

MATH 1432, SECTION 12869

SPRING 2014

HOMEWORK ASSIGNMENT 4

DUE DATE: 2/10/14 IN LAB

Name: Sol

ID: _____

INSTRUCTIONS

- Print out this file and complete the problems. You must do all the problems!
- If the problem is from the text, the section number and problem number are in parentheses.
- Use a blue or black pen or a pencil (dark).
- Write your solutions in the spaces provided. You must show work in order to receive credit for a problem.
- Remember that your homework must be complete, neatly written and stapled.
- Submit the completed assignment to your Teaching Assistant in lab on the due date.
- If you do not do all of the problems, then your recitation quiz from the previous Friday will automatically become a ZERO.

1. (Section 8.1, Problem 5)

$$\int \sec^2(1-x) dx = -\tan(1-x) + C$$

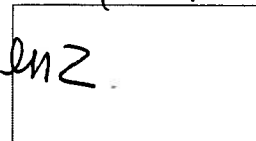


2. (Section 5.1, Problem 8)

$$\int_0^1 \frac{x^3}{1+x^4} dx = \frac{1}{4} \int_0^1 \frac{du}{1+u} = \frac{1}{4} \ln|1+u| \Big|_0^1$$

$$\begin{aligned} \text{let } u=x^4 \quad (x=0 \Rightarrow u=0) \\ \quad \quad \quad (x=1 \Rightarrow u=1) \\ du=4x^3 dx \end{aligned} = \frac{1}{4} \ln 2 - \frac{1}{4} \ln 1$$

$$= \frac{1}{4} \ln 2$$

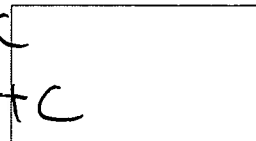


3. (Section 8.1, Problem 9)

$$\int \frac{x}{\sqrt{1-x^2}} dx = -\frac{1}{2} \int \frac{du}{\sqrt{u}} = -\frac{1}{2} \cdot \frac{\sqrt{u}}{1/2} + C$$

$$\begin{aligned} \text{let } u=1-x^2 \\ du=-2x dx \end{aligned} = -\sqrt{u} + C$$

$$= -\sqrt{1-x^2} + C$$



4. (Section 8.1, Problem 13)

$$\int_1^2 \frac{e^{1/x}}{x^2} dx = -\int_1^{\frac{1}{2}} e^u du$$

$$= -(e^{\frac{1}{2}} - e^1) = e^1 - e^{\frac{1}{2}}$$

$$\begin{aligned} \text{let } u=\frac{1}{x} \\ du=-\frac{1}{x^2} dx \end{aligned}$$



5. (Section 8.1, Problem 17)

$$\int \frac{\sec^2 \theta}{\sqrt{3 \tan \theta + 1}} d\theta = \frac{1}{3} \int \frac{du}{\sqrt{u}} = \frac{2}{3} \sqrt{u} + C.$$

$$= \frac{2}{3} \sqrt{3 \tan \theta + 1} + C$$

Let $u = 3 \tan \theta + 1$

$du = 3 \sec^2 \theta d\theta$



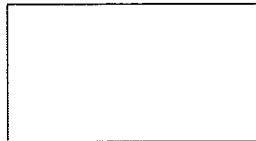
6. (Section 8.1, Problem 18)

$$\int \frac{\sin \phi}{3 - 2 \cos \phi} d\phi = \frac{1}{2} \int \frac{du}{u} = \frac{\ln|u|}{2} + C$$

$$= \frac{1}{2} \ln|3 - 2 \cos \phi| + C.$$

Let $u = 3 - 2 \cos \phi$

$du = 2 \sin \phi d\phi$



7. (Section 8.1, Problem 23)

$$\int \frac{x}{\sqrt{1-x^4}} dx = \frac{1}{2} \int \frac{du}{\sqrt{1-u^2}} = \frac{1}{2} \sin^{-1} u + C$$

$$= \frac{1}{2} \sin^{-1}(x^2) + C.$$

Let $u = x^2$

$du = 2x dx$



8. (Section 8.1, Problem 24)

