MATH 1432, SECTION 12869
SPRING 2014
HOMEWORK ASSIGNMENT 3
DUE DATE: 2/3/16 IN LAB

Name: ______________________

ID: ______________________

INSTRUCTIONS

- Print out this file and complete the problems. You must do all the problems!
- If the problem is from the text, the section number and problem number are in parantheses.
- Use a blue or black pen or a pencil (dark).
- Write your solutions in the spaces provided. You must show work in order receive credit for a problem.
- Remember that your homework must be complete, neatly written and stapled.
- Submit the completed assignment to your Teaching Assistant in lab on the due date.
- If you do not do all of the problems, then your recitation quiz from the previous Friday will automatically become a ZERO.

1. (Section 7.6, Problem 2)

Suppose the initial money is A_0 and
We want to find the time t s.t. a sum of money to $2A_0$ (r: interest rate)

$\Rightarrow 2A_0 = A_0 e^{rt} \Rightarrow e^{rt} = 2$

take "ln" $\Rightarrow$ $rt = \ln 2 \Rightarrow t = \frac{\ln 2}{r}$

(a) $r = 0.06 \Rightarrow t = \frac{\ln 2}{0.06} = \frac{0.69}{0.06} = 11.55$ years

(b) $r = 0.08 \Rightarrow t = \frac{\ln 2}{0.08} = 8.66$ years

(c) $r = 0.1 \Rightarrow t = \frac{\ln 2}{0.1} = 6.9$ years.

2. (Section 7.6, Problem 3)

Initial: A_0. $t: 20$ years. Find r s.t. $A_0 e^{20r} = 3A_0$

$\Rightarrow 3 = e^{20r} \Rightarrow \ln 3 = 20r$

$r = \frac{\ln 3}{20} = 5.5\%$.

3. (Section 7.6, Problem 5)

initial: 1000 bacteria

After 30 mins, there are 2000 $\rightarrow = 0.5$ hr

~~How~~ After 2 hrs $\rightarrow$?

Grow proportional $A(t) = A(0) e^{kt}$

$2000 = A(0.5) = A(0) \cdot e^{k \cdot 0.5} = 1000 \cdot e^{k \cdot 0.5}$

$\Rightarrow 2 = e^{\frac{k}{2}} \Rightarrow \ln 2 = \frac{k}{2} \Rightarrow k = 2\ln 2 =$

$A(2) = 1000 \cdot e^{2 \cdot 2\ln 2} = 1000 \cdot 16 = 16{,}000$.

4. (Section 7.6, Problem 6)

initial: P_0

After Every four hours $\Rightarrow$ Triple.

After 12 hours $\Rightarrow 10^6$

$3P_0 = P(4) = P_0 \cdot e^{k4} \Rightarrow 3 = e^{k4} \Rightarrow k = \frac{\ln 3}{4}$

(a) $10^6 = P(12) = P_0 \cdot e^{\frac{\ln 3}{4} \cdot 12} \Rightarrow P_0 = \frac{10^6}{3^3} = 37037$

(b) $2P_0 = P_0 e^{\frac{\ln 3}{4} \cdot t}$

$\Rightarrow 2 = e^{\frac{\ln 3}{4} t} \Rightarrow \ln 2 = \frac{\ln 3}{4} t \Rightarrow t = \frac{4\ln 2}{\ln 3} = 2.52$ hours

5. (Section 7.6, Problem 15)

Initial: 200 liters $= V(0)$

$V(5) = 200(1-20\%) = 160$

$160 = 200 \cdot e^{k\cdot 5} \Rightarrow \frac{4}{5} = e^{k\cdot 5} \Rightarrow \ln\frac{4}{5} = k5$

$\Rightarrow k = \frac{1}{5}\ln(\frac{4}{5})$

$\Rightarrow V = 200 \cdot e^{\frac{1}{5}\ln(\frac{4}{5})t}$ or $200\cdot(\frac{4}{5})^{\frac{t}{5}}$

6. (Section 7.6, Problem 16)

Initial A_0. and $A(5) = \frac{2A_0}{3}$. Find t s.t.

