MATH 1432, SECTION 12869
SPRING 2014
HOMEWORK ASSIGNMENT 7
DUE DATE: 3/3/14 IN LAB

Name: Sol

ID: ______________________

INSTRUCTIONS

- Print out this file and complete the problems. You must do all the problems!
- If the problem is from the text, the section number and problem number are in parantheses.
- Use a blue or black pen or a pencil (dark).
- Write your solutions in the spaces provided. You must show work in order receive credit for a problem.
- Remember that your homework must be complete, neatly written and stapled.
- Submit the completed assignment to your Teaching Assistant in lab on the due date.
- If you do not do all of the problems, then your recitation quiz from the previous Friday will automatically become a ZERO.

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1. (Section 9.3, Problem 9)

$x = r\cos\theta$

$y = r\sin\theta$

$[r, \theta] = [3, \frac{\pi}{2}]$

$(x, y) = (3\cos\frac{\pi}{2}, 3\sin\frac{\pi}{2})$

$= (0, 3)$

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2. (Section 9.3, Problem 11)

$(r, \theta) = (-1, -\pi)$

$(x, y) = (-1\cdot\cos(-\pi), -1\cdot\sin(-\pi))$

$= (1, 0)$

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3. (Section 9.3, Problem 12)

$[r, \theta] = [-1, \frac{\pi}{4}]$

$(x, y) = (-1\cdot\cos\frac{\pi}{4}, -1\cdot\sin\frac{\pi}{4})$

$= (-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2})$

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4. (Section 9.3, Problem 15)

$[-3, -\frac{\pi}{3}] = [r, \theta]$

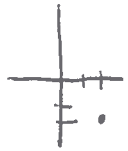
$(x, y) = (-3\cdot\cos(-\frac{\pi}{3}), -3\cdot\sin(-\frac{\pi}{3}))$

$= (-\frac{3}{2}, \frac{3\sqrt{3}}{2})$

5. (Section 9.3, Problem 21)

$r^2 = x^2 + y^2$, $\cos\theta = \dfrac{x}{r}$

$(x, y) = (2, -2) \Rightarrow$



$r^2 = 4 + 4 = 8$

$r = 2\sqrt{2}$. $\cos\theta = \dfrac{2}{2\sqrt{2}} = \dfrac{1}{\sqrt{2}}$

$[r, \theta] = [2\sqrt{2}, -\frac{\pi}{4} + 2k\pi]$

~~or $[2\sqrt{2},$~~

6. (Section 9.3, Problem 22)

$(x, y) = (3, -3\sqrt{3}) \Rightarrow$

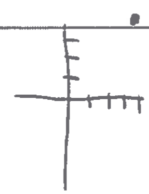
$r^2 = 9 + 27 = 36 \Rightarrow r = 6$

$\cos\theta = \dfrac{3}{6} = \dfrac{1}{2}$

$[r, \theta] = [6, -\frac{\pi}{3} + 2k\pi]$

7. (Section 9.3, Problem 23)

$(x, y) = (4\sqrt{3}, 4) \Rightarrow$



$r^2 = 48 + 16 = 64 \Rightarrow r = 8$

$\cos\theta = \dfrac{x}{r} = \dfrac{4\sqrt{3}}{8} = \dfrac{\sqrt{3}}{2}$

$[r, \theta] = [8, \frac{\pi}{6} + 2k\pi]$

8. (Section 9.3, Problem 27)

$[r, \theta] = [\frac{1}{2}, \frac{1}{6}\pi]$ $0 \le \theta < 2\pi$, $r > 0$

(a) sym. about x-axis : $[\frac{1}{2}, \frac{-\pi}{6}] = [\frac{1}{2}, \frac{11\pi}{6}]$

(b) " y-axis : $[\frac{1}{2}, \pi - \frac{\pi}{6}] = [\frac{1}{2}, \frac{5\pi}{6}]$

