Math 1432, Section 12869
Spring 2014
Homework Assignment 10
Due Date: 4/2/14 in Lab

Name: Sol

ID:

Instructions

- Print out this file and complete the problems. You must do all the problems!
- If the problem is from the text, the section number and problem number are in parantheses.
- Use a blue or black pen or a pencil (dark).
- Write your solutions in the spaces provided. You must show work in order receive credit for a problem.
- Remember that your homework must be complete, neatly written and stapled.
- Submit the completed assignment to your Teaching Assistant in lab on the due date.
- If you do not do all of the problems, then your recitation quiz from the previous Friday will automatically become a ZERO.

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1. (Section 10.4, Problem 2)

$e^{-\frac{\alpha}{n}}$ Fix α and $n \to \infty$. $-\frac{\alpha}{n} \to 0$

Since e^x is conti. and $-\frac{\alpha}{n} \to 0$

$\therefore\ e^{-\frac{\alpha}{n}} \to e^0 = 1$.

1

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2. (Section 10.4, Problem 3)

$\left(\frac{2}{n}\right)^n \Rightarrow$ for $n>3$ $\quad 0 < \left(\frac{2}{n}\right)^n < \left(\frac{2}{3}\right)^n \to 0$ by Pinching Thm.

~~Take "ln", we have~~ $\cancel{\ln\left(\frac{2}{n}\right)^n = n\ln\left(\frac{2}{n}\right)}$

0

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3. (Section 10.4, Problem 4)

$$\frac{\log_{10} n}{n} = \frac{\ln n}{\ln 10} \cdot \frac{1}{n} = \frac{1}{\ln 10} \cdot \frac{\ln n}{n} \to 0 \text{ as } n \to \infty$$

$\boxed{\text{fact: } \frac{\ln n}{n} \to 0 \text{ as } n \to \infty}$

0

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4. (Section 10.4, Problem 5)

$$\frac{\ln(n+1)}{n} = \frac{n+1}{n} \cdot \frac{\ln(n+1)}{n+1} \to 1 \cdot 0 = 0$$

as $n \to \infty$

$\frac{n+1}{n} \to 1$ as $n \to \infty$ $\qquad \frac{\ln(n+1)}{n+1} \to 0$ as $n \to \infty$

0

For each real x

$$\boxed{\frac{x^n}{n!} \to 0 \text{ as } n\to\infty}$$

5. (Section 10.4, Problem 7)

$$\frac{x^{100n}}{n!} = \frac{(x^{100})^n}{n!} \to 0 \quad \text{as } n\to\infty$$

(x^{100} is real if x is real)

$\boxed{0}$

6. (Section 10.4, Problem 11)

$$\frac{3^{n+1}}{4^{n-1}} = \frac{3^2\cdot 3^{n-1}}{4^{n-1}} = 3^2\left(\frac{3}{4}\right)^{n-1} \to 9\cdot 0 = 0 \quad \text{as } n\to\infty$$

$$\boxed{(t)^n \to 0 \text{ if } |t|<1}$$

(since $\frac{3}{4}<1$).

$\boxed{0}$

7. (Section 10.4, Problem 12)

$$\int_{-n}^{0} e^{2x}\,dx = \frac{e^{2x}}{2}\Big|_{-n}^{0} = \frac{1}{2}(1-e^{-2n})$$

$$= \frac{1}{2} - \frac{e^{-2n}}{2} \to \frac{1}{2} - 0 = \frac{1}{2}$$

$$\boxed{e^{-2n} = \frac{1}{e^{2n}} \to 0 \text{ as } n\to\infty}$$

as $n\to\infty$

8. (Section 10.4, Problem 15)

$$\int_0^n e^{-x}\,dx = -e^{-x}\Big|_0^n = 1-e^{-n} \to 1-0 = 1$$

as $n\to\infty$

$$\boxed{e^{-n} = \frac{1}{e^n} \to 0 \text{ as } n\to\infty.}$$

$\boxed{1}$

9. (Section 10.4, Problem 17)

