Math 1432, Section 12869

Fall 2013

Homework Assignment 6

Due Date: 2/21/14 in Lab

Name: Sol

ID:

Instructions

- Print out this file and complete the problems. You must do all the problems!
- If the problem is from the text, the section number and problem number are in parantheses.
- Use a blue or black pen or a pencil (dark).
- Write your solutions in the spaces provided. You must show work in order receive credit for a problem.
- Remember that your homework must be complete, neatly written and stapled.
- Submit the completed assignment to your Teaching Assistant in lab on the due date.
- If you do not do all of the problems, then your recitation quiz from the previous Friday will automatically become a ZERO.

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1. (Section 8.5, Problem 9)

$$f(x)=\frac{7}{(x-2)(x+5)}=\frac{A}{(x-2)}+\frac{B}{(x+5)}$$

$$A=\lim_{x\to 2}(x-2)f(x)=\lim_{x\to 2}\frac{7}{x+5}=1$$

$$B=\lim_{x\to -5}(x+5)\frac{7}{(x-2)(x+5)}=\lim_{x\to -5}\frac{7}{x-2}=-1$$

$$\int\frac{7}{(x+5)(x-2)}dx=\int\frac{1}{x-2}-\frac{1}{x+5}dx$$

$$=\ln|x-2|-\ln|x+5|+C$$

$$\frac{5}{x^2(x-1)}=\frac{5}{x-1}+\cancel{\frac{Ax+B}{x^2}}+\frac{A}{x}+\frac{B}{x^2}$$

$$=\frac{5}{x-1}+\frac{-5}{x}+\frac{-5}{x^2}$$

$$\begin{array}{rrrrr|l} 2 & -4 & +4 & +0 & +3 & 1 \\ & +2 & -2 & +2 & +2 & \\ \hline 2 & -2 & +2 & +2 & & 5 \end{array}$$

直接用長除法較好

2. (Section 8.5, Problem 11)

$$\int\frac{2x^4-4x^3+4x^2+3}{x^2(x-1)}dx=\int\frac{(x-1)(2x^3-2x^2+2x+2)+5}{x^2(x-1)}dx$$

$$=\int\left(\frac{2x^3-2x^2+2x+2}{x^2}+\frac{5}{x^2(x-1)}\right)dx$$

$$=\int\left(2x-2+\frac{2}{x}+\frac{2}{x^2}+\frac{5}{x-1}+\frac{-5}{x^2}+\frac{-5}{x}\right)dx$$

$$=\int\left(2x-2-\frac{3}{x}-\frac{3}{x^2}+\frac{5}{x-1}\right)dx$$

$$=x^2-2x-3\ln|x|+\frac{3}{x}+5\ln|x-1|+C$$

3. (Section 8.5, Problem 12)

let $f(x)=\dfrac{x^2+1}{x(x^2-1)}$

$$=\frac{A}{x}+\frac{B}{x+1}+\frac{C}{x-1}$$

$A=\lim_{x\to 0}xf(x)=-1$, $C=\lim_{x\to 1}(x-1)f(x)=1$

$B=\lim_{x\to -1}(x+1)f(x)=1$

$$\int f(x)dx=\int\frac{-1}{x}+\frac{1}{x+1}+\frac{1}{x-1}dx=-\ln|x|+\ln|x+1|+\ln|x-1|+C$$

4. (Section 8.5, Problem 15)

let $f(x)=\dfrac{x+3}{x^2-3x+2}=\dfrac{(x+3)}{(x-2)(x-1)}=\dfrac{A}{(x-1)}+\dfrac{B}{x-2}$

$$A=\lim_{x\to 1}(x-1)f(x)=-4$$

$$B=\lim_{x\to 2}(x-2)f(x)=5$$

$$\int f(x)dx=\int\frac{-4}{x-1}+\frac{5}{x-2}dx$$

$$=-4\ln|x-1|+5\ln|x-2|+C$$

5. (Section 8.5, Problem 18)

$$\int \frac{dx}{x^2+2x+2} = \int \frac{dx}{(x+1)^2+1} = \boxed{\tan^{-1}(x+1) + C}$$