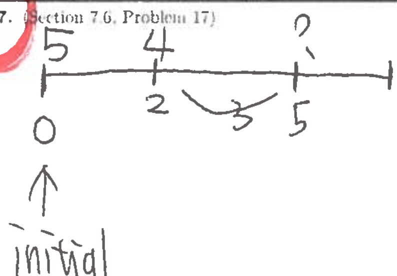
$A(t) = \frac{A_0}{2}$

$\frac{2A_0}{3} = A_0 \cdot e^{k\cdot 5} \Rightarrow \ln(\frac{2}{3}) = k\cdot 5 \Rightarrow k = \frac{1}{5}\ln(\frac{2}{3})$

$A(t) = A_0 \cdot e^{\frac{t}{5}\ln(\frac{2}{3})}$

$\frac{A_0}{2} = A_0 \cdot e^{\frac{t}{5}\ln(\frac{2}{3})} \Rightarrow \ln(\frac{1}{2}) = \frac{t}{5}\ln(\frac{2}{3}) \Rightarrow t = \frac{5\ln(\frac{1}{2})}{\ln(\frac{2}{3})}$

7. (Section 7.6, Problem 17)



$4 = 5\cdot e^{k\cdot 2} \Rightarrow k = 2^{-1}\cdot\ln(\frac{4}{5})$.

$? = 5\cdot e^{k\cdot 5} \Rightarrow = 5\cdot(\frac{4}{5})^{\frac{5}{2}} = 2.86$ grams.

$= 5\cdot e^{5\cdot 2^{-1}\cdot\ln(\frac{4}{5})} =$

8. (Section 7.6, Problem 21)

initial A_0. ② $A(1620) = \frac{A_0}{2}$.

① $A(500) = ?$ $A(t) = (1-75\%)A_0$. Find t?

$\frac{A_0}{2} = A(1620) = A_0\cdot e^{k\cdot 1620} \Rightarrow k = \frac{\ln(\frac{1}{2})}{1620}$

① $A(500) = A_0\cdot e^{\ln(\frac{1}{2})\cdot\frac{500}{1620}} = A_0\cdot(\frac{1}{2})^{\frac{500}{1620}}$

$\Rightarrow \frac{A(500)}{A_0} = (\frac{1}{2})^{\frac{500}{1620}}$

② $0.25A_0 = A_0 e^{kt}$ $t = \frac{\ln(\frac{1}{4})}{\ln(\frac{1}{2})}\cdot 1620 = 3240$.

9. (Section 7.6, Problem 22)

Initial: A_0. $A(5.13) = \frac{A_0}{2}$.

① $A(8) = ?$ ② Find A_0 if $A(3) = 100$.

$\frac{A_0}{2} = A(5.13) = A_0 e^{k\cdot 5.13} \Rightarrow k = \frac{1}{5.13}\ln(\frac{1}{2})$

① $A(8) = A_0\cdot e^{\frac{1}{5.13}\ln(\frac{1}{2})\cdot 8} = A_0(\frac{1}{2})^{\frac{8}{5.13}}$

$\frac{A(8)}{A_0} = (\frac{1}{2})^{\frac{8}{5.13}} = 0.35$

② $100 = A(3) = A_0\cdot e^{3k}$

$A_0 = \frac{100}{e^{3k}} = 148$ grams.

10. (Section 7.6, Problem 29)

Initial: 16,000. $Q(t) = 16{,}000\times e^{rt}$.

① $r = 0.05 \Rightarrow Q(3) = 16000\times e^{0.05\times 3} = 18589$.

② $r = 0.08 \Rightarrow 16000\times e^{0.08\times 3} = 20339$

③ $r = 0.12 \Rightarrow 16000\times e^{0.12\times 3} = 22933$.

sin + | All +

tan + | cos +

11. (Section 7.7, Problem 3)

(a) $\cos^{-1}\left(-\frac{1}{2}\right) = \frac{2\pi}{3}$

(b) $\sec^{-1}(-\sqrt{2}) = \frac{3\pi}{4}$

12. (Section 7.7, Problem 6)

(a) $\arcsin\left[\sin\left(\frac{11\pi}{6}\right)\right]$

$= \arcsin\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$

(b) $\arctan\left(\tan\left[\frac{11\pi}{4}\right]\right)$

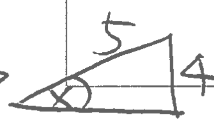
$= \arctan(-1) = -\frac{\pi}{4}$.