(c) " the origin: $[\frac{1}{2}, \pi + \frac{\pi}{6}] = [\frac{1}{2}, \frac{7\pi}{6}]$

9. (Section 9.3, Problem 28)

$[r, \theta] = [3, -\frac{5}{4}\pi]$ $0 \le \theta < 2\pi$, $r > 0$

(a) sym. about x-axis: $[3, \frac{5}{4}\pi]$

(b) " y-axis: $[3, \pi - (-\frac{5}{4}\pi)] = [3, \frac{9}{4}\pi]$
$= [3, \frac{\pi}{4}]$

(c) " the origin: $[3, -\frac{\pi}{4}] = [3, \frac{7\pi}{4}]$

10. (Section 9.3, Problem 40)

$x = r\cos\theta$
$y = r\sin\theta$ $\Rightarrow x^2 + y^2 = 9$

$\Rightarrow r^2\cos^2\theta + r^2\sin^2\theta = 9$

$\Rightarrow r^2 = 9 \Rightarrow r = 3$.

11. (Section 9.3, Problem 41)

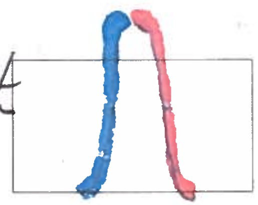
$x^2+(y-2)^2=4$

$\Rightarrow (r\cos\theta)^2+(r\sin\theta-2)^2=4$

$\Rightarrow \underline{r^2\cos^2\theta+r^2\sin^2\theta}-4r\sin\theta+4=4$

$\Rightarrow r^2-4r\sin\theta=0$

$r\neq0$ $\Rightarrow r-4\sin\theta=0 \Rightarrow r=4\sin\theta$



12. (Section 9.3, Problem 42)

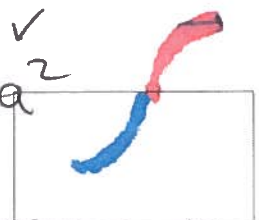
$(x-a)^2+y^2=a^2$

$(r\cos\theta-a)^2+(r\sin\theta)^2=a^2$

$r^2\cos^2\theta-2ar\cos\theta+a^2+r^2\sin^2\theta=a^2$

$r^2-2ar\cos\theta=0$

$\Rightarrow r(r-2a\cos\theta)=0 \xRightarrow{r\neq0} r-2a\cos\theta=0 \Rightarrow r=2a\cos\theta$



13. (Section 9.3, Problem 47)

$(x^2+y^2)^2=2xy$

$(r^2\cos^2\theta+r^2\sin^2\theta)^2=2(r\cos\theta)(r\sin\theta)$

$\Rightarrow (r^2)^2=2r^2\cos\theta\sin\theta$

$r=0$ $\Rightarrow r^2=2\cos\theta\sin\theta$

cancel "r^2" on both sides

14. (Section 9.3, Problem 50)

$r\cos\theta=4$

since $x=r\cos\theta \Rightarrow \boxed{x=4.}$

15. (Section 9.3, Problem 55)

$r=3\cos\theta$, by $\begin{cases} x^2+y^2=r^2 & —① \text{ and} \\ \cos\theta=\frac{x}{r} & —② \end{cases}$

$\xRightarrow{\text{by } ②} r=3\cdot\frac{x}{r} \Rightarrow r^2=3x$

$\xRightarrow{\text{by } ①} \boxed{x^2+y^2=3x}$

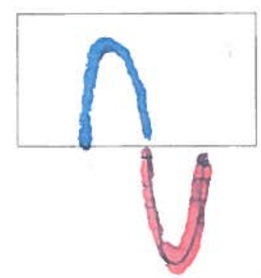
~~$x^2=3x$~~

16. (Section 9.3, Problem 58)

$r=2\sin\theta$ $\begin{cases} x^2+y^2=r^2 & —① \\ \sin\theta=\frac{y}{r} & —② \end{cases}$