~~let x+1~~

8. (Section 8.5, Problem 27)

$$\int \frac{x-3}{x^3+x^2}\,dx = \int \frac{-4}{x+1} + \frac{4}{x} + \frac{-3}{x^2}\,dx$$

$$= \boxed{-4\ln|x+1| + 4\ln|x| + \frac{3}{x} + C}$$

$$f(x) = \frac{x-3}{x^3+x^2} = \frac{x-3}{x^2(x+1)} = \frac{A}{x+1} + \frac{B}{x} + \frac{C}{x^2}$$

$A=-4$, $B=?$, $C=-3$, $-1 = f(1) = \frac{A}{2} + B + C \Rightarrow B = 4$

6. (Section 8.5, Problem 19)

$$\int \frac{x^2}{(x-1)^2(x+1)} = \int \frac{1}{4}\frac{1}{x+1} + \frac{3}{4}\frac{1}{x-1} + \frac{1}{2}\frac{1}{(x-1)^2}\,dx$$

$$= \boxed{\frac{1}{4}\ln|x+1| + \frac{3}{4}\ln|x-1| + \frac{-1}{2}\frac{1}{x-1} + C}$$

$f(x)$

$$\frac{x^2}{(x-1)^2(x+1)} = \frac{A}{x+1} + \frac{B}{x-1} + \frac{C}{(x-1)^2} = \frac{A(x-1)^2 + B(x+1)(x-1) + C(x+1)}{(x-1)^2(x+1)}$$

$$\Rightarrow x^2 = A(x-1)^2 + B(x+1)(x-1) + C(x+1)$$

$A = \lim_{x\to -1}(x+1)f(x) = \frac{1}{4}$

$C = \lim_{x\to 1}(x-1)^2 f(x) = \frac{1}{2}$

as $x=0$, we have $0 = A - B + C$

$\Rightarrow B = A + C = \frac{3}{4}$

9. (Section 8.5, Problem 31)

$$\int_0^2 \frac{x}{x^2+5x+6}\,dx = \int_0^2 \frac{3}{x+3} - \frac{2}{x+2}\,dx$$

$$= \Big[3\ln|x+3| - 2\ln|x+2|\Big]_0^2$$

$$= 3\ln 5 - 3\ln 3 - 2\ln 4 + 2\ln 2$$

$$f(x) = \frac{x}{x^2+5x+6} = \frac{x}{(x+2)(x+3)} = \frac{A}{x+2} + \frac{B}{x+3}$$

$\Rightarrow A = -2$, $B = 3$

7. (Section 8.5, Problem 21)

$$\int \frac{dx}{x^4-16} = \int \frac{-\frac{1}{8}}{x^2+4} + \frac{-\frac{1}{32}}{x+2} + \frac{\frac{1}{32}}{x-2}\,dx$$

$$= \boxed{-\frac{1}{16}\tan^{-1}\left(\frac{x}{2}\right) - \frac{1}{32}\ln|x+2| + \frac{1}{32}\ln|x-2| + C}$$

$$f(x) = \frac{1}{x^4-16} = \frac{1}{(x^2+4)(x^2-4)}$$

$$= \frac{1}{(x^2+4)(x+2)(x-2)}$$

$$= \frac{Ax+B}{x^2+4} + \frac{C}{x+2} + \frac{D}{x-2}$$

$C = \lim_{x\to -2}(x+2)f(x) = -\frac{1}{32}$

$D = \lim_{x\to 2}(x-2)f(x) = \frac{1}{32}$

$\frac{1}{-15} = f(1) = \frac{A+B}{5} + \frac{C}{3} - D$

$= \frac{A}{5} + \left(-\frac{1}{40}\right) -$ [redacted] $\frac{1}{24}$

$\Rightarrow A = 0$

$f(0) = \frac{B}{4} + \frac{C}{2} - \frac{D}{2} \Rightarrow B = -\frac{1}{8}$

10. (Section 8.5, Problem 32)

$$\int_1^3 \frac{dx}{x^3+x} = \int_1^3 \frac{1}{x} + \frac{-x}{x^2+1}\,dx = \left[\ln|x| + \frac{\ln|x^2+1|}{-2}\right]_1^3$$

$$= \ln 3 - \frac{1}{2}\ln 10 + \frac{1}{2}\ln 2$$

$$f(x) = \frac{1}{x^3+x} = \frac{1}{x(x^2+1)} = \frac{A}{x} + \frac{Bx+C}{x^2+1}$$