13. (Section 7.7, Problem 9)

(a) $\sin\left(2\cos^{-1}\left[\frac{1}{2}\right]\right)$

$= \sin\left(2\cdot\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$

(b) $\cos\left(2\sin^{-1}\left[\frac{4}{5}\right]\right)$

$\sin^{-1}\frac{4}{5} = x \Leftrightarrow \sin x = \frac{4}{5} \Rightarrow$ 

$\Rightarrow \cos x = \frac{3}{5}$

$\cos(2x) = (\cos x)^2 - (\sin x)^2$

$= \frac{9}{25} - \frac{16}{25} = \frac{-7}{25}$

14. (Section 7.7, Problem 12)

$y = \tan^{-1}\sqrt{x}$ Find y'?

$\Updownarrow$

$\sqrt{x} = \tan y \overset{\frac{d}{dx}}{\Longrightarrow} \frac{1}{2\sqrt{x}} = \sec^2 y \cdot \frac{dy}{dx}$

$\Rightarrow \frac{dy}{dx} = \frac{1}{2\sqrt{x}} \cdot \frac{1}{\sec^2 y}$

or using formula, $\frac{dy}{dx} = \frac{1}{1+(\sqrt{x})^2} \cdot (\sqrt{x})' = \frac{1}{1+(\sqrt{x})^2} \cdot \frac{1}{2\sqrt{x}}$

15. (Section 7.7, Problem 13)

$f(x) = \sec^{-1}(2x^2)$

~~$f'(x) = \frac{1}{|2x^2|\sqrt{(2x^2)^2-1}} \cdot 4x$~~

$f'(x) = \frac{1}{|2x^2|\sqrt{(2x^2)^2-1}} \cdot 4x$

$= \frac{2}{x\sqrt{4x^4-1}}$

16. (Section 7.7, Problem 14)

$f(x) = e^x \sin^{-1} x$.

$f'(x) = e^x \cdot \sin^{-1} x + \frac{e^x}{\sqrt{1-x^2}}$

17. (Section 7.7, Problem 19)

$$y=\frac{\tan^{-1}x}{x}$$

$$y'=\frac{x\cdot\frac{1}{1+x^2}-\tan^{-1}(x)}{x^2}$$

$$=\frac{x-(1+x^2)\tan^{-1}x}{x^2(1+x^2)}.$$

18. (Section 7.7, Problem 22)

$$f(x)=\ln(\tan^{-1}x)$$

$$f'(x)=\frac{1}{\tan^{-1}x}\cdot\frac{1}{1+x^2}.$$

19. (Section 7.7, Problem 23)

$$y=\tan^{-1}(\ln x).$$

$$y'=\frac{1}{1+(\ln x)^2}\cdot\frac{1}{x}$$

20. (Section 7.7, Problem 25)

$$\theta=\sin^{-1}\left(\sqrt{1-r^2}\right).$$

$$\theta'=\frac{1}{\sqrt{1-(\sqrt{1-r^2})^2}}\cdot\frac{-2r}{2(\sqrt{1-r^2})}.$$

$$=\frac{1}{\sqrt{r^2}}\cdot\frac{-r}{\sqrt{1-r^2}}=\frac{-r}{|r|\sqrt{1-r^2}}$$

21. (Section 7.7, Problem 41)

$$\int_0^{\frac{1}{\sqrt{2}}}\frac{dx}{\sqrt{1-x^2}}=\sin^{-1}x\Big|_0^{\frac{1}{\sqrt{2}}}=\frac{\pi}{4}-0=\frac{\pi}{4}.$$