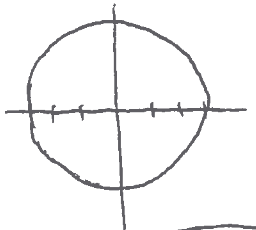
$\xRightarrow{\text{by } ②} r=2\cdot\frac{y}{r} \Rightarrow r^2=2y$

$\xRightarrow{\text{by } ①} \boxed{x^2+y^2=2y}$



17. (Section 9.4, Problem 2)

$r=-3 \Rightarrow$ circle with center o and radius 3



18. (Section 9.4, Problem 4)

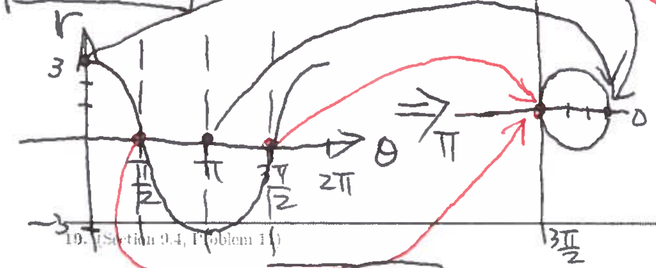
$r=3\cos\theta$ $\boxed{\text{Method I}}$ by 15

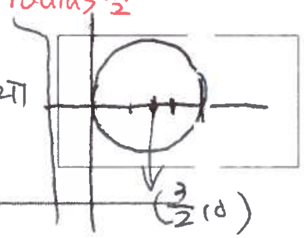
$x^2+y^2=3x \Rightarrow (x^2-3x+\frac{9}{4})+y^2=\frac{9}{4}$

$\Rightarrow (x-\frac{3}{2})^2+y^2=\frac{9}{4}$

(center $(\frac{3}{2},0)$) radius $\frac{3}{2}$

$\boxed{\text{Method 2}}$

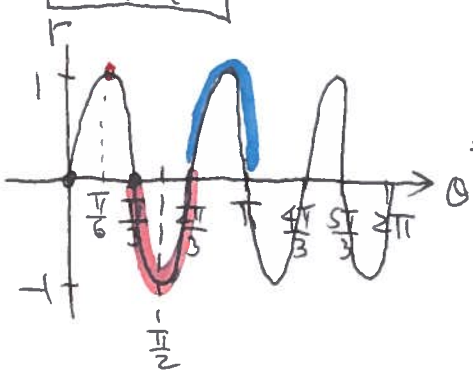


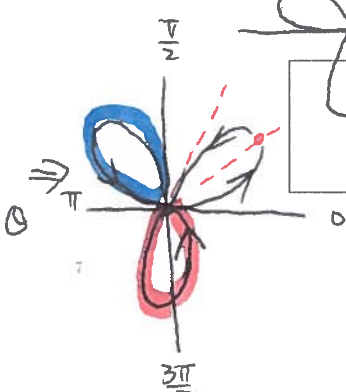


19. (Section 9.4, Problem 11)

$r=\sin 3\theta$ $\boxed{\text{Method I}}$ $\Rightarrow$ flower with 3 petals

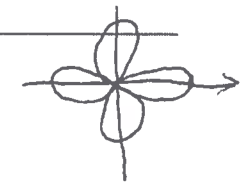
$\boxed{\text{Method 2}}$



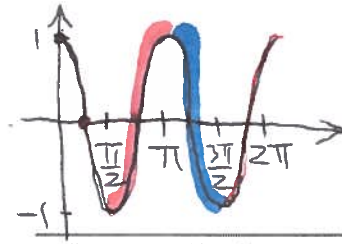


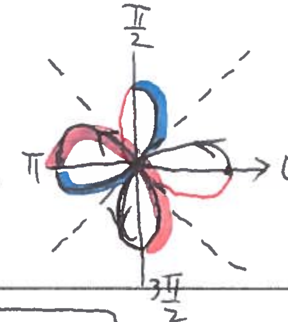
20. (Section 9.4, Problem 14)

