Math 1432, Section 12869
Spring 2014
Homework Assignment 4
Due Date: 2/10/14 in Lab

Name: Sol

ID: ______________________

Instructions

- Print out this file and complete the problems. You must do all the problems!
- If the problem is from the text, the section number and problem number are in parantheses.
- Use a blue or black pen or a pencil (dark).
- Write your solutions in the spaces provided. You must show work in order receive credit for a problem.
- Remember that your homework must be complete, neatly written and stapled.
- Submit the completed assignment to your Teaching Assistant in lab on the due date.
- If you do not do all of the problems, then your recitation quiz from the previous Friday will automatically become a ZERO.

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1. (Section 8.1, Problem 5)

$$\int \sec^2(1-x)\,dx = -\tan(1-x) + C$$

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2. (Section 8.1, Problem 8)

$$\int_0^1 \frac{x^3}{1+x^4}\,dx = \frac{1}{4}\int_0^1 \frac{du}{1+u} = \frac{1}{4}\ln|1+u|\Big|_0^1$$

let $u = x^4$ $\left(\begin{array}{l} x=0 \Rightarrow u=0 \\ x=1 \Rightarrow u=1 \end{array}\right)$

$du = 4x^3\,dx$

$$= \frac{1}{4}\ln 2 - \frac{1}{4}\ln 1$$

$$= \frac{1}{4}\ln 2$$

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3. (Section 8.1, Problem 9)

$$\int \frac{x}{\sqrt{1-x^2}}\,dx = -\frac{1}{2}\int \frac{du}{\sqrt{u}} = -\frac{1}{2}\cdot\frac{\sqrt{u}}{\cancel{2}}\cdot 2 + C$$

let $u = 1-x^2$

$du = -2x\,dx$

$$= -\sqrt{u} + C$$

$$= -\sqrt{1-x^2} + C$$

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4. (Section 8.1, Problem 13)

$$\int_1^2 \frac{e^{\frac{1}{x}}}{x^2}\,dx = -\int_1^{\frac{1}{2}} e^u\,du$$

let $u = \frac{1}{x}$

$du = \frac{-1}{x^2}\,dx$

$$= -\left(e^{\frac{1}{2}} - e^1\right) = e^1 - e^{\frac{1}{2}}$$

5. (Section 8.1, Problem 17)

$$\int \frac{\sec^2\theta}{\sqrt{3\tan\theta+1}}\,d\theta = \frac{1}{3}\int \frac{du}{\sqrt{u}} = \frac{2}{3}\sqrt{u}+C$$

$$= \frac{2}{3}\sqrt{3\tan\theta+1}+C$$

Let $u=3\tan\theta+1$

$du=3\sec^2\theta\,d\theta$

6. (Section 8.1, Problem 18)

$$\int \frac{\sin\phi}{3-2\cos\phi}\,d\phi = \frac{1}{2}\int\frac{du}{u} = \frac{\ln|u|}{2}+C$$

$$= \frac{1}{2}\ln|3-2\cos\phi|+C$$

Let $u=3-2\cos\phi$

$du=2\sin\phi\,d\phi$

7. (Section 8.1, Problem 23)

$$\int \frac{x}{\sqrt{1-x^4}}\,dx = \frac{1}{2}\int\frac{du}{\sqrt{1-u^2}} = \frac{1}{2}\sin^{-1}u+C$$

$$= \frac{1}{2}\sin^{-1}(x^2)+C$$

let $u=x^2$

$du=2x\,dx$

8. (Section 8.1, Problem 24)

$$\int \frac{e^x}{1+e^{2x}}\,dx = \tan^{-1}(e^x)+C$$

9. (Section 8.1, Problem 25) $x^2+6x+10 = (x^2+6x+9)+1 = (x+3)^2+1$

$$\int \frac{dx}{x^2+6x+10} = \int \frac{1}{1+(x+3)^2}\,dx$$

$$= \tan^{-1}(x+3)+C$$

10. (Section 8.1, Problem 30)

$$\int \cosh 2x \sinh^3 2x\,dx = \frac{1}{2}\int u^3\,du$$

$$= \frac{1}{2}\frac{u^4}{4}+C$$

$$= \frac{1}{8}\sinh^4 2x+C$$

let $u=\sinh 2x$

$du=2\cosh 2x\,dx$

u [redacted] . A. T. E.   I.L.A.T.E.

