Indeterminate form $\underbrace{\frac{0}{0}, \frac{\infty}{\infty}}_{\text{"L'"}}, 0\cdot\infty, \infty-\infty, \underbrace{0^0, 1^\infty, \infty^0}_{\text{Find "ln —" first, then "L'"}}$

MATH 1432, SECTION 12849
SPRING 2014
HOMEWORK ASSIGNMENT 11
DUE DATE: 4/7/14 IN LAB

Name: Sol.

ID:

INSTRUCTIONS

- Print out this file and complete the problems. You must do all the problems!
- If the problem is from the text, the section number and problem number are in parantheses.
- Use a blue or black pen or a pencil (dark).
- Write your solutions in the spaces provided. You must show work in order receive credit for a problem.
- Remember that your homework must be complete, neatly written and stapled.
- Submit the completed assignment to your Teaching Assistant in lab on the due date.
- If you do not do all of the problems, then your recitation quiz from the previous Friday will automatically become a ZERO.

1. (Section 10.6, Problem 3)

$$\lim_{x\to\infty} \frac{x^3}{1-x^3} \overset{\left(\frac{\infty}{\infty}\right)}{\underset{L'}{=}} \lim_{x\to\infty} \frac{3x^2}{-3x^2} = -1$$

2. (Section 10.6, Problem 5)

$$\lim_{x\to\infty} x^2 \sin\frac{1}{x} \overset{\infty\cdot 0}{=} \lim_{x\to\infty} \frac{\sin\frac{1}{x}}{\frac{1}{x^2}} \overset{\left(\frac{0}{0}\right)}{\underset{L'}{=}} \lim_{x\to\infty} \frac{-\frac{1}{x^2}\cos\frac{1}{x}}{-2\frac{1}{x^3}}$$

$$= \lim_{x\to\infty} +\frac{1}{2}x \underbrace{\cos\frac{1}{x}}_{\downarrow 1 \text{ as } n\to\infty} \Rightarrow \text{diverges}$$

(Section 10.6, Problem 6)

$$\lim_{x\to\infty} \frac{\ln x^k}{x} \overset{\left(\frac{\infty}{\infty}\right)}{\underset{L'}{=}} \lim_{x\to\infty} \frac{k\frac{1}{x}}{1} = \lim_{x\to\infty} \frac{k}{x} = 0$$

$$\frac{\ln x^k}{x} = \frac{k\ln x}{x}$$

4. (Section 10.6, Problem 11)

$$\lim_{x\to 0}\left[x(\ln|x|)^2\right] \overset{0\cdot\infty}{=} \lim_{x\to 0} \frac{(\ln|x|)^2}{\frac{1}{x}} \overset{\left(\frac{\infty}{\infty}\right)}{\underset{L'}{=}} \lim_{x\to 0} \frac{2\cdot\ln|x|\cdot\frac{1}{|x|}}{-\frac{1}{x^2}}$$

$$= \lim_{x\to 0} \frac{2\ln|x|}{-\frac{1}{x}} \overset{\left(\frac{\infty}{\infty}\right)}{=} \lim_{x\to 0} \frac{2\frac{1}{x}}{+\frac{1}{x^2}} = \lim_{x\to 0} 2x = 0$$

FTC

5. (Section 10.6, Problem 13)

$$\lim_{x\to\infty}\left(\frac{1}{x}\int_0^x e^{t^2}\,dt\right) \overset{(\frac{\infty}{\infty})}{\underset{L'}{=}} \lim_{x\to\infty}\frac{e^{x^2}}{1} \to \infty \text{ diverges}$$

$$\left(\int_0^\infty e^{t^2}\,dt \to \infty\right)$$

6. (Section 10.6, Problem 14)

$$\lim_{x\to\infty}\frac{\sqrt{1+x^2}}{x^2} = \lim_{x\to\infty}\sqrt{\frac{1+x^2}{x^4}} = \lim_{x\to\infty}\sqrt{\frac{1}{x^4}+\frac{1}{x^2}} = 0$$

