Using IBP ⊛ $\int \sec^3\theta\, d\theta = \sec\theta\tan\theta - \int \tan^2\theta \sec\theta\, d\theta$

$u = \sec\theta \quad dV = \sec^2\theta\, d\theta$

$du = \sec\theta\tan\theta \quad V = \tan\theta$

Math 1432, Section 12869
Spring 2014
Homework Assignment 5
Due Date: 2/17/14 in Lab

Name: Sol.

ID: ______

Instructions

- Print out this file and complete the problems. You must do all the problems!
- If the problem is from the text, the section number and problem number are in parantheses.
- Use a blue or black pen or a pencil (dark).
- Write your solutions in the spaces provided. You must show work in order receive credit for a problem.
- Remember that your homework must be complete, neatly written and stapled.
- Submit the completed assignment to your Teaching Assistant in lab on the due date.
- If you do not do all of the problems, then your recitation quiz from the previous Friday will automatically become a ZERO.

1. (Section 8.4, Problem 1)

$$\int \frac{dx}{\sqrt{a^2-x^2}} = \int \frac{a\cos\theta\, d\theta}{a\cos\theta} = \int d\theta = \theta + C$$

$$= \sin^{-1}\left(\frac{x}{a}\right) + C$$

Let $x = a\sin\theta$

$dx = a\cos\theta\, d\theta$

$(a^2 - a^2\sin^2\theta = a^2\cos^2\theta)$

2. (Section 8.4, Problem 3)

$$\int \sqrt{x^2-1}\, dx = \int \tan\theta \cdot \sec\theta \tan\theta\, d\theta$$

$\Downarrow$

$\tan\theta = \sqrt{x^2-1}$

$\Leftarrow X = \sec\theta$

$dx = \sec\theta\tan\theta\, d\theta$

$(\sec^2\theta - 1 = \tan^2\theta)$

$$= \int \tan^2\theta \sec\theta\, d\theta = \int \sec^3\theta - \sec\theta\, d\theta$$

$$\overset{⊛}{=} \sec\theta\tan\theta - \int \tan^2\theta\sec\theta\, d\theta - \ln|\sec\theta + \tan\theta| + C$$

$$\Rightarrow \int \tan^2\theta \sec\theta\, d\theta = \frac{1}{2}\sec\theta\tan\theta - \frac{1}{2}\ln|\sec\theta+\tan\theta| + C$$

$$\boxed{= \frac{1}{2} x \cdot \sqrt{x^2-1} - \frac{1}{2}\ln\left|x + \sqrt{x^2-1}\right| + C}$$

3. (Section 8.4, Problem 4)

$$\int \frac{x}{\sqrt{4-x^2}}\, dx = -\frac{1}{2}\int \frac{du}{\sqrt{u}}$$

$$= -\frac{1}{2}\cdot 2\sqrt{u} + C$$

$$= -\sqrt{4-x^2} + C$$

$\boxed{\text{Let } u = 4-x^2 \quad du = -2x\, dx}$

4. (Section 8.4, Problem 5)

$$\int \frac{x^2}{\sqrt{4-x^2}}\, dx = \int \frac{4\sin^2\theta}{\cancel{2\sin\theta}\ 2\cos\theta} \cdot 2\cos\theta\, d\theta = \int 4\sin^2\theta\, d\theta$$

$$= \int 2 - 2\cos 2\theta\, d\theta$$

$$= 2\theta - \sin 2\theta + C$$

$$= 2\sin^{-1}\left(\frac{x}{2}\right) - \frac{x(\sqrt{4-x^2})}{2} + C$$

~~$= 4\int \sin\theta\cos\theta\, d\theta$~~

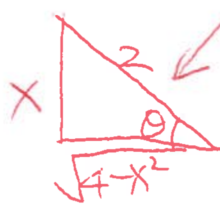
~~$= 4\int \frac{1}{2}\sin 2\theta\, d\theta$~~

~~$= -2\,\frac{\cos 2\theta}{2} + C$~~

~~$= -\cos 2\theta$~~

Let $x = 2\sin\theta$.