$A = 1$.

$\frac{1}{2} = f(1) = \frac{B}{2} + \frac{C}{2} + A \Rightarrow B + C = -1$

$-\frac{1}{2} = f(-1) = \frac{-B}{2} + \frac{C}{2} - A \Rightarrow -B + C = 1$

$\Rightarrow B = -1$, $C = 0$

$\int_0^1 \sin^2\pi x\,dx = \int_0^1 \frac{1}{2} + \frac{1}{2}\cos 2\pi x\,dx = \frac{x}{2} - \frac{1}{4\pi}\sin 2\pi x \Big|_0^1 = \frac{1}{2}$

$u^2 - 2u - 8 = u^2 - 2u + 1 - 9 = (u-1)^2 - 9$

11. (Section 8.5, Problem 35)

$$\int \frac{\cos\theta}{\sin^2\theta - 2\sin\theta - 8}\,d\theta = \int \frac{du}{u^2 - 2u - 8} = -\int \frac{du}{9-(u-1)^2}$$

let $u = \sin\theta$, $du = \cos\theta\,d\theta$

$$= \int \frac{du}{(u-4)(u+2)} = \int \frac{\frac{1}{6}}{u-4} + \frac{-\frac{1}{6}}{u+2}\,du$$

$$= \frac{1}{6}\ln|u-4| - \frac{1}{6}\ln|u+2| + C$$

$$= \frac{1}{6}\ln|\sin\theta - 4| - \frac{1}{6}\ln|\sin\theta + 2| + C$$

12. (Section 8.5, Problem 36)

$$\int \frac{e^t}{e^{2t} + 5e^t + 6}\,dt = \int \frac{du}{u^2 + 5u + 6} = \int \frac{du}{(u+2)(u+3)}$$

let $u = e^t$, $du = e^t dt$

$$= \int \frac{1}{u+2} + \frac{-1}{u+3}\,du$$

$$= \ln|u+2| - \ln|u+3| + C$$

$$= \ln|e^t + 2| - \ln|e^t + 3| + C$$

13. (Section 8.7, Problem 1)

$\int_0^{12} x^2 dx = \frac{x^3}{3}\Big|_0^{12} = \frac{12^3}{3} = 576.$ $\quad b = 12,\ a = 0,\ n = 12$

For (a) (b)

	x_0	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	x_{12}
x	0	1	2	3	4	5	6	7	8	9	10	11	12
$f(x)$	0	1	4	9	16	25	36	49	64	81	100	121	144
	$\frac{1}{2}$	$\frac{5}{2}$	$\frac{13}{2}$	$\frac{25}{2}$	$\frac{41}{2}$	$\frac{61}{2}$	$\frac{85}{2}$	$\frac{113}{2}$	$\frac{145}{2}$	$\frac{181}{2}$	$\frac{221}{2}$	$\frac{265}{2}$	

(a) $L_n = \frac{12-0}{12}[0+1+4+9+16+25+36+49+64+81+100+121] = \frac{11 \cdot 12 \cdot 23}{6} = 506$

(b) $R_n = \frac{12-0}{12}[1+4+9+16+25+36+49+64+81+100+121+144] = \frac{12 \cdot 13 \cdot 25}{6} = 650$