7. (Section 10.6, Problem 17)

$$\lim_{x\to 1} x^{\frac{1}{x-1}} \overset{(1^\infty)}{=} \lim_{x\to 1} e^{\ln x^{\frac{1}{x-1}}} = e \qquad \left(\lim_{x\to 1}\ln x^{\frac{1}{x-1}} = 1 \text{ and } e^x \text{ is conti.}\right)$$

$$\boxed{\lim_{x\to 1}\ln x^{\frac{1}{x-1}} = \lim_{x\to 1}\frac{1}{x-1}\ln x \overset{(\frac{0}{0})}{\underset{L'}{=}} \lim_{x\to 1}\frac{\frac{1}{x}}{1} = 1}$$

8. (Section 10.6, Problem 19)

$$\lim_{x\to\infty}\left(\cos\frac{1}{x}\right)^x \overset{1^\infty}{=} \lim_{x\to\infty} e^{\ln(\cos\frac{1}{x})^x} = e^0 = 1$$

$$\boxed{\begin{aligned}\lim_{x\to\infty}\ln\left(\cos\tfrac{1}{x}\right)^x &= \lim_{x\to\infty} x\ln\left(\cos\tfrac{1}{x}\right) \overset{(\infty\cdot 0)}{=} \\ &= \lim_{x\to\infty}\frac{\ln(\cos\frac{1}{x})}{\frac{1}{x}} \overset{(\frac{0}{0})}{\underset{L'}{=}} \lim_{x\to\infty}\frac{\frac{1}{\cos\frac{1}{x}}\cdot -\sin\frac{1}{x}\cdot\left(-\frac{1}{x^2}\right)}{\frac{-1}{x^2}} = \lim_{x\to\infty} -\tan\frac{1}{x} = 0\end{aligned}}$$

9. (Section 10.6, Problem 21)

$$\lim_{x\to 0}\left[\frac{1}{\ln(1+x)} - \frac{1}{x}\right] = \lim_{x\to 0}\left[\frac{x-\ln(1+x)}{x\cdot\ln(1+x)}\right]$$

$$\overset{(\frac{0}{0})}{\underset{L'}{=}} \lim_{x\to 0}\left[\frac{1-\frac{1}{1+x}}{\ln(1+x)+\frac{x}{x+1}}\right] \overset{(\frac{0}{0})}{\underset{L'}{=}} \lim_{x\to 0}\left[\frac{\frac{1}{(1+x)^2}}{\frac{1}{1+x}+\frac{1}{(1+x)^2}}\right] = \frac{1}{2}$$

10. (Section 10.6, Problem 25)

$$\lim_{x\to 0}\left(\frac{1}{x} - \cot x\right) = \lim_{x\to 0}\left(\frac{1}{x} - \frac{\cos x}{\sin x}\right)$$

$$= \lim_{x\to 0}\left(\frac{\sin x - x\cos x}{x\cdot\sin x}\right) \overset{(\frac{0}{0})}{\underset{L'}{=}} \lim_{x\to 0}\frac{\cos x - \cos x + x\sin x}{\sin x + x\cos x}$$

$$= 0$$

11. (Section 10.6, Problem 26)

$$\lim_{x\to\infty} \frac{1}{x}\int_0^x \sin\left(\frac{1}{t+1}\right)dt \overset{(\frac{\infty}{\infty})}{\underset{L'}{=}} \lim_{x\to\infty} \frac{\sin\left(\frac{1}{x+1}\right)}{1} = 0$$

14. (Section 10.6, Problem 32)

$$\lim_{x\to 0}(e^x+3x)^{\frac{1}{x}} \overset{1^\infty}{=} \lim_{x\to 0} e^{\ln(e^x+3x)^{\frac{1}{x}}} = e^4$$

$$\text{Find } \lim_{x\to 0}\ln(e^x+3x)^{\frac{1}{x}} = \lim_{x\to 0}\frac{1}{x}\ln(e^x+3x) \overset{(\frac{\infty}{\infty})}{=} \lim_{x\to 0}\frac{\frac{1}{e^x+3x}(e^x+3)}{1}$$

$$= \lim_{x\to 0}\frac{e^x+3}{e^x+3x} = 4$$

12. (Section 10.6, Problem 27)

$$\lim_{x\to\infty}(x^3+1)^{\frac{1}{\ln x}} \overset{\infty^0}{=} \lim_{x\to\infty} e^{\ln(x^3+1)^{\frac{1}{\ln x}}} = e^3$$