$dx = 2\cos\theta\, d\theta$



$\sin\theta = \frac{x}{2}$

$\Rightarrow \cos\theta = \frac{\sqrt{4-x^2}}{2}$

$\Rightarrow \sin 2\theta = 2\sin\theta\cos\theta$

$= \frac{x\sqrt{4-x^2}}{2}$

$\tan\theta = \frac{x}{2}$

$\sec\theta = \frac{\sqrt{4+x^2}}{2}$

$\frac{8\cdot 2\sqrt{2}}{(2\sqrt{2})^3} = 1.$

5. (Section 8.4, Problem 8)

$$\int \frac{x^2}{\sqrt{4+x^2}}\,dx = \int \frac{4\tan^2\theta}{2\sec\theta}\cdot 2\sec^2\theta\, d\theta$$

$$= 4\int \tan^2\theta\sec\theta\, d\theta$$

let $x = 2\tan\theta$

$dx = 2\sec^2\theta\, d\theta$

$$\text{(see 2)} = \cancel{4}^{2}\left(\frac{1}{\cancel{2}}\sec\theta\tan\theta - \frac{1}{\cancel{2}}\ln|\sec\theta+\tan\theta|\right)+C$$

$$= 2\sec\theta\tan\theta - 2\ln|\sec\theta+\tan\theta| + C$$

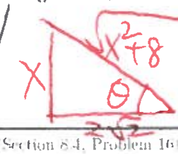
$$= 2\cdot\frac{\sqrt{4+x^2}}{2}\cdot\frac{x}{2} - 2\ln\left|\frac{\sqrt{4+x^2}}{2}+\frac{x}{2}\right| + C$$

8. (Section 8.4, Problem 15)

$$\int \frac{x^2}{(x^2+8)^{\frac{3}{2}}}\,dx = \int \frac{8\tan^2\theta}{(2\sqrt{2})^{3}\cdot\sec^3\theta}\, 2\sqrt{2}\sec^2\theta\, d\theta$$

Let $x = 2\sqrt{2}\tan\theta$

$dx = 2\sqrt{2}\sec^2\theta\, d\theta$



$$= \int \frac{\tan^2\theta}{\sec\theta}\,d\theta = \int \frac{\sec^2\theta - 1}{\sec\theta}\,d\theta$$

$$= \int \sec\theta\, d\theta - \int \frac{d\theta}{\sec\theta}$$

$$= \ln|\sec\theta+\tan\theta| - \sin\theta + C$$

$$= \ln\left|\frac{\sqrt{x^2+8}}{2\sqrt{2}} + \frac{x}{2\sqrt{2}}\right| - \frac{x}{\sqrt{x^2+8}} + C$$

6. (Section 8.4, Problem 9)

直接做較好

$$\int_0^{\frac{1}{2}} \frac{x^2}{(1-x^2)^{\frac{3}{2}}}\,dx \Rightarrow \int \frac{x^2}{(1-x^2)^{\frac{3}{2}}}\,dx = \int \frac{\sin^2\theta}{\cos^3\theta}\cdot\cos\theta\, d\theta$$

$$= \int \tan^2\theta\, d\theta \quad (\sec^2\theta - 1)$$

Let $x = \sin\theta$

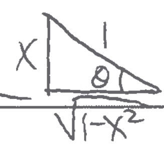
$dx = \cos\theta\, d\theta$

$$= \int \sec^2\theta\, d\theta - \int d\theta$$

$$= \tan\theta - \theta + C = \frac{x}{\sqrt{1-x^2}} - \sin^{-1}x + C.$$

$$= \frac{x}{\sqrt{1-x^2}} - \sin^{-1}x\,\Big|_0^{\frac{1}{2}}$$

$$= \frac{1}{\sqrt{3}} - \frac{\pi}{6}$$



9. (Section 8.4, Problem 16)