22. (Section 7.7, Problem 42)

$$\int_0^1\frac{dx}{\sqrt{4-x^2}}=\sin^{-1}\frac{x}{2}\Big|_0^1=\frac{\pi}{6}.$$

$\uparrow$ a^2 $\;=\sin^{-1}\frac{x}{a}.$

23. (Section 7.7, Problem 43)

$$\int_0^5 \frac{dx}{25+x^2} = \frac{1}{5}\tan^{-1}\frac{x}{5}\Big|_0^5 = \frac{1}{5}\cdot\left(\frac{\pi}{4}\right) = \frac{\pi}{20}$$

a^2

$$\frac{1}{a}\tan^{-1}\frac{x}{a}$$

24. (Section 7.7, Problem 45)

$$\int_0^{\frac{3}{2}} \frac{dx}{9+4x^2} = \int_0^{\frac{3}{2}} \frac{dx}{4(\frac{9}{4}+x^2)} = \frac{1}{4}\int_0^{\frac{3}{2}} \frac{dx}{\frac{9}{4}+x^2}$$

a^2

$$= \frac{1}{4}\cdot\frac{2}{3}\cdot\tan^{-1}\left(\frac{2}{3}x\right)\Big|_0^{\frac{3}{2}}$$

$$= \frac{1}{6}\cdot\left(\frac{\pi}{4}-0\right) = \frac{\pi}{24}$$

25. (Section 7.7, Problem 46)

$$\int_2^5 \frac{dx}{9+(x-2)^2} = \frac{1}{3}\tan^{-1}\left(\frac{x-2}{3}\right)\Big|_2^5$$

a^2

$$= \frac{1}{3}\cdot\left(\frac{\pi}{4}-0\right) = \frac{\pi}{12}$$

let $u=e^x$, $du=e^x dx$

26. (Section 7.7, Problem 51)

$$\int_0^{\ln 2} \frac{e^x}{1+e^{2x}}dx = \int_1^2 \frac{du}{1+u^2} = \tan^{-1}(u)\Big|_1^2$$

$$= \tan^{-1}2 - \frac{\pi}{4}.$$

let $u=x^2$, $du=2x\,dx$

27. (Section 7.7, Problem 53)

$$\int \frac{x}{\sqrt{1-x^4}}dx = \frac{1}{2}\int \frac{du}{\sqrt{1-u^2}} = \frac{1}{2}\sin^{-1}u + C.$$

$$= \frac{1}{2}\sin^{-1}x^2 + C.$$

let $u=\tan x$, $du=\sec^2 x\,dx$

28. (Section 7.7, Problem 57)

$$\int \frac{\sec^2 x}{9+\tan^2 x}dx = \int \frac{1}{9+u^2}du$$

$$= \frac{1}{3}\tan^{-1}\left(\frac{u}{3}\right)+C$$

$$= \frac{1}{3}\tan^{-1}\left(\frac{\tan x}{3}\right)+C$$

EMCF $\int_{-3}^{3} \frac{x}{\sqrt{1+3x^2}}\,dx$

29. (Section 7.7, Problem 59)

$\int \frac{\sin^{-1}x}{\sqrt{1-x^2}}\,dx = \int u\,du = \frac{u^2}{2} + C$

$= \frac{(\sin^{-1}x)^2}{2} + C.$

Let $u = \sin^{-1}x$.

$du = \frac{dx}{\sqrt{1-x^2}}$

30. (Section 7.7, Problem 62)

$\int \frac{dx}{x[1+(\ln x)^2]} = \int \frac{du}{1+u^2} = \tan^{-1}u + C$

$= \tan^{-1}(\ln x) + C.$

Let $u = \ln x$

$du = \frac{dx}{x}$

31. (Section 7.7, Problem 63)

$\int_0^3 \frac{1}{\sqrt{1-x^2}}\,dx$

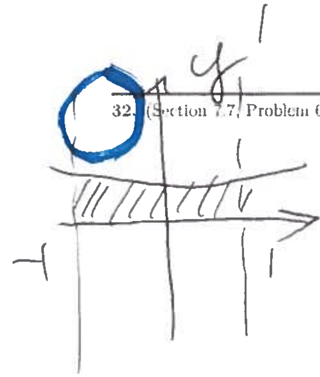
$1-x^2 < 0$ as $x > 1$,

so $\sqrt{1-x^2}$ DNE as $x > 1$.

$\Rightarrow \frac{1}{\sqrt{1-x^2}}$ isn't continuous on $[0,3]$.

$\Rightarrow$ We can't use The Fundamental Thm of Cal.