$$\int_{-n}^{n}\frac{dx}{1+x^2} = 2\int_0^n \frac{dx}{1+x^2} = 2\arctan x\Big|_0^n$$

$$= 2\arctan n - 0 = 2\arctan n \to 2\cdot\frac{\pi}{2} = \pi$$

as $n\to\infty$

$$\boxed{\arctan n \to \frac{\pi}{2} \text{ as } n\to\infty}$$

$\boxed{\pi}$

10. (Section 10.4, Problem 20)

$$n^2 \sin n\pi = n^2\cdot 0 = 0$$

$n\in\mathbb{N}$, $\sin n\pi = 0$

$\boxed{0}$

11. (Section 10.4, Problem 21)

$$\frac{\ln n^2}{n} = 2\frac{\ln n}{n} \to 2\cdot 0 = 0 \text{ as } n\to\infty$$

$$\boxed{\frac{\ln n}{n} \to 0 \quad \text{as } n\to\infty}$$

$$\boxed{0}$$

12. (Section 10.4, Problem 23)

$$n^2 \sin\frac{\pi}{n} = n^2\cdot\frac{\pi}{n}\cdot\frac{\sin(\frac{\pi}{n})}{(\frac{\pi}{n})} = n\pi\cdot\frac{\sin(\frac{\pi}{n})}{\frac{\pi}{n}} \Rightarrow \text{diverge}$$

$$\boxed{\lim_{x\to 0}\frac{\sin x}{x} \to 1} \Rightarrow \boxed{\begin{array}{l}\frac{\pi}{n}\to 0 \text{ as } n\to\infty \\ \frac{\sin(\frac{\pi}{n})}{\frac{\pi}{n}} \to 1 \text{ as } n\to\infty\end{array}}$$

13. (Section 10.4, Problem 24)

$$\frac{n!}{2n} = \frac{n(n-1)!}{2n} = \frac{(n-1)!}{2} \Rightarrow \text{diverge.}$$

14. (Section 10.4, Problem 28)

$$\int_{\frac{1}{n}}^{1}\frac{dx}{\sqrt{x}} = 2\sqrt{x}\Big|_{\frac{1}{n}}^{1} = 2 - 2\sqrt{\frac{1}{n}} \to 2 \quad \text{as } n\to\infty$$

$$\boxed{\begin{array}{l}\frac{1}{n}\to 0 \text{ and } \sqrt{x} \text{ is conti.} \\ \Rightarrow \sqrt{\frac{1}{n}} \to 0\end{array}}$$

$$\boxed{2}$$

15. (Section 10.4, Problem 31)

$$\frac{n^n}{2^{n^2}} = \frac{n^n}{(2^n)^n} = \left(\frac{n}{2^n}\right)^n \to 0 \quad \text{as } n\to\infty$$

$$\boxed{\begin{array}{l}\text{when } n\geq 2. \\ 0 < \left(\frac{n}{2^n}\right)^n < \left(\frac{1}{2}\right)^n\end{array}}$$

$$\boxed{0}$$

16. (Section 10.4, Problem 35)

$$\left(1+\frac{x}{2n}\right)^{2n} = \left[\left(1+\frac{\frac{x}{2}}{n}\right)^n\right]^2 \to \left[e^{\frac{x}{2}}\right]^2 = e^x$$

as $n\to\infty$.

$$\boxed{\begin{array}{l}\text{For each real } x \\ \left(1+\frac{x}{n}\right)^n \to e^x \\ \text{as } n\to\infty\end{array}}$$

$$\boxed{\begin{array}{l}\text{since } x^2 \text{ conti.} \\ \text{and } \left(1+\frac{\frac{x}{2}}{n}\right)^n \to e^{\frac{x}{2}}\end{array}}$$

$$\boxed{e^x}$$

$a_n = \cos(n^2\pi) = (-1)^n$.