$$\text{Find } \lim_{x\to\infty}\ln(x^3+1)^{\frac{1}{\ln x}} = \lim_{x\to\infty}\frac{1}{\ln x}\ln(x^3+1) \overset{(\frac{\infty}{\infty})}{\underset{L'}{=}} \lim_{x\to\infty}\frac{\frac{1}{x^3+1}\cdot 3x^2}{\frac{1}{x}}$$

$$= \lim_{x\to\infty}\frac{3x^3}{x^3+1} = 3$$

15. (Section 10.6, Problem 33)

$$\lim_{x\to 1}\left(\frac{1}{\ln x} - \frac{x}{x-1}\right) = \lim_{x\to 1}\left(\frac{x-1-x\ln x}{(x-1)\ln x}\right)$$

$$\overset{(\frac{0}{0})}{\underset{L'}{=}} \lim_{x\to 1}\frac{\left(1-\ln x - x\cdot\frac{1}{x}\right)\cdot x}{\left(\ln x + \frac{x-1}{x}\right)\cdot x} = \lim_{x\to 1}\frac{-x\ln x}{x\ln x + x - 1} \overset{(\frac{0}{0})}{\underset{L'}{=}} \lim_{x\to 1}\frac{-\ln x - 1}{\ln x + 1 + 1}$$

$$= -\frac{1}{2}$$

13. (Section 10.6, Problem 28)

$$\lim_{x\to\infty}(e^x+1)^{\frac{1}{x}} \overset{\infty^0}{=} \lim_{x\to\infty} e^{\ln(e^x+1)^{\frac{1}{x}}} = e$$

$$\text{Find } \lim_{x\to\infty}\ln(e^x+1)^{\frac{1}{x}} = \lim_{x\to\infty}\frac{1}{x}\ln(e^x+1) \overset{(\frac{\infty}{\infty})}{\underset{L'}{=}} \lim_{x\to\infty}\frac{\frac{e^x}{e^x+1}}{1}$$

$$= \lim_{x\to\infty}\frac{e^x}{e^x+1} = \lim_{x\to\infty} 1 - \frac{1}{e^x+1} = 1$$

16. (Section 10.6, Problem 40)

$$\lim_{n\to\infty}\frac{n^k}{2^n} \overset{(\frac{\infty}{\infty})}{\underset{L'}{=}} \lim_{n\to\infty}\frac{k\,n^{k-1}}{\ln 2\cdot 2^n}$$

$$\overset{(\frac{0}{0})}{\underset{L'}{=}} \lim_{n\to\infty}\frac{k(k-1)\,n^{k-2}}{(\ln 2)^2\cdot 2^n}$$

$$= \cdots \overset{(\frac{0}{0})}{\underset{L'}{=}} \lim_{n\to\infty}\frac{k!}{(\ln 2)^k\cdot 2^n} = 0$$

← Fixed numbers

17. (Section 10.6, Problem 37)

$$\lim_{n\to\infty} (\ln n)^{\frac{1}{n}} \overset{\infty^0}{=} \lim_{n\to\infty} e^{\ln(\ln n)^{\frac{1}{n}}} = e^{0} = 1$$

Find $\lim_{n\to\infty} \ln(\ln n)^{\frac{1}{n}} = \lim_{n\to\infty} \frac{1}{n}\ln(\ln n) \overset{(\frac{\infty}{\infty})}{=} \lim_{n\to\infty} \frac{\frac{1}{\ln n}\cdot\frac{1}{n}}{1}$

$$= \lim_{n\to\infty} \frac{1}{\ln n}\cdot\frac{1}{n} = 0\cdot 0 = 0$$

18. (Section 10.7, Problem 3)

$$\int_0^\infty \frac{dx}{4+x^2} = \lim_{b\to\infty}\int_0^b \frac{dx}{4+x^2} = \lim_{b\to\infty} \frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right)\Big|_0^b$$

$$= \lim_{b\to\infty} \frac{1}{2}\tan^{-1}\frac{b}{2} = \frac{1}{2}\cdot\frac{\pi}{2} = \frac{\pi}{4}$$