32. (Section 7.7, Problem 65)



$A = \int_{-1}^{1} \frac{dx}{\sqrt{4-x^2}} = \sin^{-1}\left(\frac{x}{2}\right)\Big|_{-1}^{1}$

$= \frac{\pi}{6} - \left(-\frac{\pi}{6}\right) = \frac{\pi}{3}$

33. (Section 7.8, Problem 1)

$y = \sinh x^2 \Rightarrow y' = 2x\cosh x^2$

(or by def. $\sinh x^2 = \frac{e^{x^2} - e^{-x^2}}{2}$

$\overset{\frac{d}{dx}}{\Rightarrow} (\sinh x^2)' = \frac{2xe^{x^2} + 2xe^{-x^2}}{2}$

$= 2x\left(\frac{e^{x^2} + e^{-x^2}}{2}\right) = 2x\cosh x^2$)

34. (Section 7.8, Problem 3)

$y = \sqrt{\cosh ax} = (\cosh ax)^{\frac{1}{2}}$

$y' = \frac{1}{2} \cdot a \frac{1}{\sqrt{\cosh ax}}$

35. (Section 7.8, Problem 4)

$y=(\sinh ax)(\cosh ax)$ (Using product rule)

$y' = a(\cosh ax)^2 + a(\sinh ax)^2$

$(\overset{or}{=} a\cosh 2ax)$

36. (Section 7.8, Problem 5)

$y = \dfrac{\sinh x}{\cosh x - 1}$

$y' = \dfrac{(\cosh x)(\cosh x - 1) - (\sinh x)^2}{(\cosh x - 1)^2}$

$= \dfrac{(\cosh x)^2 - (\sinh x)^2 - \cosh x}{(\cosh x - 1)^2} = \dfrac{1-\cosh x}{(\cosh x - 1)^2} = \boxed{\dfrac{1}{1-\cosh x}}$

by identity $(\cosh^2 x - \sinh^2 x = 1)$

37. (Section 7.8, Problem 12)

$y = \cosh(\ln x^3)$ (Using chain rule)

$y' = [\sinh(\ln x^3)] \cdot \dfrac{3x^2}{x^3}$

$= \dfrac{3}{x}[\sinh(\ln x^3)]$

38. (Section 7.8, Problem 14)

$y = \tan^{-1}(\sinh x)$

$y' = \dfrac{1}{1+(\sinh x)^2} \cdot (\cosh x)$

39. (Section 7.8, Problem 15)

$y = \ln(\cosh x)$

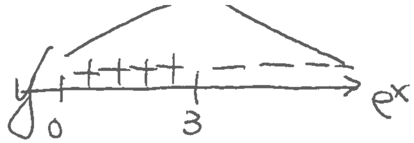
$y' = \dfrac{(\sinh x)}{(\cosh x)}$

40. (Section 7.8, Problem 17)

$y = (\sinh x)^x = e^{\ln(\sinh x)^x} = e^{x\ln(\sinh x)}$

$y' = [x \cdot \ln(\sinh x)]' \cdot e^{x\ln(\sinh x)}$

$= \left[\ln(\sinh x) + \dfrac{x\cdot(\cosh x)}{\sinh x}\right] \cdot (\sinh x)^x$



41. (Section 7.8, Problem 27)

Find abs. extr. value.

$y = -5\cosh x + 4\sinh x$

$= \frac{-5(e^x+e^{-x})}{2} + \frac{4(e^x-e^{-x})}{2}$

$= \frac{-e^x-9e^{-x}}{2} \Rightarrow y' = \frac{-e^x+9e^{-x}}{2}$

$y'=0 \Rightarrow -e^x+9e^{-x}=0$

$\Rightarrow e^x = 9e^{-x} \Rightarrow e^{2x}=9.$

$\Rightarrow e^x = \pm 3$ (but $e^x>0$).

$\Rightarrow e^x = 3$

$\Downarrow$

y has max value $\frac{-3-\frac{9}{3}}{2} = -3$ when $e^x = 3$.