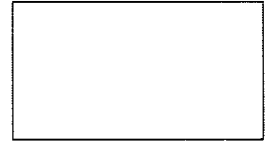
$$\int \frac{e^x}{1+e^{2x}} dx = \tan^{-1}(e^x) + C$$



9. (Section 8.1, Problem 25) ~~$(x^2+6x+10) = (x^2+6x+9) + 1 = (x+3)^2 + 1$~~

$$\int \frac{dx}{x^2+6x+10} \stackrel{\downarrow}{=} \int \frac{1}{1+(x+3)^2} dx$$

$$= \tan^{-1}(x+3) + C$$



10. (Section 8.1, Problem 30)

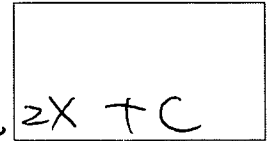
$$\int \cosh 2x \sinh^3 2x dx = \frac{1}{2} \int u^3 du$$

$$= \frac{1}{2} \frac{u^4}{4} + C$$

Let $u = \sinh 2x$

$du = 2 \cosh 2x dx$

$= \frac{1}{8} \sinh^4 2x + C$

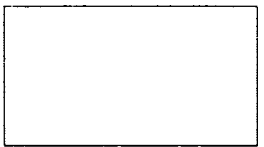


I.L.A.T.E.
 Inverse log poly th exp.

11. (Section 8.1, Problem 31)

$$\int \frac{\sin^{-1} x}{\sqrt{1-x^2}} dx = \frac{(\sin^{-1} x)^2}{2} + C$$

or let $u = \sin^{-1} x$, $du = \frac{dx}{\sqrt{1-x^2}}$
 $= \int u du = \frac{u^2}{2} + C$



12. (Section 8.2, Problem 4)

$$\int x \ln x^2 dx = (\ln x^2) \frac{x^2}{2} - \int x dx = \frac{x^2}{2} \ln x^2 - \frac{x^2}{2} + C$$

$u = \ln x^2$, $dv = x dx$ (or $= x^2 \ln x - \frac{x^2}{2} + C$)

$du = \frac{2}{x} dx$, $v = \frac{x^2}{2}$

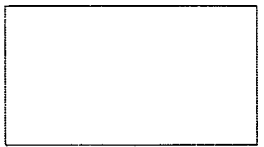


13. (Section 8.2, Problem 5)

$$\int_0^1 \frac{x^2 e^{-x}}{e} dx = -x e^{-x} - 2x e^{-x} - 2e^{-x} \Big|_0^1$$

$$= -e^{-1} - 2e^{-1} - 2e^{-1} + 0 + 0 + 2e^0 = 2 - 5e^{-1}$$

(u) (dv)
 $x^2 e^{-x}$
 $2x e^{-x}$
 $2 e^{-x}$
 $0 - e^{-x}$
 stop!



$$\int_0^{\frac{\pi}{2}} x \cos \pi x dx = x \frac{\sin \pi x}{\pi} - \frac{\cos \pi x}{\pi^2} \Big|_0^{\frac{\pi}{2}}$$

(Section 8.2, Problem 19)

$$\int_0^{\frac{1}{2}} x \cos \pi x dx = +x \frac{\sin \pi x}{\pi} - \frac{\cos \pi x}{\pi^2} \Big|_0^{\frac{1}{2}}$$

$$= +\frac{1}{2\pi} + 0 + 0 + \frac{1}{\pi^2} = \frac{1}{2\pi} + \frac{1}{\pi^2}$$

$u = x$, $dv = \cos \pi x$
 $du = dx$, $v = \frac{\sin \pi x}{\pi}$



(Section 8.2, Problem 21)

$$\int_0^{\frac{\pi}{2}} x^2 \sin x dx = -x^2 \cos x + 2x \sin x + 2 \cos x \Big|_0^{\frac{\pi}{2}}$$

$$= -0 + \pi + 0 + 0 - 0 - 2 = \pi - 2$$

$u = x^2$, $dv = \sin x$
 $du = 2x dx$, $v = -\cos x$
 $2x \sin x$
 $2 - \sin x$
 $0 \cos x$



(Section 8.2, Problem 24)

$$\int e^x \sin x dx = e^x \sin x - \int e^x \cos x dx$$

$$= e^x \sin x - e^x \cos x + \int e^x \sin x dx$$

$$\Rightarrow \int e^x \sin x dx = \frac{1}{2} (e^x \sin x - e^x \cos x) + C$$

17. (Section 8.2, Problem 29)

$$\int x^3 \sin x^2 dx = \int x^2 \cdot \sin x^2 \cdot x dx = \frac{1}{2} \int u \sin u du$$

$u = x^2$
 $dv = \sin x^2$
 $du = 2x dx$
 $v = -\cos x^2$

Let $u = x^2$, $du = 2x dx$

I.B.P.