(c) $n = 6$. $x_0\ x_1\ x_2\ x_3\ x_4\ x_5\ x_6$: $0\ 2\ 4\ 6\ 8\ 10\ 12$

$\frac{x_n + x_{n+1}}{2}$: $1\ 3\ 5\ 7\ 9\ 11$

$M_n = \frac{12-0}{6}[1+9+25+49+81+121] = 572$

(d) $T_n = \frac{12}{12}\left[\frac{1}{2}(1+5+13+\cdots+265)\right] = 578$

(e) $n=6$

$S_6 = \frac{1}{3}T_6 + \frac{2}{3}M_6$

$= \frac{1}{3}(292 \cdot 2) + \frac{2}{3} 572$

$= 576.$

$f(x) = \sin^2\pi x$

14. (Section 8.7, Problem 2)

(a) $n = 3$. $x_0\ x_1\ x_2\ x_3$: $0\ \frac{1}{3}\ \frac{2}{3}\ 1$; midpoints $\frac{1}{6}\ \frac{1}{2}\ \frac{5}{6}$

$M_3 = \frac{1-0}{3}[f(\frac{1}{6}) + f(\frac{1}{2}) + f(\frac{5}{6})] = \frac{1}{3}[\frac{1}{4} + 1 + \frac{1}{4}] = \frac{6}{12} = \boxed{\frac{1}{2}}$

(b) $n = 6$

	x_0	x_1	x_2	x_3	x_4	x_5	x_6
x	0	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{5}{6}$	1
$f(x)$	0	$\frac{1}{4}$	$\frac{3}{4}$	1	$\frac{3}{4}$	$\frac{1}{4}$	0
$\frac{f(x_n)+f(x_{n+1})}{2}$	$\frac{1}{8}$	$\frac{1}{2}$	$\frac{7}{8}$	$\frac{7}{8}$	$\frac{1}{2}$	$\frac{1}{8}$	

$T_6 = \frac{1-0}{6}[\frac{1}{8} + \frac{1}{2} + \frac{7}{8} + \frac{7}{8} + \frac{1}{2} + \frac{1}{8}] = \frac{3}{6} = \boxed{\frac{1}{2}}$

(c) $S_3 = \frac{1}{3}T_3 + \frac{2}{3}M_3 = \frac{1}{3}[\frac{1}{2}] + \frac{2}{3}[\frac{1}{2}] = \boxed{\frac{1}{2}}$

15. (Section 8.7, Problem 6)

(a) $f(x) = \frac{1}{\sqrt{4+x^3}}$

$n = 4$. x_0, x_1, x_2, x_3, x_4

x	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2
$f(x)$	$\frac{1}{2}$	$\sqrt{\frac{8}{33}}$	$\frac{1}{\sqrt{5}}$	$\sqrt{\frac{8}{60}}$	$\frac{1}{\sqrt{12}}$

$T_4 = \frac{2-0}{4}\left[\frac{1}{2}\left(\frac{1}{2} + \sqrt{\frac{8}{33}}\right) + \left(\sqrt{\frac{8}{33}} + \frac{1}{\sqrt{5}}\right) + \left(\frac{1}{\sqrt{5}} + \sqrt{\frac{8}{60}}\right) + \left(\sqrt{\frac{8}{60}} + \frac{1}{\sqrt{12}}\right)\right]$

$\frac{f(x_n)+f(x_{n+1})}{2}$

(b) $n=2$, $S_2 = \frac{1}{3}T_2 + \frac{2}{3}M_2$.

$x_0\ x_1\ x_2$: $0\ 1\ 2$

$\frac{x_n + x_{n+1}}{2}$: $\frac{1}{2}\ \frac{3}{2} \Rightarrow M_2 = \frac{2}{2}[f(\frac{1}{2}) + f(\frac{3}{2})]$

$f(x)$: $f(0) = \frac{1}{2}$, $f(1) = \frac{1}{\sqrt{5}}$, $f(2) = \frac{1}{\sqrt{12}}$

$T_2 = \frac{2}{2}\left[\frac{1}{2}\left(\frac{1}{2} + \frac{1}{\sqrt{5}}\right) + \frac{1}{2}\left(\frac{1}{\sqrt{5}} + \frac{1}{\sqrt{12}}\right)\right]$