Inverse log poly tri exp.

11. (Section 8.1, Problem 33)

$$\int \frac{\sin^{-1}x}{\sqrt{1-x^2}}\,dx = \frac{(\sin^{-1}x)^2}{2} + C$$

or let $u=\sin^{-1}x$, $du = \frac{dx}{\sqrt{1-x^2}}$

$$= \int u\,du = \frac{u^2}{2} + C.$$

12. (Section 8.2, Problem 4)

$$\int \underset{A}{x}\,\underset{L}{\ln x^2}\,dx = (\ln x^2)\frac{x^2}{2} - \int x\,dx = \frac{x^2}{2}\ln x^2 - \frac{x^2}{2} + C$$

(or $= x^2\ln x - \frac{x^2}{2} + C$)

u	dv
$\ln x^2$	x
$\frac{2}{x}$	$\frac{x^2}{2}$

$u = \ln x^2 \leftarrow dv = x\,dx$

$du = \frac{2}{x}dx \leftarrow v = \frac{x^2}{2}$

13. (Section 8.2, Problem 5)

$$\int_0^1 \underset{A}{x^2}\,\underset{E}{e^{-x}}\,dx = -x^2e^{-x} - 2xe^{-x} - 2e^{-x}\Big|_0^1$$

$$= -e^{-1} - 2e^{-1} - 2e^{-1} + 0 + 0 + 2e^0$$

$$= 2 - 5e^{-1}.$$

(u)	(dv)	
x^2	e^{-x}	+
[redacted]		
0	$-e^{-x}$	+

stop!

$$\int_0^{\frac{1}{2}} x\cos\pi x\,dx = x\frac{\sin\pi x}{\pi} - \frac{\cos\pi x}{\pi^2}\Big|_0^{\frac12}$$

$$= \boxed{\frac{1}{2\pi} - \frac{1}{\pi^2}}$$

[redacted]

14. (Section 8.2, Problem 19)

$$\int_0^{\frac12} \underset{A}{x}\,\underset{T}{\cos\pi x}\,dx = +x\cdot\frac{\sin\pi x}{\pi} + \frac{\cos\pi x}{\pi^2}\Big|_0^{\frac12}$$

$$= +\frac{1}{2\pi} + 0 + 0 + \frac{(-1)}{\pi^2}$$

$$= \cancel{\pi^2 \quad \frac{\pi}{2}}$$

$$=$$

(u)	(dv)	
x	$\cos\pi x$	+
[redacted]		

15. (Section 8.2, Problem 20)

$$\int_0^{\frac{\pi}{2}} \underset{A}{x^2}\,\underset{T}{\sin x}\,dx = -x^2\cos x + 2x\sin x + 2\cos x\Big|_0^{\frac{\pi}{2}}$$

$$= -0 + \pi + 0 + 0 - 0 - 2$$

$$= \pi - 2$$

u	dv	
x^2	$\sin x$	+
$2x$	$-\cos x$	−
2	$-\sin x$	+
0	$\cos x$	−

16. (Section 8.2, Problem 23)

$$\int \underset{E}{e^x}\,\underset{T}{\sin x}\,dx = e^x\sin x - \int e^x\cos x\,dx$$