$r=\cos 2\theta$ $\boxed{\text{Method 1}}$ 4 petals



$\boxed{\text{Method 2}}$





21. (Section 9.4, Problem 19)

$r=-1+\sin\theta$ $\boxed{\text{Method 1}}$ $a=-1,\ b=1$ Cardioid $|a|=|b|$

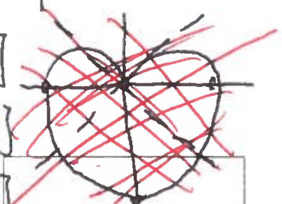
$[r,\theta]=[-1,0]$

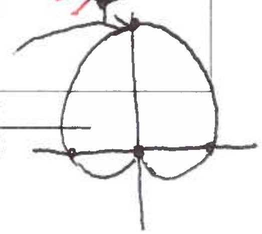
$[0,\frac{\pi}{2}]$

$[-1,\pi]$

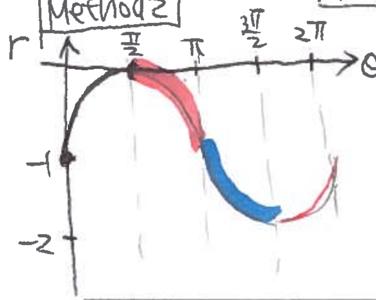
$[-2,\frac{3\pi}{2}]$

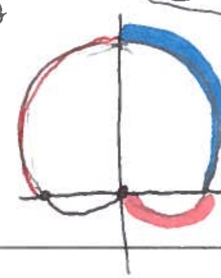
$[-1,2\pi]$





$\boxed{\text{Method 2}}$

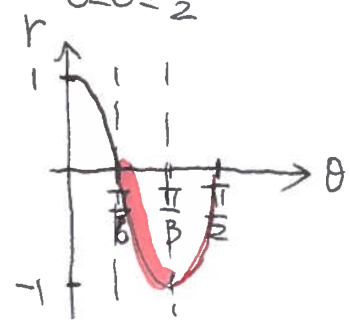


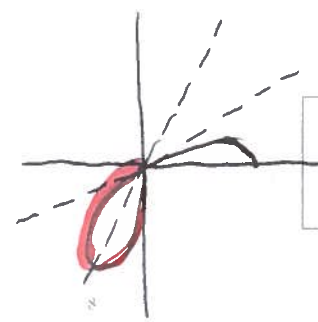


22. (Section 9.4, Problem 22)

