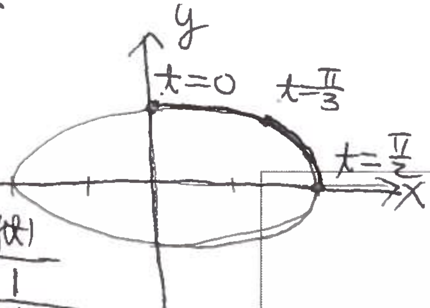
$x(t) = 2\sin t$, $y(t) = \cos t$ $\quad 0 \le t \le \frac{\pi}{2}$.

$\frac{x}{2} = \sin t$

$\left(\frac{x}{2}\right)^2 + y^2 = 1$

(elliptic curve)

t	$x(t)$	$y(t)$
0	0	1
$\frac{\pi}{3}$	$\sqrt{3}$	$\frac{1}{2}$
$\frac{\pi}{2}$	2	0



12. (Section 9.6, Problem 23)

~~$x(t) = \cos(At+\theta)$, $y(t) = \sin(At+\theta)$~~

Formula

$x(t) = \cos(at+\theta)$, $y(t) = \sin(at+\theta)$ counterclockwise

$x(t) = \cos(-at+\theta)$, $y(t) = \sin(at+\theta) \Rightarrow$ clockwise.

a: period. θ: starting angle. (as $t=0$)

(a) Traverses circle once $\Rightarrow \boxed{2\pi = a}$

begins at $(0,1) \xrightarrow{\text{at } t=0} (0,1) = (x(0), y(0)) = (\cos\theta, \sin\theta) \Rightarrow \boxed{\theta = \frac{\pi}{2}}$.

~~13. (Section 9.6, Problem 27)~~

$\Rightarrow x(t) = \cos(2\pi t + \frac{\pi}{2})$, $y(t) = \sin(2\pi t + \frac{\pi}{2})$, $t \in [0,1]$

(or $x = \cos 2\pi t \cos\frac{\pi}{2} - \sin 2\pi t \cdot \sin\frac{\pi}{2} = -\sin 2\pi t$, $y(t) = \sin 2\pi t \cos\frac{\pi}{2} + \cos 2\pi t \sin\frac{\pi}{2} = \cos 2\pi t$)

(b) begin at $(0,1) \Rightarrow \boxed{\theta = \frac{\pi}{2}}$. Traverses circle twice $\Rightarrow \boxed{a = 4\pi}$.

$x(t) = \cos(-4\pi t + \frac{\pi}{2}) = \sin 4\pi t$, $y(t) = \sin(-4\pi t + \frac{\pi}{2}) = \cos 4\pi t$ $\quad t \in [0,1]$

(c) quarter $\Rightarrow \boxed{\frac{\pi}{2} = a}$ from $(1,0) \to (0,1) \Rightarrow$ counter clockwise

and $(1,0) = (x(0), y(0)) \Rightarrow \boxed{\theta = 0}$

$x(t) = \cos(\frac{\pi}{2}t)$, $y(t) = \sin(\frac{\pi}{2}t)$ $\quad t \in [0,1]$

13. Way from the class $\quad (3,7) \to (8,5)$ $\quad x_0, y_0 \quad x_1, y_1$

$x(t) = 3 + t(8-3) = 3+5t$

$y(t) = 7 + t(5-7) = 7-2t$ $\quad t \in [0,1]$

14. (Section 9.6, Problem 28)

Way from the class $\quad (2,6) \to (6,3)$

$x(t) = 2 + t(6-2) = 2+4t$ $\quad t \in [0,1]$

$y(t) = 6 + t(3-6) = 6-3t$

or Method 2 $\quad (x(t), y(t)) = \boxed{t(x_1, y_1) + (1-t)(x_0, y_0)}$

$= t(6,3) + (1-t)(2,6)$

$= (2+4t, 6-3t)$ $\quad t \in [0,1]$

15. (Section 9.7, Problem 3)

$x(t) = 2t$, $y(t) = \cos \pi t$. $t=0$

point: $(x(0), y(0)) = (0, 1)$.