(since $\frac{1}{4+x^2} < \frac{1}{x^2}$ and $\int \frac{dx}{x^2}$ exists so $\int_0^\infty \frac{dx}{4+x^2}$ exists)

19. (Section 10.7, Problem 4)

$$\int_0^\infty e^{-px}\,dx = \lim_{b\to\infty}\int_0^b e^{-px}\,dx$$

$$= \lim_{b\to\infty} -\frac{1}{p}\cdot\frac{1}{e^{px}}\Big|_0^b = \lim_{b\to\infty}\left(-\frac{1}{p}\cdot\frac{1}{e^{pb}} + \frac{1}{p}\cdot\frac{1}{e^0}\right)$$

$$\cancel{= -\frac{1}{p} + \lim_{b\to\infty}\frac{1}{p}} \qquad = \lim_{b\to\infty}\left(-\underbrace{\frac{1}{p}\cdot\frac{1}{e^{pb}}}_{\to 0} + \frac{1}{p}\right)$$

$$= \frac{1}{p}$$

20. (Section 10.7, Problem 7)

$$\int_0^8 \frac{dx}{x^{\frac{2}{3}}} = \lim_{a\to 0}\int_a^8 \frac{dx}{x^{\frac{2}{3}}} = \lim_{a\to 0} 3x^{\frac{1}{3}}\Big|_a^8$$

$\frac{1}{x^{\frac{2}{3}}}$ is discontinuous @ $x=0$

$$= \lim_{a\to 0} 3\left(8^{\frac{1}{3}} - \underbrace{a^{\frac{1}{3}}}_{\to 0}\right)$$

$$= 3\cdot 2 = 6$$

21. (Section 10.7, Problem 8)

$$\int_0^1 \frac{dx}{x^2} = \lim_{a\to 0}\int_a^1 \frac{dx}{x^2} = \lim_{a\to 0} -\frac{1}{x}\Big|_a^1$$

$\frac{1}{x^2}$ is discontinuous @ $x=0$

$$= \lim_{a\to 0} -\left(1 - \underbrace{\frac{1}{a}}_{\to\infty}\right)$$

$\Rightarrow$ Divergent!

22. (Section 10.7, Problem 9)

$$\int_0^1 \frac{dx}{\sqrt{1-x^2}} = \lim_{b\to 1}\int_0^b \frac{dx}{\sqrt{1-x^2}} = \lim_{b\to 1} \sin^{-1}x\Big|_0^b$$

$\frac{1}{\sqrt{1-x^2}}$ is discontinuous @ $x=1$

$$= \lim_{b\to 1} \sin^{-1}x\Big|_0^b$$

$$= \lim_{b\to 1}\sin^{-1}b - \sin^{-1}0$$

$$= \frac{\pi}{2}$$

$$\lim_{a\to0} -\frac{a^2}{2}\ln a = \lim_{a\to 0}\frac{\ln a}{-2a^{-2}} \overset{L'}{=} \lim_{a\to0}\frac{\frac{1}{a}}{4a^{-3}} = 0$$

$u = \ln x$, $dv = x$; $\frac{1}{x}$, $\frac{x^2}{2}$

23. (Section 10.7, Problem 11)

$$\int_0^2 \frac{x}{\sqrt{4-x^2}}\,dx = \lim_{b\to2}\int_0^b \frac{x}{\sqrt{4-x^2}}\,dx = \lim_{b\to2} -\sqrt{4-x^2}\Big|_0^b$$

$\boxed{\frac{x}{\sqrt{4-x^2}} \to \infty \text{ as } x\to 2}$

$$= \lim_{b\to2}\left[-\sqrt{4-b^2}+\sqrt{4}\right]$$

$$= -0+2 = 2$$

24. (Section 10.7, Problem 13)

$$\int_e^\infty \frac{\ln x}{x}\,dx = \lim_{b\to\infty}\int_e^b \frac{\ln x}{x}\,dx = \lim_{b\to\infty} \frac{(\ln x)^2}{2}\Big|_e^b$$

$$= \lim_{b\to\infty} \underbrace{\left(\frac{\ln b}{2}\right)^2}_{\to\infty} - \frac{1}{2} \Rightarrow \text{diverges}$$