$r=\cos 3\theta$

$0\le\theta\le\frac{\pi}{2}$





$r = 2 - \cos\theta$ $\boxed{\text{Method 1}}$ $a=2,\ b=-1,\ |a|>|b|$ dimple

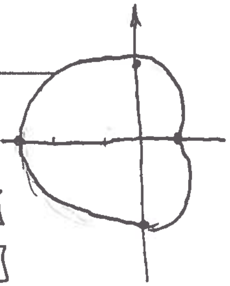
$[r,\theta] = [1, 0]$

$[2, \frac{\pi}{2}]$

$[3, \pi]$

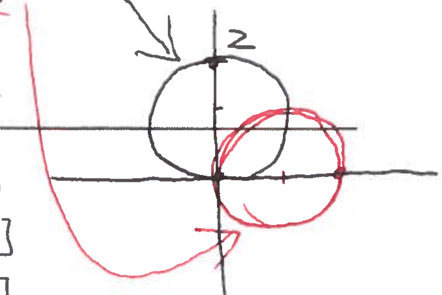
$[2, \frac{3\pi}{2}]$

$[1, 2\pi]$



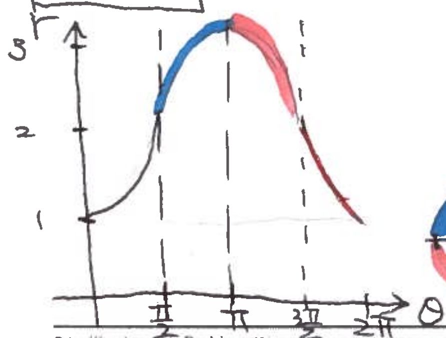
$r = 2\sin\theta$, $r = 2\cos\theta$

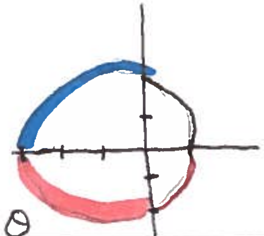
$m=1 \downarrow$ circle $\quad m=1 \downarrow$ circle



23. (Section 9.4, Problem 28)

$\boxed{\text{Method 2}}$





26. (Section 9.4, Problem 42)

$[r,\theta] = [0, 0] \qquad [2, 0]$

$[2, \frac{\pi}{2}] \qquad [0, \frac{\pi}{2}]$

$[0, \pi] \qquad [-2, \pi]$

$[-2, \frac{3\pi}{2}] \qquad [0, \frac{3\pi}{2}]$

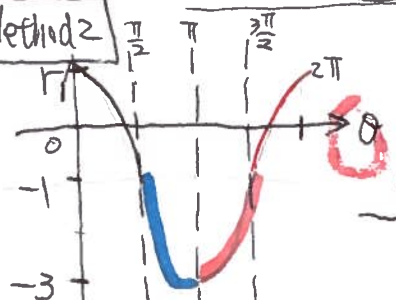
$[0, 2\pi] \qquad [2, 2\pi]$

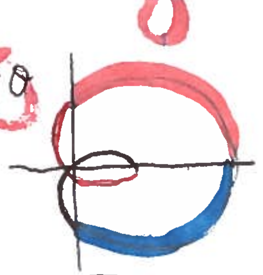
point $(0,0), (1,1)$

24. (Section 9.4, Problem 31)

$r = -1 + 2\cos\theta$ $\boxed{\text{Method 1}}$ $a=-1,\ b=2\ \ |a|<|b|$ loop

$\boxed{\text{Method 2}}$





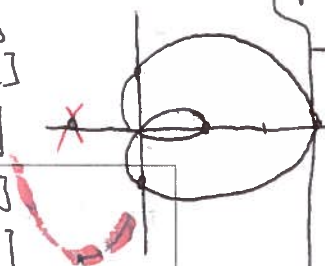
$[r,\theta] = [1, 0]$

$[-1, \frac{\pi}{2}]$

$[-3, \pi]$

$[-1, \frac{3\pi}{2}]$

$[1, 2\pi]$



27. (Section 9.5, Problem 1)

$r = a\cos\theta$

$$A = \int_\alpha^\beta \frac{1}{2}[f(\theta)]^2 d\theta$$

$$A = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{1}{2} a^2\cos^2\theta\, d\theta$$

$$= \frac{a^2}{2}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{1}{2} + \frac{\cos 2\theta}{2} d\theta =$$

$$\frac{a^2}{2}\left[\frac{\theta}{2} + \frac{1}{4}\sin 2\theta\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}}$$

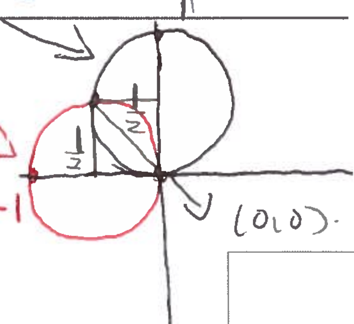
$$= \frac{a^2}{2}\left[\frac{\pi}{2} + 0\right]$$

$$= \frac{\pi}{4}a^2$$

25. (Section 9.4, Problem 39)

$r = \sin\theta$ $\quad r = -\cos\theta$

$m=1 \downarrow$ circle $\quad \downarrow$ circle.