$\frac{d}{du}$	u	$\sin u$	\downarrow	$+$
		$-\cos u$	$-$	
		$-\sin u$	$+$	

$$= -\frac{1}{2} u \cos u + \frac{1}{2} \sin u + C$$

$$= -\frac{1}{2} x^2 \cos x^2 + \frac{1}{2} \sin x^2 + C$$

18. (Section 8.2, Problem 31)

$$\int_0^{\frac{1}{4}} \frac{1}{\sqrt{1-4x^2}} dx = x \cdot \sin^{-1} 2x \Big|_0^{\frac{1}{4}} - \int_0^{\frac{1}{4}} \frac{x}{\sqrt{1-4x^2}} dx$$

$u = \sin^{-1} 2x$
 $dv = 1 \cdot dx$
 $du = \frac{2}{\sqrt{1-4x^2}}$
 $v = x$

$$= \frac{1}{4} \cdot \frac{\pi}{6} + \frac{1}{4} \sqrt{1-4x^2} \Big|_0^{\frac{1}{4}}$$

$$= \frac{\pi}{24} + \frac{1}{4} \left(\frac{\sqrt{3}}{2} - 1 \right)$$

19. (Section 8.2, Problem 33)

$$\int_0^1 x \tan^{-1} x^2 dx = \frac{x^2}{2} \tan^{-1} x^2 \Big|_0^1 - \frac{1}{2} \int_0^1 \frac{2x^3}{1+x^4} dx$$

$u = \tan^{-1} x^2$
 $dv = x dx$
 $du = \frac{2x}{1+x^4} dx$
 $v = \frac{x^2}{2}$

$$= \frac{1}{2} \cdot \frac{\pi}{4} - \frac{1}{4} \ln|1+x^4| \Big|_0^1$$

$$= \frac{\pi}{8} - \frac{1}{4} \ln 2$$

20. (Section 8.2, Problem 35)

$$\int x^2 \cosh 2x dx = \frac{x^2}{2} \sinh 2x - \frac{x}{2} \cosh 2x + \frac{1}{4} \sinh 2x + C$$

u	dv	
x^2	$\cosh 2x$	$+$
$2x$	$\frac{\sinh 2x}{2}$	$-$
2	$\frac{\cosh 2x}{4}$	$+$
0	$\frac{\sinh 2x}{4}$	$-$

21. (Section 8.2, Problem 38)

$$\int \cos(\ln x) dx = x \cos(\ln x) + \int \sin(\ln x) dx$$

$u = \cos(\ln x)$
 $dv = 1 \cdot dx$
 $du = -\frac{\sin(\ln x)}{x} dx$
 $v = x$

$$= x \cos(\ln x) + x \sin(\ln x) - \int \cos(\ln x) dx$$

$u = \sin(\ln x)$
 $dv = 1 \cdot dx$
 $du = \frac{\cos(\ln x)}{x} dx$
 $v = x$

$$\Rightarrow \int \cos(\ln x) dx = \frac{1}{2} (x \cos(\ln x) + x \sin(\ln x)) + C$$

22. (Section 8.2, Problem 39)

$$\int \sin(\ln x) dx = x \sin(\ln x) - \int \cos(\ln x) dx$$

Use ② of above (21)

$$= x \sin(\ln x) - x \cos(\ln x) + (-\int \sin(\ln x) dx)$$

Use ① of above.