$n=1$. $a_1=-1$, $a_2=1$, $a_3=-1$

17. (Section 10.4, Problem 38)

(a) $\frac{n^2\cos(n^2\pi)}{n^2+1} = \frac{n^2}{n^2+1}\cdot\cos(n^2\pi) = \frac{n^2}{n^2+1}(-1)^n$

$\Rightarrow$ diverge.

(b) $n(2^{\frac{1}{n}}-1)$

(c) $n^e e^{-n}$.

$\boxed{}$

20. (Section 10.5, Problem 5)

$\lim_{x\to\frac{\pi}{2}} \frac{\cos x}{\sin 2x} \overset{(\frac{0}{0})}{\underset{L'}{=}} \lim_{x\to\frac{\pi}{2}} \frac{-\sin x}{2\cos 2x} = \frac{-1}{-2} = \frac{1}{2}$.

$\boxed{\frac{1}{2}}$

18. (Section 10.5, Problem 1)

$\lim_{x\to0^+} \frac{\sin x}{\sqrt{x}} \overset{(\frac{0}{0})}{\underset{L'}{=}} \lim_{x\to0^+} \frac{\cos x}{\frac{1}{2}\frac{1}{\sqrt{x}}} = \lim_{x\to0^+} 2\sqrt{x}\cos x = 0\cdot1 = 0$

~~For little positive x, we have~~

$\cancel{0 < \frac{\sin x}{\sqrt{x}} < \frac{\sin x}{x}}$

$\boxed{0}$

21. (Section 10.5, Problem 6)

$\lim_{x\to a} \frac{x-a}{x^n-a^n} \overset{(\frac{0}{0})}{\underset{L'}{=}} \lim_{x\to a} \frac{1}{nx^{n-1}} = \frac{1}{n\cdot a^{n-1}}$

constant

$\boxed{\frac{1}{na^{n-1}}}$

19. (Section 10.5, Problem 3)

$\lim_{x\to0} \frac{e^x-1}{\ln(1+x)} \overset{(\frac{0}{0})}{\underset{L'}{=}} \lim_{x\to0} \frac{e^x}{\frac{1}{1+x}} = \lim_{x\to0}(1+x)e^x = 1\cdot1 = 1$.

$\boxed{1}$

22. (Section 10.5, Problem 7)

$\lim_{x\to0} \frac{2^x-1}{x} \overset{(\frac{0}{0})}{\underset{L'}{=}} \lim_{x\to0} \frac{(\ln 2)\cdot 2^x}{1} = \ln 2$.

$(2^x-1)' = (e^{\ln 2^x}-1)'$

$= (e^{x\ln 2}-1)' = \ln 2\cdot e^{x\ln 2} = (\ln 2)\cdot 2^x$

$\boxed{\ln 2}$

23. (Section 10.5, Problem 8)

$$\lim_{x\to 0}\frac{\tan^{-1}x}{x} \underset{\text{by } L'}{\overset{(\frac{0}{0})}{=}} \lim_{x\to 0}\frac{\frac{1}{1+x^2}}{1} = 1.$$

$$\boxed{(\tan^{-1}x)' = \frac{1}{1+x^2}}$$

$\boxed{1}$

24. (Section 10.5, Problem 10)

$$\lim_{x\to 0}\frac{e^x-1}{\underbrace{x(1+x)}_{=\,x+x^2}} \underset{\text{by } L'}{\overset{(\frac{0}{0})}{=}} \lim_{x\to 0}\frac{e^x}{1+2x} = \frac{1}{1} = 1.$$

$\boxed{1}$

25. (Section 10.5, Problem 15)

$$\lim_{x\to 0}\frac{e^x+e^{-x}-2}{1-\cos 2x} \underset{\text{by } L'}{\overset{(\frac{0}{0})}{=}} \lim_{x\to 0}\frac{e^x-e^{-x}}{+2\sin 2x}$$

$$\underset{\text{by } L'}{\overset{(\frac{0}{0})}{=}} \lim_{x\to 0}\frac{e^x+e^{-x}}{4\cos 2x} = \frac{2}{4} = \boxed{\frac{1}{2}}.$$