$$\int_0^a \sqrt{a^2-x^2}\,dx = \int_0^{\frac{\pi}{2}} a^2\cos^2\theta\, d\theta$$

$$= a^2\int_0^{\frac{\pi}{2}} \frac{1}{2} + \frac{1}{2}\cos 2\theta\, d\theta$$

let $x = a\sin\theta$

$dx = a\cos\theta\, d\theta$

$$= a^2\left[\frac{\theta}{2} + \frac{\sin 2\theta}{4}\right]_0^{\frac{\pi}{2}} = \frac{\pi}{4}a^2$$

as $x = a \Rightarrow \theta = \frac{\pi}{2}$

as $x = 0 \Rightarrow \theta = 0$

Q. why $\int_0^a \sqrt{a^2-x^2}\,dx$ = area ?

7. (Section 8.4, Problem 12)

$$\int_0^2 \frac{x^3}{\sqrt{16-x^2}}\,dx = \int_0^{\frac{\pi}{6}} 4^3\frac{\sin^3\theta}{\cancel{4\cos\theta}}\cdot\cancel{4\cos\theta}\, d\theta$$

Let $x = 4\sin\theta$

$dx = 4\cos\theta\, d\theta$

$$= 4^3\int_0^{\frac{\pi}{6}} (1-\cos^2\theta)\sin\theta\, d\theta$$

as $x = 2 \Rightarrow \theta = \frac{\pi}{6}$

$(\sin\theta = \frac{1}{2})$

$$= 4^3\left[\cos\theta - \frac{\cos^3\theta}{3}\right]_{\frac{\pi}{6}}^{0}$$

as $x = 0 \Rightarrow \theta = 0$

$$= 4^3\left[\frac{\sqrt{3}}{2} - 1\right] - \frac{4^3}{3}\left[\frac{3}{8}\sqrt{3} - 1\right]$$

10. (Section 8.4, Problem 21)

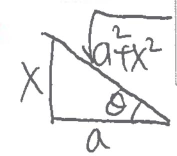
$$\int \frac{dx}{x^2\sqrt{a^2+x^2}} = \int \frac{a\sec^2\theta\, d\theta}{a^2\tan^2\theta\cdot a\sec\theta} = \frac{1}{a^2}\int \frac{\sec\theta}{\tan^2\theta}\,d\theta$$

let $x = a\tan\theta$

$dx = a\sec^2\theta\, d\theta$

$$\cancel{= \frac{1}{a^2}\int \frac{\sec\theta}{\sec^2\theta - 1}\,d\theta}$$

$$\cancel{= \frac{1}{a^2}\int \int}$$



$\left(\sec\theta = \frac{1}{\cos\theta}\right.$

$\left.\tan\theta = \frac{\sin\theta}{\cos\theta}\right)$

$\frac{\sec\theta}{\tan^2\theta} = \frac{1}{\cos\theta}\cdot\frac{\cos^2\theta}{\sin^2\theta}$

$$= \frac{1}{a^2}\int \frac{\cos\theta}{\sin^2\theta}\,d\theta = \frac{-1}{a^2}\frac{1}{\sin\theta} + C = -\frac{1}{a^2}\cdot\frac{\sqrt{a^2+x^2}}{x} + C$$

(or let $u = \sin\theta$ $du = \cos\theta\, d\theta$)

Times "$\frac{e^x}{e^x}$"

11. (Section 8.4, Problem 24)

$$\int \frac{dx}{e^x\sqrt{4+e^{2x}}} \overset{\downarrow}{=} \int \frac{e^x\,dx}{e^{2x}\sqrt{4+e^{2x}}} = \int \frac{2\sec^2\theta\,d\theta}{(2\tan\theta)^2\cdot 2\sec\theta}$$