$(0,0)$.

$(-\frac{1}{2}, \frac{1}{2})$.

$[r,\theta] = [0, 0] \qquad [r,\theta] = [-1, 0]$

$[1, \frac{\pi}{2}] \qquad [0, \frac{\pi}{2}]$

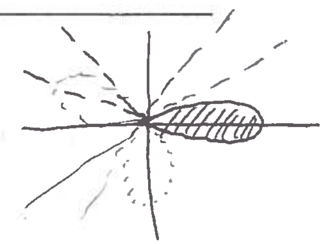
$[0, \pi] \qquad [1, \pi]$

$[-1, \frac{3\pi}{2}] \qquad [0, \frac{3\pi}{2}]$

$[0, 2\pi] \qquad [-1, 2\pi]$

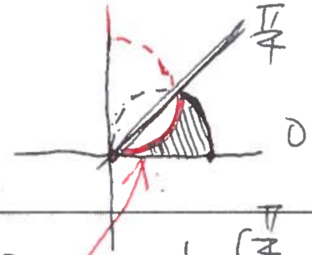
28. (Section 9.5, Problem 2)

$$A = \int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} \frac{1}{2} a^2\cos^2 3\theta\, d\theta$$



$$= \frac{a^2}{2}\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} \frac{1}{2} + \frac{1}{2}\cos 6\theta\, d\theta$$

$$= \frac{a^2}{2}\left[\frac{\theta}{2} + \frac{1}{12}\sin 6\theta\right]_{-\frac{\pi}{6}}^{\frac{\pi}{6}} = \frac{a^2}{4}\cdot\frac{\pi}{3} = \frac{a^2}{12}\pi$$



29. (Section 9.5, Problem 8) $0 \le \theta \le \frac{\pi}{4}$

$\underline{r=\cos\theta}$ $\quad$ $\underline{r=\sin\theta}$

$[r,\theta]=[1,0]$ $\quad$ $[r,\theta]=[0,0]$

$=[\frac{\sqrt3}{2},\frac{\pi}{6}]$ $\quad$ $=[\frac12,\frac{\pi}{6}]$

$=[\frac{\sqrt2}{2},\frac{\pi}{4}]$ $\quad$ $[\frac{\sqrt2}{2},\frac{\pi}{4}]$

$$A=\frac12\int_0^{\frac{\pi}{4}}\cos^2\theta-\sin^2\theta\,d\theta$$

$$=\frac12\int_0^{\frac{\pi}{4}}\cos2\theta\,d\theta$$

$$=\frac12\cdot\frac12\sin2\theta\Big|_0^{\frac{\pi}{4}}=\boxed{\frac14}$$

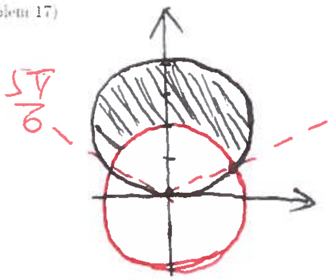
30. (Section 9.5, Problem 17)

$r=4\sin\theta$

$[r,\theta]=[0,0]$

$[4,\frac{\pi}{2}]$

$[0,\pi]$



$$A=\frac12\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}}(4\sin\theta)^2-(2)^2\,d\theta$$

$$=\frac12\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}}16\sin^2\theta-4\,d\theta$$

$$=\frac12\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}}16\left(\frac12-\frac12\cos2\theta\right)-4\,d\theta$$

$$=\frac12\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}}8-8\cos2\theta-4\,d\theta$$