Slope: $\left.\frac{y'(t)}{x'(t)}\right|_{t=0} = \left.\frac{-\pi \sin \pi t}{2}\right|_{t=0} = 0 \Rightarrow$ horizontal line.

$\Rightarrow y = 1$.

16. (Section 9.7, Problem 4)

$x(t) = 2t-1$, $y(t) = t^4$, $t=1$

point: $(x(1), y(1)) = (1,1)$

Slope: $\left.\frac{y'(t)}{x'(t)}\right|_{t=1} = \left.\frac{4t^3}{2}\right|_{t=1} = \frac{4}{2} = 2$

$(y-1) = 2(x-1)$.

(d) three-quarter $\Rightarrow \boxed{\frac{3\pi}{2} = a}$ $\quad (1,0) \to (0,1)$ $\quad$ clockwise

$\Rightarrow \boxed{\theta = 0}$

$x(t) = \cos(-\frac{3\pi}{2}t)$ $\quad y(t) = \sin(-\frac{3\pi}{2}t)$ $\quad t \in [0,1]$

17. (Section 9.7, Problem 6)

$X(t)=\frac{1}{t},\ y(t)=t^2+1,\ t=1.$

point $(X(1),y(1))=(1,2)$

slope $\left.\frac{y'(t)}{x'(t)}\right|_{t=1}=\left.\frac{2t}{-\frac{1}{t^2}}\right|_{t=1}=-2t^3\Big|_{t=1}=-2.$

tangent line: $y-2=-2(X-1)$

18. (Section 9.7, Problem 7)

$X(t)=\cos^3 t,\ y(t)=\sin^3 t \qquad t=\frac{\pi}{4}.$

point $(X(\frac{\pi}{4}),y(\frac{\pi}{4}))=(\frac{\sqrt{2}}{4},\frac{\sqrt{2}}{4}).$

slope $\left.\frac{y'(t)}{X'(t)}\right|_{t=\frac{\pi}{4}}=\left.\frac{3\cdot\sin^2 t\cdot\cos t}{3\cos^2 t\cdot(-\sin t)}\right|_{t=\frac{\pi}{4}}=\left.\frac{\sin t}{-\cos t}\right|_{t=\frac{\pi}{4}}=-1$

tangent line: $(y-\frac{\sqrt{2}}{4})=-(X-\frac{\sqrt{2}}{4}).$

19. (Section 9.7, Problem 9)

$X=r\cos\theta,\ y=r\sin\theta,$ and $r=4-2\sin\theta,\ \theta=0$

$\Rightarrow X(\theta)=(4-2\sin\theta)(\cos\theta),\ y(\theta)=(4-2\sin\theta)\sin\theta$

point $(X(0),y(0))=(4,0).$

slope $\left.\frac{y'(\theta)}{X'(\theta)}\right|_{\theta=0}=\left.\frac{(-2\cos\theta)\sin\theta+(4-2\sin\theta)(\cos\theta)}{(-2\cos\theta)\cos\theta+(4-2\sin\theta)(-\sin\theta)}\right|_{\theta=0}$

$=\frac{0+4}{-2+0}=-2$

tangent line. $y=-2(X-4)$

20. (Section 9.7, Problem 10)

$r=4\cos 2\theta,\ \theta=\frac{\pi}{2}$

$X(\theta)=(4\cos2\theta)\cos\theta,\ y(\theta)=(4\cos2\theta)\sin\theta$

point $(X(\frac{\pi}{2}),y(\frac{\pi}{2}))=(0,-4).$

slope $\left.\frac{y'(\theta)}{x'(\theta)}\right|_{\theta=\frac{\pi}{2}}=\left.\frac{-8\sin2\theta\sin\theta+4\cos2\theta\cos\theta}{-8\sin2\theta\cos\theta+4\cos2\theta(-\sin\theta)}\right|_{\theta=\frac{\pi}{2}}=\frac{0}{4}=0$

$\Rightarrow$ horizontal line

$\Rightarrow y=-4$

21. (Section 9.7, Problem 15)