$u = \sin x \leftarrow dv = e^x dx$

$du = \cos x\,dx \leftarrow v = e^x$

$u = \cos x \leftarrow dv = e^x dx$

$du = -\sin x\,dx \leftarrow v = e^x$

$$= e^x\sin x - e^x\cos x + \left(-\int e^x\sin x\,dx\right)$$

$$\Rightarrow \int e^x\sin x\,dx = \frac12\left(e^x\sin x - e^x\cos x\right) + C.$$

17. (Section 8.2, Problem 29)

$\int x^3 \sin x^2 \, dx = \int x^2 \cdot \sin x^2 \cdot x\,dx = \frac{1}{2}\int u \sin u \, du$

(u-sub)

Let $u = x^2$, $du = 2x\,dx$

u	dv
x^3	$\sin x^2$
$3x^2$	$2x \cos x^2$

I.B.P.

$\frac{d}{du}$ u	$\int$ $\sin u$	
1	$-\cos u$	$+$
0	$-\sin u$	$-$
		$+$

$= -\frac{1}{2} u \cos u + \frac{1}{2}\sin u + C$

$= -\frac{1}{2} x^2 \cos x^2 + \frac{1}{2}\sin x^2 + C$

20. (Section 8.2, Problem 35)

$\int x^2 \cosh 2x \, dx = \frac{x^2}{2}\sinh 2x - \frac{x}{2}\cosh 2x + \frac{1}{4}\sinh 2x + C$

u	dV	
x^2	$\cosh 2x$	$+$
$2x$	$\frac{\sinh 2x}{2}$	$-$
2	$\frac{\cosh 2x}{4}$	$+$
0	$\frac{\sinh 2x}{8}$	$-$

[redacted]

18. (Section 8.2, Problem 31)

$\int_0^{\frac14} 1 \cdot \sin^{-1} 2x \, dx = x \cdot \sin^{-1} 2x \Big|_0^{\frac14} - \int_0^{\frac14} \frac{x}{\sqrt{1-4x^2}} dx$

$u = \sin^{-1} 2x \quad dV = 1 \cdot dx$

$du = \frac{1}{\sqrt{1-4x^2}} \quad V = x$

$= \frac{1}{4}\cdot\frac{\pi}{6} + \frac{1}{4}\sqrt{1-4x^2}\Big|^{\frac14}$

$= \boxed{\frac{\pi}{24} + \frac{1}{4}\left(\frac{\sqrt{3}}{2} - 1\right)}$

[redacted]

19. (Section 8.2, Problem 33)

$\int_0^1 x \tan^{-1} x^2 \, dx = \frac{x^2}{2}\tan^{-1} x^2 \Big|_0^1 - \frac{1}{\cancel{2}}\int_0^1 \frac{\cancel{2}x^3}{1+x^4} dx$

$u = \tan^{-1} x^2 \quad dV = x\,dx$

$du = \frac{2x}{1+x^4} dx \quad V = \frac{x^2}{2}$

$= \frac{1}{2}\cdot\frac{\pi}{4} - \frac{1}{4}\ln|1+x^4| \Big|_0^1$

$= \boxed{\frac{\pi}{8} - \frac{1}{4}\ln 2}$

21. (Section 8.2, Problem 38)

$\int 1 \cdot \cos(\ln x)\, dx = x\cos(\ln x) + \int 1 \cdot \sin(\ln x)\, dx$

①

u	dv
$\cos(\ln x)$	1
$-\frac{\sin(\ln x)}{x}$	x

$= x\cos(\ln x) + x\sin(\ln x) - \int \cos(\ln x)\, dx$

②

u	dv
$\sin(\ln x)$	1
$\frac{\cos(\ln x)}{x}$	x

$\Rightarrow \int \cos(\ln x)\, dx = \frac{1}{2}(x\cos(\ln x) + x\sin(\ln x)) + C$

22. (Section 8.2, Problem 39)

$\int \sin(\ln x)\, dx = x\sin(\ln x) - \int \cos(\ln x)\, dx$

Use ② of above (21)

use ① of above $= x\sin(\ln x) - x\cos(\ln x) + \left(-\int \sin(\ln x)\, dx\right)$