25. (Section 10.7, Problem 14)

$$\int_e^\infty \frac{dx}{x\ln x}\,dx = \lim_{b\to\infty}\int_e^b \frac{dx}{x\ln x} = \lim_{b\to\infty} \ln(\ln x)\Big|_e^b$$

$$= \lim_{b\to\infty} \ln(\ln b) - \ln(\ln e)$$

$$= \lim_{b\to\infty} \underbrace{\ln(\ln b)}_{\to\infty} - 0 \Rightarrow \text{diverges}$$

26. (Section 10.7, Problem 15)

$$\int_0^1 x\ln x\,dx = \lim_{a\to0}\int_a^1 x\ln x\,dx =$$

$\boxed{@x=0 \text{ we have } x\cdot\ln x = 0\cdot(-\infty)}$

$$= \lim_{a\to0}\left(\frac{x^2}{2}\ln x\Big|_a^1 - \int_a^1 \frac{x}{2}\,dx\right)$$

$$= \lim_{a\to0}\left(\underbrace{\frac{1}{2}\ln 1}_{0} - \frac{a^2}{2}\ln a - \frac{x^2}{4}\Big|_a^1\right)$$

$$= \lim_{a\to0} \underbrace{-\frac{a^2}{2}\ln a}_{0\cdot(-\infty)\to L'} - \frac{1}{4} + \underbrace{\frac{a^2}{4}}_{\to 0} \overset{L'}{=} 0 - \frac{1}{4} + 0 = -\frac{1}{4}$$

27. (Section 10.7, Problem 19)

$$\int_{-\infty}^{\infty}\frac{dx}{x^2} \underset{\text{symmetric}}{=} 2\int_0^\infty \frac{dx}{x^2} = 2\left[\lim_{a\to0}\lim_{b\to\infty}\int_a^b \frac{dx}{x^2}\right]$$

$\boxed{@x=0,\ \frac{1}{x^2}\to\infty}$

$$= 2\left[\lim_{a\to0}\lim_{b\to\infty} -\frac{1}{x}\Big|_a^b\right]$$

$$= 2\left[\lim_{a\to0}\lim_{b\to\infty} \underbrace{\frac{1}{a}}_{\to\infty} - \frac{1}{b}\right]$$

Diverges.

28. (Section 10.7, Problem 20)

$$\int_{\frac{1}{3}}^3 \frac{dx}{\sqrt[3]{3x-1}} = \lim_{a\to\frac{1}{3}}\int_a^3 \frac{dx}{\sqrt[3]{3x-1}} = \lim_{a\to\frac13}\frac{1}{2}(3x-1)^{\frac{2}{3}}\Big|_a^3$$

$\boxed{@x=\frac{1}{3},\ \frac{1}{\sqrt[3]{3x-1}}\to\infty}$

$$= \lim_{a\to\frac{1}{3}}\left[\frac{1}{2}8^{\frac{2}{3}} - \underbrace{\frac{1}{2}(3a-1)^{\frac{2}{3}}}_{\to 0}\right]$$

$$= \frac{1}{2}\cdot 8^{\frac{2}{3}} = \frac{1}{2}\cdot 2^2 = 2.$$

$$\int \frac{dx}{x^2-4} = \int \frac{\frac{1}{4}}{x-2} - \frac{\frac{1}{4}}{x+2}\,dx =$$

$$\frac{1}{4}\ln|x-2| - \frac{1}{4}\ln|x+2|$$

29. (Section 10.7, Problem 27)

$$\int_{-3}^{1} \frac{dx}{x^2-4} = \int_{-3}^{-2}\frac{dx}{x^2-4} + \int_{-2}^{1}\frac{dx}{x^2-4} = \lim_{a\to -2^-}\left[\int_{-3}^{a}\frac{dx}{x^2-4}\right] + \lim_{a\to -2^+}\left[\int_{a}^{1}\frac{dx}{x^2-4}\right]$$

$x^2-4=0 \Rightarrow x=\pm 2$

$@\,x=\pm 2 \quad \frac{1}{x^2-4}\to\infty$

$$= \lim_{a\to -2^-}\left[\frac{1}{4}\left(\ln|x-2| - \ln|x+2|\right)\Big|_{-3}^{a}\right] + \lim_{a\to -2^+}\left[\frac{1}{4}\left(\ln|x-2| - \ln|x+2|\right)\Big|_{a}^{1}\right]$$

$$= \lim_{a\to -2^-}\left[\frac{1}{4}\left(\ln|a-2| - \underbrace{\ln|a+2|}_{\to\infty} - \ln|5| + \ln|1|\right)\right] + \cdots$$

$\Rightarrow$ Divergent!