$y=x^3,$ let $x(t)=t,\ y(t)=t^3$

and $[x'(t)]^2+[y'(t)]^2=[1]^2+(3t^2)^2\neq 0$

point $(0,0)=(x(t),y(t))\Rightarrow t=0$

slope $\left.\frac{y'(t)}{x'(t)}\right|_{t=0}=\left.\frac{3t^2}{1}\right|_{t=0}=0\Rightarrow$ horizontal

tangent line: $y=0.$

22. (Section 9.7, Problem 17)

$y^5=x^3,$ let $x(t)=t,\ y(t)=t^{\frac{3}{5}}$ $\quad (x')^2+(y')^2\neq0$

point $(0,0)=(x(t),y(t))\Rightarrow t=0$

slope $\left.\frac{y'(t)}{x'(t)}\right|_{t=0}=\left.\frac{\frac{3}{5}t^{-\frac{2}{5}}}{1}\right|_{t=0}=\left.\frac{3}{5}\frac{1}{t^{\frac{2}{5}}}\right|_{t=0}$ $\Rightarrow$ vertical

tangent line $x=0$

29. (Section 9.8, Problem 5)

$f(x)=\frac{1}{3}\sqrt{x}(x-3)=\frac{1}{3}x^{\frac{3}{2}}-x^{\frac{1}{2}}$, $x\in[0,3]$. $f'(x)=\frac{1}{2}x^{\frac{1}{2}}-\frac{1}{2}x^{-\frac{1}{2}}$

$[f'(x)]^2=\frac{1}{4}x-\frac{1}{2}+\frac{1}{4}x^{-1}$, length $=\int_0^3\sqrt{1+\frac{x}{4}-\frac{1}{2}+\frac{1}{4x}}\,dx$

$=\int_0^3\sqrt{\frac{x}{4}+\frac{1}{2}+\frac{1}{4x}}\,dx=\int_0^3 \frac{1}{2}x^{\frac{1}{2}}+\frac{1}{2}x^{-\frac{1}{2}}dx=\frac{1}{3}x^{\frac{3}{2}}+x^{\frac{1}{2}}\Big|_0^3=2\sqrt{3}.$

distance from $(\overset{x=0}{0},\ 0)$ to $(\overset{x=3}{3},0)$ $\Rightarrow$ distance $=3$

30. (Section 9.8, Problem 9)

$f(x)=\frac{x^2}{4}-\frac{\ln x}{2}$, $x\in[1,5]$. $f'(x)=\frac{x}{2}-\frac{1}{2x}$

$1+[f'(x)]^2=\left(\frac{x}{2}+\frac{1}{2x}\right)^2\Rightarrow$ length $\int_1^5\sqrt{\left(\frac{x}{2}+\frac{1}{2x}\right)^2}\,dx$

$=\int_1^5\frac{x}{2}+\frac{1}{2x}\,dx=\frac{x^2}{4}+\frac{\ln x}{2}\Big|_1^5=6+\frac{1}{2}\ln 5$

distance from $(1,\frac{1}{4})$ to $(5,\frac{25}{4}-\frac{\ln 5}{2})$ is $\sqrt{4^2+(6-\frac{\ln 5}{2})^2}$

31. (Section 9.8, Problem 10)

$f(x)=\frac{x^2}{8}-\ln x$, $x\in[1,4]$, $f'(x)=\frac{x}{4}-\frac{1}{x}$

$1+[f'(x)]^2=1+\left(\frac{x}{4}-\frac{1}{x}\right)^2=\left(\frac{x}{4}+\frac{1}{x}\right)^2$

$\Rightarrow$ length: $\int_1^4\sqrt{\left(\frac{x}{4}+\frac{1}{x}\right)^2}\,dx=\int_1^4\frac{x}{4}+\frac{1}{x}\,dx=\frac{x^2}{8}+\ln x\Big|_1^4$

$=\ldots+\ln 4$

$\frac{15}{8}+\ln 4$

distance from $(1,\frac{1}{8})$ to $(4,\ 2-\ln 4)$ is $\sqrt{3^2+(\frac{15}{8}-\ln 4)^2}$

32. (Section 9.8, Problem 13)

$f(x)=\ln(\sec x)$, $x\in[0,\frac{\pi}{4}]$, $f'(x)=\frac{\sec x\tan x}{\sec x}=\tan x$