$\Rightarrow \int \sin(\ln x)\, dx = \frac{1}{2}(x\sin(\ln x) - x\cos(\ln x)) + C$

① $\sin^2 x + \cos^2 x = 1$

② $\cos^2 x = \frac{1}{2} + \frac{1}{2}\cos 2x$ 1. $\sin^2 x+\cos^2 x=1$

③ $\sin^2 x = \frac{1}{2} - \frac{1}{2}\cos 2x$

23. (Section 8.3, Problem 1)

$$\int \sin^3 x\,dx = \int \sin x \cdot \sin^2 x\,dx$$

$$\overset{\text{by ①}}{=} \int \sin x \cdot (1 \cancel{+} \overline{\ }\cos^2 x)\,dx$$

$$= -\int 1 - u^2\,du = \int u^2 - 1\,du$$

set $u = \cos x$, $du = -\sin x\,dx$

$$= \frac{u^3}{3} - u + C = \frac{(\cos x)^3}{3} - \cos x + C$$

24. (Section 8.3, Problem 2)

$$\int_0^{\frac{\pi}{8}} \cos^2 4x\,dx \overset{\text{by ②}}{=} \int_0^{\frac{\pi}{8}} \frac{1}{2} + \frac{1}{2}\cos 8x\,dx$$

$$= \frac{x}{2} + \frac{1}{16}\sin 8x \Big|_0^{\frac{\pi}{8}}$$

$$= \frac{\pi}{16} + \frac{1}{16}\cdot 0 - 0 = \frac{\pi}{16}$$

25. (Section 8.3, Problem 3)

$$\int_0^{\frac{\pi}{6}} \sin^2 3x\,dx \overset{\text{by ③}}{=} \int_0^{\frac{\pi}{6}} \frac{1}{2} - \frac{1}{2}\cos 6x\,dx$$

$$= \frac{x}{2} - \frac{1}{12}\sin 6x \Big|_0^{\frac{\pi}{6}}$$

$$= \frac{\pi}{12} - 0 + 0 = \frac{\pi}{12}.$$

26. (Section 8.3, Problem 5)

$$\int \cos^4 x \cdot \sin^3 x\,dx = \int \cos^4 x \cdot \sin^2 x \cdot \sin x\,dx$$

$$\overset{\text{by ①}}{=} \int \cos^4 x \cdot (1 - \cos^2 x) \cdot \sin x\,dx$$

Set $u = \cos x$, $du = -\sin x\,dx$

$$= -\int u^4 (1 - u^2)\,du$$

$$= \int u^6 - u^4\,du = \frac{u^7}{7} - \frac{u^5}{5} + C = \frac{(\cos x)^7}{7} - \frac{(\cos x)^5}{5} + C$$

27. (Section 8.3, Problem 9)

$$\int \sec^2 \pi x\,dx = \frac{1}{\pi}\tan \pi x + C$$

28. (Section 8.3, Problem 10)

$$\int \csc^2 2x\,dx = -\frac{1}{2}\cot 2x + C$$

④ $1+\tan^2 x=\sec^2 x$

⑤ $=$ ③ $1-2\sin^2 x=\cos 2x$

29. (Section 8.3, Problem 11)

$$\int \tan^3 x\,dx = \int \tan x \cdot \tan^2 x\,dx$$

$$\overset{④}{=} \int \tan x \cdot (\sec^2 x - 1)\,dx = \int \tan x \cdot \sec^2 x\,dx - \int \tan x\,dx$$

$$= \frac{\tan^2 x}{2} - \ln|\sec x| + C$$

32. (Section 8.3, Problem 17)

$$\int \tan^2 x \sec^2 x\,dx = \int u^2\,du = \frac{u^3}{3} + C = \frac{(\tan x)^3}{3} + C$$