$r=2$

$\begin{cases} r=4\sin\theta \\ r=2\end{cases} \Rightarrow 2=4\sin\theta,\ \frac12=\sin\theta,\ \theta=\frac{\pi}{6},\frac{5\pi}{6}$

$$=\frac12\left[4\theta-4\sin2\theta\right]_{\frac{\pi}{6}}^{\frac{5\pi}{6}}$$

$$=\frac{4\pi}{3}-2\left[-\frac12-\frac12\right]$$

$$=\frac{4\pi}{3}+2$$

31. (Section 9.5, Problem 20)

$r=4\cos\theta$

$[r,\theta]=[4,0]$

$[2\sqrt3,\frac{\pi}{6}]$

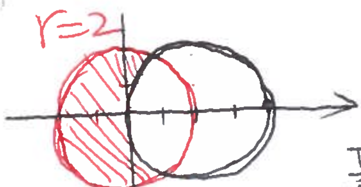
$[2,\frac{\pi}{3}]$

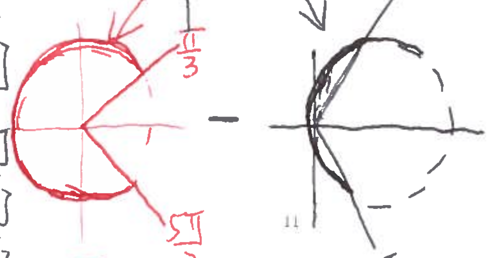
$[0,\frac{\pi}{2}]$

$[-2,\frac{2\pi}{3}]$

$[2\sqrt3,\frac{5\pi}{6}]$

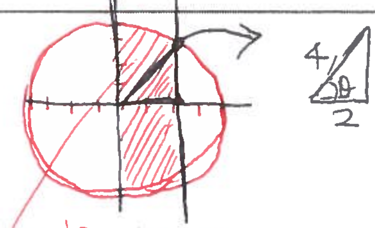
$[4,\pi]$





$$\int_{\pi}^{\frac{5\pi}{3}}2^2\,d\theta-\int_{\frac{\pi}{3}}^{\frac{2\pi}{3}}(4\cos\theta)^2\,d\theta$$

32. (Section 9.5, Problem 21)

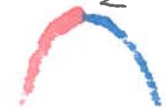


$r=2\sec\theta=2\frac{1}{\cos\theta}$

$=2\cdot\frac{r}{x}$

$\Rightarrow x=2$

$\cos\theta=\frac12\Rightarrow\theta=\frac{\pi}{3}$



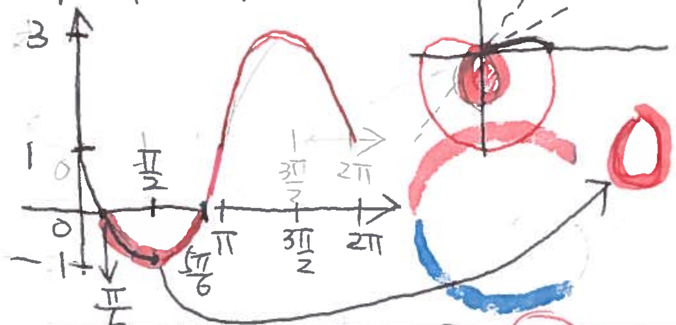
$r=4$

$2\times\left(\ \frac{\pi}{2}\ \frac{\pi}{3}\ +\ \frac{\pi}{3}\ 0\ \right)$

$$2\times\left(\int_{\frac{\pi}{3}}^{\frac{\pi}{2}}\frac{4^2}{2}\,d\theta+\int_0^{\frac{\pi}{3}}\frac{(2\sec\theta)^2}{2}\,d\theta\right)$$

33. (Section 9.5, Problem 22)

$r=1-2\sin\theta$



$$=\frac12\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}}(1-2\sin\theta)^2\,d\theta$$

34. (Section 9.5, Problem 23)