$1+[f'(x)]^2=1+\tan^2x=\sec^2x$,

length: $\int_0^{\frac{\pi}{4}}\sqrt{\sec^2x}\,dx=\int_0^{\frac{\pi}{4}}\sec x\,dx=\ln|\sec x+\tan x|\Big|_0^{\frac{\pi}{4}}=\ln(\sqrt{2}+1)$

distance from $(0,0)$ to $(\frac{\pi}{4},\ln\sqrt{2})$ is $\sqrt{(\frac{\pi}{4})^2+(\ln\sqrt{2})^2}$

33. (Section 9.8, Problem 16)

$f(x)=\cosh x$, $x\in[0,\ln 2]$, $f'(x)=\sinh x$

$1+[f'(x)]^2=1+\sinh^2x=\cosh^2x$,

length: $\int_0^{\ln 2}\sqrt{\cosh^2x}\,dx=\int_0^{\ln 2}\cosh x\,dx=\sinh x\Big|_0^{\ln 2}$

$=\frac{e^x-e^{-x}}{2}\Big|_0^{\ln 2}=\frac{3}{4}$. $\boxed{\sinh x=\frac{e^x-e^{-x}}{2}}$

distance from $(0,1)$ $(\ln 2,\frac{5}{4})$ is $\sqrt{(\ln 2)^2+(\frac{1}{4})^2}$

34. (Section 9.8, Problem 21)

$x=t^2$, $y=t^3$ from $t=0$ to $t=1$.

Formula of ~~velocity~~ speed

$V(t)=\sqrt{[x'(t)]^2+[y'(t)]^2}$

$=\sqrt{(2t)^2+(3t^2)^2}$

initial speed $(t=0)$: $V(0)=0$

terminal speed $(t=1)$: $V(1)=\sqrt{13}$

u-sub. let $u=4+9t^2$
$du=18t\,dt$

length: $\int_0^1\sqrt{4t^2+9t^4}\,dt=\int_0^1 t\sqrt{4+9t^2}\,dt\overset{\Downarrow}{=}\frac{1}{18}\cdot\frac{2}{3}\,13^{\frac{3}{2}}-\frac{8}{27}$

23. (Section 9.7, Problem 20)

$x(t)=t^2-2t,\ y(t)=t^3-12t$ $\quad [x']^2+[y']^2=(2t-2)^2+(3t^2-12)^2\neq 0$ ✓

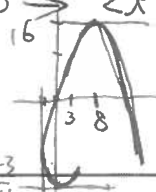
(a) horizontal $\Rightarrow y'(t)=0 \rightarrow 3t^2-12=0 \Rightarrow t=2$ or -2

point: $(x(2),y(2))=(0,-16)$, $(x(-2),y(-2))=(8,16)$ $\quad t=\cancel{1}$

(b) vertical $\Rightarrow x'(t)=0 \rightarrow 2t-2=0.\ t=1!$ $\quad (x,y)=(x(1),y(1))$ $=(-1,-11)$

$t=0,\ (x,y)=(0,0)$

$t=-1,\ (x,y)=(3,11)$



24. (Section 9.7, Problem 21) $\sin t=\frac{x-3}{-4}$ $\quad \cos t=\frac{y-4}{3} \Rightarrow \left(\frac{x-3}{-4}\right)^2+\left(\frac{y-4}{3}\right)^2=1$

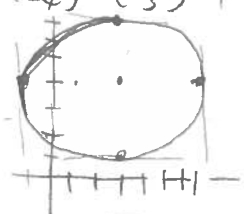
$x(t)=3-4\sin t,\ y(t)=4+3\cos t$

$\Rightarrow$ (a) $y'(t)=0 \Rightarrow -3\sin t=0 \Rightarrow t=0$ or π

point $(x(0),y(0))=(3,7)$, $(x(\pi),y(\pi))=(3,1)$.