42. (Section 7.8, Problem 31)

$(*)\begin{cases} y''-9y=0 \\ y(0)=2 \\ y'(0)=1 \end{cases}$

$y = A\cosh cx + B\sinh cx \quad —(1) \quad (c>0)$

$y' = Ac\sinh cx + Bc\cosh cx$

$y'' = Ac^2\cosh cx + Bc^2\sinh cx \quad —(2)$

by $(*)$, $(2)-9\times(1)=0 \Rightarrow (Ac^2-9A)\cosh cx + \cancel{(Bc^2-9B)\sinh cx = 0 \Rightarrow Ac^2-9A=0}$ $\cancel{Bc^2-9B=0}$

$y(0)=2 \Rightarrow 2=y(0)=A\cosh 0 + B\sinh 0 = A \Rightarrow \boxed{A=2}$

$y'(0)=1 \Rightarrow 1=y'(0)=Bc$ $\cancel{\Rightarrow 2c^2-9=0 \quad c=\frac{3}{2}\sqrt{2}. \quad B=\frac{\sqrt{2}}{3}.}$

$Ac^2-9A=0 \Rightarrow c^2-9=0$ (since $A\neq 0$)

$\Rightarrow \boxed{c=3}$ (since $c>0$).

$\boxed{B=\frac{1}{3}}$ (since $Bc=1$

43. (Section 7.8, Problem 33)

$\int \cosh ax\,dx = \frac{\sinh ax}{a} + C.$

44. (Section 7.8, Problem 36)

$\int(\sinh ax)\cdot(\cosh^2 ax)\,dx$

$= \frac{(\cosh ax)^3}{3a} + C$

(or let $u=(\cosh ax)$, $du = a\sinh ax\,dx$

$\frac{1}{a}\int u^2\,du = \frac{u^3}{3a} + C = \frac{(\cosh ax)^3}{3a} + C$)

45. (Section 7.8, Problem 38)

$\int \frac{\cosh ax}{\sinh ax}\,dx = \frac{1}{a}\ln|\sinh ax| + C$

(or let $u=\sinh ax$, $du = a\cosh dx$.

$\frac{1}{a}\int \frac{du}{u} = \frac{1}{a}\ln|u| + C$

$= \frac{1}{a}\ln|\sinh ax| + C$)

46. (Section 7.8, Problem 39)

$\int \frac{\sinh ax}{\cosh^2 ax}\,dx = \frac{1}{a}\int \frac{du}{u^2} = -\frac{1}{a}\frac{1}{u} + C$

$= -\frac{1}{a}\frac{1}{\cosh ax} + C$

let $u=(\cosh ax)$

$du = a(\sinh ax)\,dx$

47. (Section 7.8, Problem 43)

$$\int \frac{\sinh\sqrt{x}}{\sqrt{x}}\,dx = 2\cosh\sqrt{x} + C$$

$$\left(\begin{array}{l} \text{let } \sqrt{x} = u. \quad du = \frac{1}{2}\frac{1}{\sqrt{x}}dx \\ 2\int \frac{\sinh u}{1}\,du = 2\cosh u + C \\ \qquad\qquad\qquad = 2\cosh\sqrt{x} + C \end{array}\right)$$

48. (Section 7.8, Problem 45)

Average value of fn on $[a,b]$ $= \boxed{\frac{1}{b-a}\int_a^b f(x)\,dx}$.

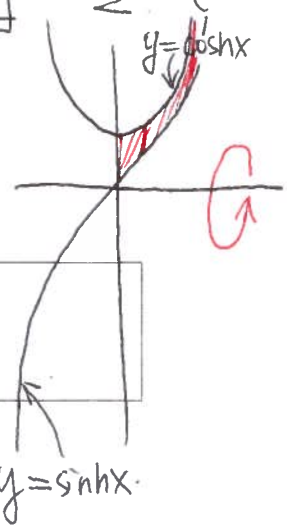
$f(x) = \cosh x \quad , x \in [-1, 1]$

$$A.V. = \frac{1}{1-(-1)}\int_{-1}^{1} \cosh x\,dx$$

$$= \frac{1}{2}\sinh x\Big|_{-1}^{1} = \frac{1}{2}\left[\left(\frac{e-e^{-1}}{2}\right) - \left(\frac{e^{-1}-e^{1}}{2}\right)\right] = \frac{e-e^{-1}}{2} \left(= \sinh(1)\right)$$

49. (Section 7.8, Problem 49)

$$\cancel{\int_0^1 \cosh x - \sinh x\,dx}$$



By Washer

$$\pi\int_0^1 (\cosh^2 x) - (\sinh^2 x)\,dx$$

$$= \pi\int_0^1 1\,dx = \pi$$

identity