16. (Section 8.7, Problem 13)

(a) $E_n^T = -\frac{(b-a)^3}{12n^2} f''(c)$, $c \in (a,b)$

$\int_1^4 \sqrt{x}\,dx$, $f(x) = \sqrt{x}$, $f''(x) = -\frac{1}{4}x^{-3/2}$

$|f''(x)| < \frac{1}{4}$, $c \in$

$|E_n| = \left|\frac{(4-1)^3}{12n^2}\right| \cdot \frac{1}{4} < 0.01$

$\Rightarrow \frac{27}{12 \cdot 4}\frac{1}{n^2} < 0.01$

$n^2 > \frac{2700}{12 \cdot 4} = \frac{2700}{48} = 56.25$

$n > 8$

(b)

(a) $|E_n^T| = \left|\frac{(b-a)^3}{12n^2}\right| |f''(c)|$ $\quad c\in[1,3]$

$f(x)=e^x \Rightarrow f''(x)=e^x \quad |e^x|<e^3$

$f^{(4)}(x)=e^x$

17. (Section 8.7, Problem 19)

$|E_n| \le \left|\frac{(3-1)^3}{12n^2}\right|\cdot e^3 < 0.01$

$\Rightarrow \frac{8}{12}\cdot e^3\cdot 100 \le n^2$

$\Rightarrow n^2 > 1339.035$

$n > 36.59$

$n = 37$

(b) $|E_n^S| \le \left|\frac{(3-1)^5}{2880n^4}\right| |e^3| < 0.01$

$\frac{2^5}{2880}\cdot e^3\cdot 100 < n^4$

$\Rightarrow n^4 > 22.31726$

$\Rightarrow n > 2.17$

$\Rightarrow$ $n=3$

18. (Section 8.7, Problem 20)

$\int_1^e \ln x\,dx \quad \varepsilon = 0.01$

$f(x)=\ln x$. $\quad |f''(x)| = \left|\frac{-1}{x^2}\right| < 1 \quad f^{(4)}(x) = \left|\frac{-6}{x^4}\right| < 6$

(a)

$|E_n| \le \left|\frac{(e-1)^3}{12n^2}\right|\cdot|1| < 0.01$

$n^2 > \frac{(e-1)^3\cdot 100}{12} > \frac{5.073214\cdot 100}{12} = 42.27067$

$n \ge 7$

(b) $|E_n| \le \left|\frac{(e-1)^5}{2880n^4}\right| |6| < 0.01$

$n^4 > \frac{6(e-1)^5\cdot 100}{2880} = 3.12054$

$n \ge 2$

$\sum_{k=1}^{n} k = 1+2+3+4+\cdots+n = \frac{n(n+1)}{2}$

(Proof: let $S = 1+2+3+4+\cdots+n$

$+)\ S = n+(n-1)+(n-2)+\cdots+1$

$2S = (n+1)+(n+1)+\cdots+(n+1)$

$\Rightarrow S = \frac{n(n+1)}{2}$)

$\sum_{k=1}^{n} k^2 = 1^2+2^2+3^2+4^2+\cdots+n^2 = \frac{n(n+1)(2n+1)}{6}$

(use $(k+1)^3 = k^3+3k^2+3k+1$)

16. (b) $E_n^S = -\frac{(b-a)^5}{2880n^4} f^{(4)}(c)$

$|f^{(4)}(x)| = \left|\frac{15}{16}\cdot x^{-\frac{2}{2}}\right| < \frac{15}{16} \quad x\in(1,4)$.

$|E_n^S| \le \left|\frac{(4-1)^5}{2880n^4}\right| \cdot \frac{15}{16} < 0.01$

$\Rightarrow \frac{3^5}{2880\cdot n^4}\cdot\frac{15}{16} < 0.01$

$\Rightarrow n^4 > \frac{3^5\cdot 100\cdot 15}{2880\cdot 16} = 7.91$

$n \ge 2 \quad (2^4 = 16)$