26. (Section 10.5, Problem 18)

$$\lim_{x\to 0}\frac{\cos x - 1 + \frac{x^2}{2}}{x^4} \underset{\text{by } L'}{\overset{(\frac{0}{0})}{=}} \lim_{x\to 0}\frac{-\sin x + x}{4x^3}$$

$$\underset{\text{by } L'}{\overset{(\frac{0}{0})}{=}} \lim_{x\to 0}\frac{-\cos x+1}{12x^2} \underset{\text{by } L'}{\overset{(\frac{0}{0})}{=}} \lim_{x\to 0}\frac{\sin x}{24x} \underset{L'}{\overset{(\frac{0}{0})}{=}} \lim_{x\to 0}\frac{\cos x}{24} = \boxed{\frac{1}{24}}.$$

27. (Section 10.5, Problem 19)

$$\lim_{x\to 0}\frac{1+x-e^x}{\underbrace{x(e^x-1)}_{xe^x-x}} \underset{\text{by } L'}{\overset{(\frac{0}{0})}{=}} \lim_{x\to 0}\frac{1-e^x}{e^x+xe^x-1} \underset{\text{by } L'}{\overset{(\frac{0}{0})}{=}} \lim_{x\to 0}\frac{-e^x}{e^x+e^x+xe^x}$$

$$= -\frac{1}{2}.$$

$\boxed{-\frac{1}{2}}$

28. (Section 10.5, Problem 21)

$$\lim_{x\to 0}\frac{x-\tan x}{x-\sin x} \underset{\text{by } L'}{\overset{(\frac{0}{0})}{=}} \lim_{x\to 0}\frac{1-\sec^2 x}{1-\cos x}$$

$$\underset{\text{by } L'}{\overset{(\frac{0}{0})}{=}} \lim_{x\to 0}\frac{-2\sec x\cdot\sec x\underbrace{\tan x}_{=\,\frac{\sin x}{\cos x}}}{\sin x} \overset{\text{simplify}}{=} \lim_{x\to 0}\frac{-2\sec^2 x}{\cos x}$$

$$= \frac{-2}{1} = -2.$$

$\boxed{-2}$

29. (Section 10.5, Problem 22)

$$\lim_{x\to 0}\frac{xe^{nx}-x}{1-\cos nx}\ \underset{\text{by } L'}{\overset{(\frac{0}{0})}{=}}\ \lim_{x\to 0}\frac{e^{nx}+nxe^{nx}-1}{+n\sin nx}$$

$$\underset{L'}{\overset{(\frac{0}{0})}{=}}\ \lim_{x\to 0}\frac{ne^{nx}+ne^{nx}+n^2xe^{nx}}{+n^2\cos nx}=\frac{2n}{+n^2}=\frac{2}{n}.$$

$\boxed{\frac{2}{n}}$

30. (Section 10.5, Problem 24)

$$\lim_{x\to 0}\frac{2x-\sin\pi x}{4x^2-1}=\frac{0-0}{-1}=0$$

$\boxed{0}$

31. (Section 10.5, Problem 27)

$$\lim_{x\to 0}\frac{\cos x-\cos 3x}{\sin(x^2)}\ \underset{L'}{\overset{(\frac{0}{0})}{=}}\ \lim_{x\to 0}\frac{-\sin x+3\sin 3x}{2x\cos(x^2)}$$

$$\underset{L'}{\overset{(\frac{0}{0})}{=}}\ \lim_{x\to 0}\frac{-\cos x+9\cos 3x}{2\cos(x^2)-4x^2\sin x^2}=\frac{8}{2}=4$$

$\boxed{4}$

32. (Section 10.5, Problem 31)

$$\lim_{x\to 0}\frac{\tan^{-1}x}{\tan^{-1}2x}\ \underset{L'}{\overset{(\frac{0}{0})}{=}}\ \lim_{x\to 0}\frac{\frac{1}{1+x^2}}{\frac{2}{1+(2x)^2}}=\frac{1}{2}.$$