(b) $x'(t)=0 \Rightarrow -4\cos t=0 \Rightarrow t=\frac{\pi}{2},\frac{3\pi}{2}$

point $(x(\frac{\pi}{2}),y(\frac{\pi}{2}))=(-1,4)$, $(x(\frac{3\pi}{2}),y(\frac{3\pi}{2}))=(7,4)$



25. (Section 9.7, Problem 22)

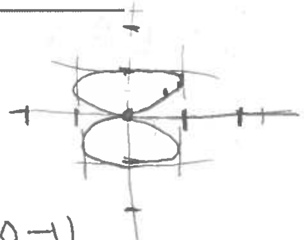
$x(t)=\sin 2t = 2\sin t\cos t,\ y(t)=\sin t$

(a) $y'(t)=0 \rightarrow \cos t=0 \Rightarrow t=\frac{\pi}{2},\frac{3\pi}{2}$.

$(x(\frac{\pi}{2}),y(\frac{\pi}{2}))=(0,1)$ $\quad (x(\frac{3\pi}{2}),y(\frac{3\pi}{2}))=(0,-1)$

(b) $x'(t)=0 \Rightarrow 2\cos 2t=0 \Rightarrow t=\frac{\pi}{4},\frac{3\pi}{4},\frac{5\pi}{4},\frac{7\pi}{4}$

$(x(\frac{\pi}{4}),y(\frac{\pi}{4}))=(1,\frac{\sqrt{2}}{2})$, $(-1,\frac{\sqrt{2}}{2})$ $(1,-\frac{\sqrt{2}}{2})$ $(-1,-\frac{\sqrt{2}}{2})$



$t=0,\ (x,y)=(0,0)$ $\quad t=\frac{\pi}{3},\ (x,y)=(\frac{\sqrt{3}}{2},\frac{\sqrt{3}}{2})$

$t=\frac{\pi}{6}$ $(x,y)=(\frac{\sqrt{3}}{2},\frac{1}{2})$ $\quad t=$

$x(t)=t^2-2t,\ y(t)=t^3-3t^2+2t.$

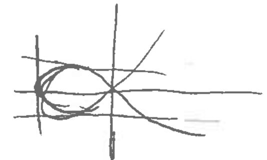
(a) $y'(t)=0 \Rightarrow 3t^2-6t+2=0 \Rightarrow t=\frac{6\pm\sqrt{12}}{6}=1\pm\frac{\sqrt{3}}{3}$

26. (Section 9.7, Problem 23)

$\Rightarrow (x(1+\frac{\sqrt{3}}{3}),y(1+\frac{\sqrt{3}}{3}))=(-\frac{2}{3},-\frac{2}{9}\sqrt{3})$

$(x(1-\frac{\sqrt{3}}{3}),y(1-\frac{\sqrt{3}}{3}))=(-\frac{2}{3},\frac{2}{9}\sqrt{3})$

(b) $x'(t)=2t-2.\ t=1,\ (x(1),y(1))=(-1,0)$



27. (Section 9.7, Problem 25)

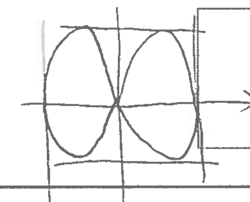
$x(t)=\cos t,\ y(t)=\sin 2t.$

(a) $y'(t)=2\cos 2t=0$ $\quad t=\frac{\pi}{4},\frac{3\pi}{4},\frac{5\pi}{4},\frac{7\pi}{4}.$

$(x(\frac{\pi}{4}),y(\frac{\pi}{4}))=(\frac{\sqrt{2}}{2},1)$, $(-\frac{\sqrt{2}}{2},-1)$ $(-\frac{\sqrt{2}}{2},1)$ $(\frac{\sqrt{2}}{2},-1)$

(b) $x'(t)=-\sin t$, $t=0,\pi$

$(x,y)=(1,0)$, $(-1,0)$



28. (Section 9.8, Problem 1)

Formula: $\int_a^b \sqrt{1+[f'(x)]^2}\,dx.$

f(x) is given: $f(x)=\cancel{2}x+3,\ x\in[0,1]$, $f'(x)=2$

length: $\int_0^1 \sqrt{1+2^2}\,dx=\sqrt{5}$

distance from $(0,f(0))$ to $(1,f(1))$

$(0,3)$ $\quad (1,5)$

$\Rightarrow \sqrt{(5-3)^2+(1-0)^2}=\sqrt{5}.$

35. (Section 9.8, Problem 25)