Let $u=\tan x$

$du = \sec^2 x\,dx$

30. (Section 8.3, Problem 14)

$$\int \cos^3 x \cos 2x\,dx = \int \cos x \cdot \cos^2 x \cdot \cos 2x\,dx$$

$$\overset{⑤}{=} \int \cos x \cdot (1-\sin^2 x)\cdot(1-2\sin^2 x)\,dx$$

$$= \int \cos x \cdot (1 - 3\sin^2 x + 2\sin^4 x)\,dx$$

$$= \frac{2}{5}(\sin x)^5 - \sin^3 x + \sin x + C$$

33. (Section 8.3, Problem 24)

$$\int \tan^4 x\,dx \overset{④}{=} \int \tan^2 x \cdot \tan^2 x\,dx = \int \tan^2 x \cdot (\sec^2 x - 1)\,dx$$

$$= \int \tan^2 x \sec^2 x\,dx - \int \tan^2 x\,dx \overset{④}{=} \int \tan^2 x \sec^2 x\,dx - \int \sec^2 x\,dx + \int dx$$

$$= \frac{(\tan x)^3}{3} - \tan x + x + C$$

31. (Section 8.3, Problem 15)

$$\int \sin 2x \cos 3x\,dx = \frac{1}{2}\int \sin 5x - \sin x\,dx$$

$\sin(3x+2x) = (\sin 3x)(\cos 2x) + (\cos 3x)(\sin 2x)$

$\sin(3x-2x) = \quad " \quad - \quad "$

$$= -\frac{1}{10}\cos 5x + \frac{1}{2}\cos x + C$$

34. (Section 8.3, Problem 27)

$$\int \sin 5x \sin 2x\,dx = \frac{1}{2}\int \cos 3x - \cos 7x\,dx$$

$\cos 3x = \cos(5x-2x) = (\cos 5x)(\cos 2x) + (\sin 5x)(\sin 2x)$

$\cos 7x = \cos(5x+2x) = (\cos 5x)(\cos 2x) - (\sin 5x)(\sin 2x)$

$$= \frac{1}{2}\cdot\frac{\sin 3x}{3} - \frac{1}{2}\cdot\frac{\sin 7x}{7} + C$$

$$= \frac{\sin 3x}{6} - \frac{\sin 7x}{14} + C$$

35. (Section 8.3, Problem 28)

$$\int \sec^4 3x\,dx \overset{④}{=} \int \sec^2 3x\cdot(1+\overset{\sec^2 3x}{\overset{\parallel}{\tan^2 3x}})\,dx$$

$$= \int \sec^2 3x\,dx + \int \sec^2 3x \tan^2 3x\,dx$$

$$= \frac{1}{3}\tan 3x + \frac{1}{3}\cdot\frac{(\tan 3x)^3}{3} + C = \boxed{\frac{(\tan 3x)^3}{9} + \frac{\tan 3x}{3}} + C$$

36. (Section 8.3, Problem 31)

$$\int \tan^5 3x\,dx = \int \tan^3 3x\cdot \underline{\tan^2 3x}\,dx$$

$$= \int \tan^3 3x\cdot(\underline{\sec^2 3x - 1})\,dx$$

$$= \int \tan^3 3x\cdot \sec^2 3x\,dx \boxed{-\int \tan^3 3x\,dx} \rightarrow \tan 3x\cdot \underline{\tan^2 3x}$$

$$= \int \tan^3 3x\cdot \sec^2 3x\,dx - \int \tan 3x\,(\underline{\sec^2 3x - 1})\,dx$$

$$= \int \tan^3 3x \sec^2 3x\,dx - \int \tan 3x \sec^2 3x + \int \tan 3x\,dx$$

$$= \frac{1}{3}\frac{(\tan 3x)^4}{4} - \frac{1}{3}\frac{(\tan 3x)^2}{2} + \frac{1}{3}\ln|\sec 3x| + C$$