$$\Rightarrow \int \sin(\ln x) dx = \frac{1}{2} (x \sin(\ln x) - x \cos(\ln x)) + C$$

$$\textcircled{1} \sin x + \cos x = 1$$

$$\textcircled{2} \cos^2 x = \frac{1}{2} + \frac{1}{2} \cos 2x \quad 1. \sin^2 x + \cos^2 x = 1$$

$$\textcircled{3} \sin^2 x = \frac{1}{2} - \frac{1}{2} \cos 2x$$

23. (Section 8.3, Problem 1)

$$\int \sin^3 x \, dx = \int \sin x \cdot \sin^2 x \, dx$$

$$\text{by } \textcircled{1} = \int \sin x \cdot (1 - \cos^2 x) \, dx$$

$$= -\int 1 - u^2 \, du = \int u^2 - 1 \, du$$

$$\begin{array}{l} \text{let } u = \cos x \\ du = -\sin x \, dx \end{array} = \frac{u^3}{3} - u + C = \frac{(\cos x)^3}{3} - \cos x + C$$

24. (Section 8.3, Problem 2)

$$\int_0^{\frac{\pi}{8}} \cos^2 4x \, dx \stackrel{\text{by } \textcircled{2}}{=} \int_0^{\frac{\pi}{8}} \frac{1}{2} + \frac{1}{2} \cos 8x \, dx$$

$$= \frac{x}{2} + \frac{1}{16} \sin 8x \Big|_0^{\frac{\pi}{8}}$$

$$= \frac{\pi}{16} + \frac{1}{16} \cdot 0 - 0 = \frac{\pi}{16}$$

25. (Section 8.3, Problem 3)

$$\int_0^{\frac{\pi}{6}} \sin^2 3x \, dx \stackrel{\text{by } \textcircled{3}}{=} \int_0^{\frac{\pi}{6}} \frac{1}{2} - \frac{1}{2} \cos 6x \, dx$$

$$= \frac{x}{2} - \frac{1}{12} \sin 6x \Big|_0^{\frac{\pi}{6}}$$

$$= \frac{\pi}{12} - 0 + 0 = \frac{\pi}{12}$$

26. (Section 8.3, Problem 5)

$$\int \cos^4 x \cdot \sin^3 x \, dx = \int \cos^4 x \cdot \sin^2 x \cdot \sin x \, dx$$

$$\text{by } \textcircled{1} = \int \cos^4 x \cdot (1 - \cos^2 x) \cdot \sin x \, dx$$

$$\begin{array}{l} \text{let } u = \cos x \\ du = -\sin x \, dx \end{array} = -\int u^4 (1 - u^2) \, du = \int u^6 - u^4 \, du = \frac{u^7}{7} - \frac{u^5}{5} + C = \frac{(\cos x)^7}{7} - \frac{(\cos x)^5}{5} + C$$

27. (Section 8.3, Problem 9)

$$\int \sec^2 \pi x \, dx = \frac{1}{\pi} \tan \pi x + C$$

28. (Section 8.3, Problem 10)

$$\int \csc^2 2x \, dx = -\frac{1}{2} \cot 2x + C$$

$$\textcircled{4} 1 + \tan^2 x = \sec^2 x$$

$$\textcircled{5} = \textcircled{3} 1 - 2\sin^2 x = \cos 2x$$

29. (Section 8.3, Problem 11)

$$\int \tan^3 x dx = \int \tan x \cdot \tan^2 x dx$$

$$\begin{aligned} \textcircled{4} \int \tan x \cdot (\sec^2 x - 1) dx &= \int \tan x \cdot \sec^2 x dx - \int \tan x dx \\ &= \frac{\tan^2 x}{2} - \ln|\sec x| + C \end{aligned}$$



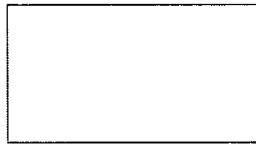
30. (Section 8.3, Problem 14)

$$\int \cos^3 x \cos 2x dx = \int \cos x \cdot \cos^2 x \cdot \cos 2x dx$$

$$\textcircled{5} \int \cos x \cdot (1 - \sin^2 x) \cdot (1 - 2\sin^2 x) dx$$

$$= \int \cos x \cdot (1 - 3\sin^2 x + 2\sin^4 x) dx$$

$$= \frac{2}{5}(\sin x)^5 - \sin^3 x + \sin x + C$$



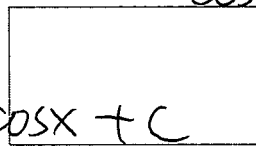
31. (Section 8.3, Problem 15)