$r=2\sin3\theta$

$[r,\theta]=[0,0]$

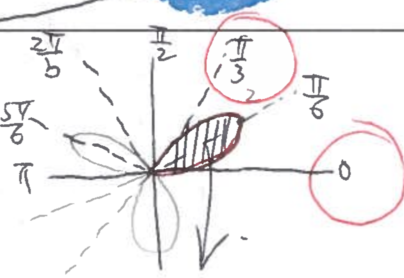
$[2,\frac{\pi}{6}]$

$[0,\frac{\pi}{3}]$

$[-2,\frac{\pi}{2}]$

$[0,\frac{2\pi}{3}]$

$[\ \ ,\frac{5\pi}{6}]$



$$\frac12\int_0^{\frac{\pi}{3}}(2\sin3\theta)^2\,d\theta$$

35. (Section 9.5, Problem 25)

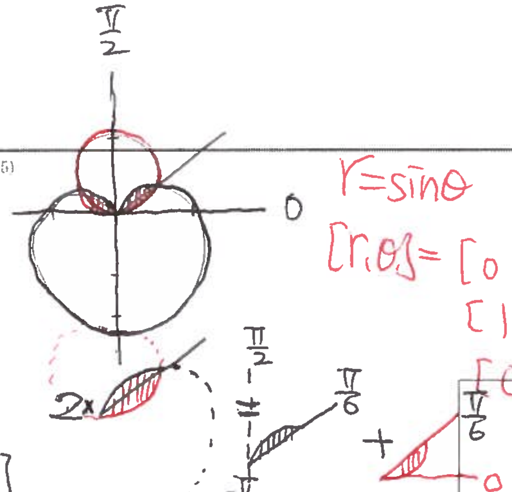
$r = 1 - \sin\theta$

$[r, \theta]$: $[1, 0]$, $[0, \frac{\pi}{2}]$, $[1, \pi]$, $[2, \frac{3\pi}{2}]$, $[0, 2\pi]$

$r = \sin\theta$

$[r, \theta] = [0, 0]$, $[1, \frac{\pi}{2}]$, $[0, \pi]$

$$\begin{cases} r = 1 - \sin\theta \\ r = \sin\theta \end{cases} \Rightarrow 1 = 2\sin\theta \Rightarrow \sin\theta = \frac{1}{2}. \quad \theta = \frac{\pi}{6}$$



$$\frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} (1-\sin\theta)^2\, d\theta + \frac{1}{2}\int_0^{\frac{\pi}{6}} (\sin\theta)^2\, d\theta$$

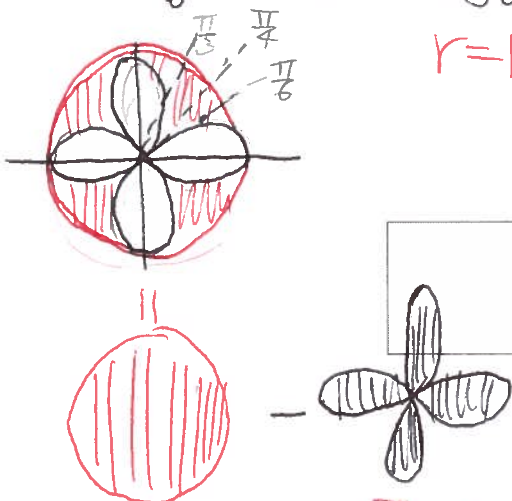
36. (Section 9.5, Problem 27)

$r = \cos 2\theta$

$[r, \theta]$: $[1, 0]$, $[\frac{1}{2}, \frac{\pi}{6}]$, $[-\frac{1}{2}, \frac{\pi}{3}]$, $[-1, \frac{\pi}{2}]$

$[0, \frac{\pi}{4}]$

$r = 1$



$$\pi - 4\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{(\cos 2\theta)^2}{2}\, d\theta$$

$$\text{or } \pi - 8\int_0^{\frac{\pi}{4}} \frac{(\cos 2\theta)^2}{2}\, d\theta$$