30. (Section 10.7, Problem 33)

$$\int_0^1 \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx = \lim_{a\to 0} 2e^{\sqrt{x}}\Big|_a^1 = \lim_{a\to 0}\left[2e - 2e^{\sqrt{a}}\right]$$

$$= 2e - 2$$

(Section 10.7, Problem 34)

$$\int_0^{\frac{\pi}{2}} \frac{\cos x}{\sqrt{\sin x}}\,dx = \lim_{a\to 0}\int_a^{\frac{\pi}{2}}\frac{\cos x}{\sqrt{\sin x}}\,dx$$

$$= \lim_{a\to 0} 2\sqrt{\sin x}\Big|_a^{\frac{\pi}{2}} = \lim_{a\to 0} 2\sqrt{1} - 2\underbrace{\sqrt{\sin a}}_{\to 0}$$

$$= 2$$

u	dv
$\sin^{-1}x$	dx
$\frac{1}{\sqrt{1-x^2}}$	x

(Section 10.7, Problem 37)

$$\int_0^1 \sin^{-1}x\,dx = x\cdot\sin^{-1}x\Big|_0^1 - \int_0^1 \frac{dx}{\sqrt{1-x^2}}x$$

$$= \frac{\pi}{2} - 0 - \lim_{b\to 1}\int_0^b \frac{dx}{\sqrt{1-x^2}}x$$

$$= \frac{\pi}{2} - \lim_{b\to 1}\left[-\sqrt{1-x^2}\Big|_0^b\right]$$

$$= \frac{\pi}{2} - \lim_{b\to 1}\left[-\underbrace{\sqrt{1-b^2}}_{\to 0} + 1\right] = \frac{\pi}{2} - 1.$$

33. (Section 11.1, Problem 1)

$$\sum_{k=0}^{2}(3k+1)$$

$$= \underbrace{1}_{k=0} + \underbrace{4}_{k=1} + \underbrace{7}_{k=2} = 12.$$

34. (Section 11.1, Problem 6)

$$\sum_{k=2}^{4}\frac{1}{3^{k-1}} = \frac{1}{3^{2-1}} + \frac{1}{3^{3-1}} + \frac{1}{3^{4-1}}$$

$$= \frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3}$$

$$= \frac{3^2+3+1}{3^3} = \frac{13}{27}$$

35. (Section 11.1, Problem 7)

$1+3+5+7+\cdots+21$

pattern is $\underline{\underline{2k-1}}$ $\quad k=1,\ k=2,\ \ldots,\ k=11$

$\quad 1,\ 3,\ \ldots,\ 21$

$$=\sum_{k=1}^{11}(2k-1)$$

36. (Section 11.1, Problem 13)

$$\frac{1}{2^3}+\frac{1}{2^4}+\cdots+\frac{1}{2^{10}}$$

$$\sum_{k=3}^{10}\frac{1}{2^k} \qquad \sum_{k=0}^{7}\frac{1}{2^{k+3}}$$

37. (Section 11.1, Problem 26)

$$\sum_{k=3}^{\infty}\frac{1}{k^2-k}=\sum_{k=3}^{\infty}\frac{1}{k(k-1)}=\sum_{k=3}^{\infty}\frac{1}{k-1}-\frac{1}{k}$$

telescope

$$=\left(\frac{1}{2}-\cancel{\frac{1}{3}}\right)+\left(\cancel{\frac{1}{3}}-\cancel{\frac{1}{4}}\right)+\left(\cancel{\frac{1}{4}}-\frac{1}{5}\right)+\cdots\cdots 0)$$

Final term goes to 0 since $\frac{1}{k}\to 0$ as $k\to\infty$

$$=\frac{1}{2}.$$