$x(t)=e^t\sin t$, $y(t)=e^t\cos t$ $t=0\sim\pi$

$v(t)=\sqrt{(x')^2+(y')^2}=\sqrt{(e^t\sin t+e^t\cos t)^2+(e^t\cos t-e^t\sin t)^2}$

$=\sqrt{2e^{2t}}=\sqrt{2}e^t$

$v(0)=\sqrt{2}$, $v(\pi)=\sqrt{2}e^{\pi}$

length: $\int_0^{\pi}\sqrt{2}e^t\,dt=\sqrt{2}e^{\pi}-\sqrt{2}$

36. (Section 9.8, Problem 29)

$r=1$ from $\theta=0$ to $\theta=2\pi$

Formula for $[r,\theta]$ equation: $\int_\alpha^\beta\sqrt{[\rho(\theta)]^2+[\rho'(\theta)]^2}\,d\theta$

$\Rightarrow \rho(\theta)=1$. $\rho'(\theta)=0$. $\Rightarrow \int_0^{2\pi}\sqrt{1}\,d\theta=2\pi$

(circumference of a circle with radius one)

37. (Section 9.8, Problem 33)

$r=e^{2\theta}$. $\theta=0\sim2\pi$, $r'=2e^{2\theta}$ $\left(\frac{dr}{d\theta}\right)$

$\int_0^{2\pi}\sqrt{e^{4\theta}+4e^{4\theta}}\,d\theta$

$=\int_0^{2\pi}\sqrt{5}e^{2\theta}\,d\theta=\frac{\sqrt{5}}{2}e^{2\theta}\Big|_0^{2\pi}$

$=\frac{\sqrt{5}}{2}(e^{4\pi}-1)$

* $\sin^2\theta=\frac{1}{2}-\frac{1}{2}\cos2\theta$

$\boxed{\cos^2\theta=\frac{1}{2}+\frac{1}{2}\cos2\theta}$ $\Rightarrow 4\cdot\left(\frac{1}{2}+\frac{1}{2}\cos\theta\right)=4\cdot\cos^2\frac{\theta}{2}$

38. (Section 9.8, Problem 34)

$r=1+\cos\theta$, $r'=-\sin\theta$, $\theta=0\sim2\pi$

$2\int_0^{\cancel{2}\pi}\sqrt{(1+\cos\theta)^2+(\sin\theta)^2}\,d\theta$

$=2\int_0^{\cancel{2}\pi}\sqrt{1+2\cos\theta+\cos^2\theta+\sin^2\theta}\,d\theta=\int_0^{2\pi}\sqrt{2+2\cos\theta}\,d\theta$

$\Rightarrow 2\int_0^{\cancel{2}\pi}\sqrt{4\cos^2\frac{\theta}{2}}\,d\theta=2\int_0^{\cancel{2}\pi}2\cos\frac{\theta}{2}\,d\theta=2\cdot4\sin\frac{\theta}{2}\Big|_0^{\cancel{2}\pi}$

$=8$

39. (Section 9.8, Problem 35)

$r=1-\cos\theta$, $r'=\sin\theta$, $\theta=0\sim\frac{\pi}{2}$

$\int_0^{\frac{\pi}{2}}\sqrt{2-2\cos\theta}\,d\theta$

$=\int_0^{\frac{\pi}{2}}2\sin\frac{\theta}{2}\,d\theta=-4\cos\frac{\theta}{2}\Big|_0^{\frac{\pi}{2}}=4-2\sqrt{2}$

$=$

38.

$\sqrt{4\cos^2\frac{\theta}{2}}=\begin{cases}2\cos\frac{\theta}{2} & \theta\in[0,\pi]\\ -2\cos\frac{\theta}{2} & \theta\in[\pi,2\pi]\end{cases}$

$\int_0^{2\pi}\sqrt{4\cos^2\frac{\theta}{2}}\,d\theta=\int_0^{\pi}2\cos\frac{\theta}{2}\,d\theta-\int_{\pi}^{2\pi}2\cos\frac{\theta}{2}\,d\theta$

$=4\sin\frac{\theta}{2}\Big|_0^{\pi}-4\sin\frac{\theta}{2}\Big|_{\pi}^{2\pi}=4(1-0)-4(0-1)$

$=8$