Let $e^x = 2\tan\theta$

$e^x dx = 2\sec^2\theta\, d\theta$

$$= \frac{1}{4}\int \frac{\sec\theta}{\tan^2\theta}\,d\theta$$

$$\overset{\text{by (10)}}{=} -\frac{1}{4}\,\frac{1}{\sin\theta} + C = -\frac{1}{4}\cdot\frac{\sqrt{4+e^{2x}}}{e^x} + C.$$

12. (Section 8.4, Problem 32)

$$\int \frac{x+2}{\sqrt{x^2-2x+3}}\,dx = \int \frac{x+2}{\sqrt{(x-1)^2+2}}\,dx \overset{\uparrow}{=} \int \frac{\sqrt{2}\tan\theta - 1 + 2}{\sqrt{(\sqrt{2}\tan\theta)^2+2}}\,dx = \int \frac{(\sqrt{2}\tan\theta+1)(\sqrt{2}\sec^2\theta)}{\sqrt{2}\sec\theta}\,d\theta$$

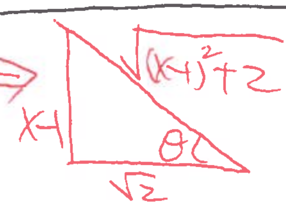
$\sqrt{2}\sec^2\theta\, d\theta$

Note: x^2-2x+3
$= x^2-2x+1+2$
$= (x-1)^2+2$

Let $x-1 = \sqrt{2}\tan\theta \Rightarrow x = \sqrt{2}\tan\theta - 1$

$dx = \sqrt{2}\sec^2\theta\, d\theta$

$\tan\theta = \frac{x-1}{\sqrt{2}} \Rightarrow$



$\Downarrow$

$\sec\theta = \frac{\sqrt{(x-1)^2+2}}{\sqrt{2}}$

$$= \int (\sqrt{2}\tan\theta + 1)\sec\theta\, d\theta$$

$$= \int \sqrt{2}\tan\theta\sec\theta + \sec\theta\, d\theta$$

$$= \sqrt{2}\sec\theta + \ln|\sec\theta + \tan\theta| + C$$

$$= \sqrt{2}\cdot\frac{\sqrt{(x-1)^2+2}}{\sqrt{2}} + \ln\left|\frac{\sqrt{(x-1)^2+2}}{\sqrt{2}} + \frac{x-1}{\sqrt{2}}\right| + C$$

$$= \sqrt{(x-1)^2+2} + \ln\left|\frac{\sqrt{(x-1)^2+2}}{\sqrt{2}} + \frac{x-1}{\sqrt{2}}\right| + C$$

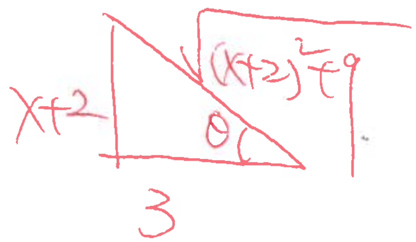
Sorry! This is not 8.4 Problem ~~24~~ 32!

12. $$\int \frac{x+2}{\sqrt{x^2+4x+13}}\,dx = \int \frac{x+2}{\sqrt{(x+2)^2+9}}\,dx = \int \frac{3\tan\theta}{3\sec\theta}\cdot 3\sec^2\theta\,d\theta = 3\int \tan\theta\sec\theta\,d\theta$$

Note: $$\boxed{\begin{aligned} &x^2+4x+13 \\ &=(x^2+4x+4)+9 \\ &=(x+2)^2+9 \end{aligned}}$$

let $$\boxed{\begin{aligned} &x+2 = 3\tan\theta \Rightarrow x = 3\tan\theta - 2 \\ &dx = 3\sec^2\theta\,d\theta \end{aligned}}$$

$$\tan\theta = \frac{x+2}{3}$$



$$= 3\cdot\sec\theta + C$$

$$= 3\cdot\frac{\sqrt{(x+2)^2+9}}{3} + C$$

$$= \sqrt{x^2+4x+13} + C.$$