$$\int \sin 2x \cos 3x dx = \frac{1}{2} \int \sin 5x - \sin x dx$$

$$\sin(3x+2x) = \sin 3x \cos 2x + \cos 3x \sin 2x$$

$$\sin(3x-2x) = \sin 3x \cos 2x - \cos 3x \sin 2x$$

$$= -\frac{1}{10} \cos 5x + \frac{1}{2} \cos x + C$$



32. (Section 8.3, Problem 17)

$$\int \tan^2 x \sec^2 x dx = \int u^2 du$$

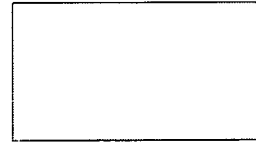
$$\begin{aligned} \text{Let } u = \tan x \quad &= \frac{u^3}{3} + C = \frac{(\tan x)^3}{3} + C \\ du = \sec^2 x dx & \end{aligned}$$



33. (Section 8.3, Problem 24)

$$\int \tan^4 x dx \stackrel{\textcircled{4}}{=} \int \tan^2 x \cdot \tan^2 x dx = \int \tan^2 x \cdot (\sec^2 x - 1) dx$$

$$\begin{aligned} &= \int \tan^2 x \sec^2 x dx - \int \tan^2 x dx \stackrel{\textcircled{4}}{=} \int \tan^2 x \sec^2 x dx - \int \sec^2 x dx + \int dx \\ &= \frac{(\tan x)^3}{3} - \tan x + x + C \end{aligned}$$



34. (Section 8.3, Problem 27)

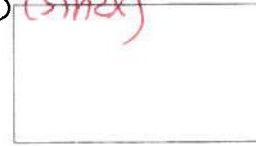
$$\int \sin 5x \sin 2x dx = \frac{1}{2} \int \cos 3x - \cos 7x dx$$

$$\cos 3x = \cos(5x-2x) = \cos 5x \cos 2x + \sin 5x \sin 2x$$

$$\cos 7x = \cos(5x+2x) = \cos 5x \cos 2x - \sin 5x \sin 2x$$

$$= \frac{1}{2} \cdot \frac{\sin 3x}{3} - \frac{1}{2} \cdot \frac{\sin 7x}{7} + C$$

$$= \frac{\sin 3x}{6} - \frac{\sin 7x}{14} + C$$



35. (Section 8.3, Problem 28)

$$\begin{aligned}
 \int \sec^4 3x \, dx &\stackrel{\textcircled{4}}{=} \int \sec^2 3x \cdot \overbrace{(1 + \tan^2 3x)}^{\sec^2 3x} \, dx \\
 &= \int \sec^2 3x \, dx + \int \sec^2 3x \tan^2 3x \, dx \\
 &= \frac{1}{3} \tan 3x + \frac{1}{3} \cdot \frac{(\tan 3x)^3}{3} + C = \boxed{\frac{(\tan 3x)^3}{9} + \frac{\tan 3x}{3}} + C
 \end{aligned}$$

36. (Section 8.3, Problem 31)

$$\begin{aligned}
 \int \tan^5 3x \, dx &= \int \tan^3 3x \cdot \tan^2 3x \, dx \\
 &= \int \tan^3 3x \cdot (\sec^2 3x - 1) \, dx \\
 &= \int \tan^3 3x \cdot \sec^2 3x \, dx - \int \tan^3 3x \, dx \quad \rightarrow \tan 3x \cdot \underline{\tan^2 3x} \\
 &= \int \tan^3 3x \cdot \sec^2 3x \, dx - \int \tan 3x (\sec^2 3x - 1) \, dx \\
 &= \int \tan^3 3x \sec^2 3x \, dx - \int \tan 3x \sec^2 3x + \int \tan 3x \, dx \\
 &= \frac{1}{3} \frac{(\tan 3x)^4}{4} - \frac{1}{3} \frac{(\tan 3x)^2}{2} + \frac{1}{3} \ln |\sec 3x| + C
 \end{aligned}$$