$\boxed{\frac{1}{2}}$

33. (Section 10.5, Problem 33)

$$\lim_{n\to\infty}\frac{\frac{\pi}{2}-\tan^{-1}n}{\frac{1}{n}}\ \underset{L'}{\overset{(\frac{0}{0})}{=}}\ \lim_{n\to\infty}\frac{-\frac{1}{1+n^2}}{-\frac{1}{n^2}}$$

$$\overset{\text{simplify}}{=}\ \lim_{n\to\infty}\frac{n^2}{1+n^2}=1$$

leading coefficient

$\boxed{1}$

34. (Section 10.5, Problem 35)

$$\lim_{n\to\infty}\frac{1}{n[\ln(n+1)-\ln n]}=\lim_{n\to\infty}\frac{\frac{1}{n}}{\ln\left(\frac{n+1}{n}\right)}$$

$$\left[\begin{array}{l}\frac{1}{n}\to 0 \text{ as } n\to\infty\\ \ln\left(\frac{n+1}{n}\right)\to\ln 1=0 \text{ as } n\to\infty\end{array}\right]\to\ \underset{L'}{\overset{(\frac{0}{0})}{=}}\ \lim_{n\to\infty}\frac{-\frac{1}{n^2}}{\frac{-\frac{1}{n^2}}{1+\frac{1}{n}}}=-\frac{1}{n^2}\cdot\frac{1+\frac{1}{n}}{-\frac{1}{n^2}}$$

Find derivative which depends on "n".

$$\left[\ln\left(\frac{n+1}{n}\right)\right]'=\left[\ln\left(1+\frac{1}{n}\right)\right]'=\frac{-\frac{1}{n^2}}{1+\frac{1}{n}}$$

$$\overset{\text{simplify}}{=}\ \lim_{n\to\infty}1+\frac{1}{n}=1.$$

$\boxed{1}$

35. (Section 10.5, Problem 45)

Find a, b s.t. $\lim_{x\to 0} \frac{\cos ax - b}{2x^2} = -4$.

First, plug "0" in we get $\frac{1-b}{0}$, but we know this limit exists. so "$1-b=0$" $\Rightarrow \boxed{b=1}$.

If $b=1$, we have $(\frac{0}{0})$ form. Then, by L'HÔPITAL'S RULE, we have $\lim_{x\to 0} \frac{\cos ax - 1}{2x^2} \overset{(\frac{0}{0})}{\underset{L'}{=}} \lim_{x\to 0} \frac{-a\sin ax}{4x}$

$\overset{(\frac{0}{0})}{\underset{L'}{=}} \lim_{x\to 0} \frac{-a^2\cos ax}{4} = \frac{-a^2}{4} = -4$

so $-a^2 = -16 \Rightarrow \boxed{a = \pm 4}$

$$\boxed{\begin{array}{l} a = \pm 4 \\ b = 1 \end{array}}$$

36. (Section 10.5, Problem 47)

Calculate $\lim_{x\to 0} \frac{(1+x)^{\frac{1}{x}} - e}{x} \overset{(\frac{0}{0})}{\underset{L'}{=}} \lim_{x\to 0} (1+x)^{\frac{1}{x}} \cdot \left(\frac{-(1+x)\ln(1+x) + x}{x^2+x^3}\right) = \lim_{x\to 0} (1+x)^{\frac{1}{x}} \cdot \lim_{x\to 0} \left(\frac{-(1+x)\ln(1+x)+x}{x^2+x^3}\right)$

$= e \cdot (-\frac{1}{2}) = -\frac{e}{2}$

$$\boxed{-\frac{e}{2}}$$

Since we know $\lim_{x\to 0} (1+x)^{\frac{1}{x}} = e$

and try to find the derivative of $(1+x)^{\frac{1}{x}}$.

$\left[(1+x)^{\frac{1}{x}}\right]' = \left[e^{\ln(1+x)^{\frac{1}{x}}}\right]' = \left[e^{\frac{1}{x}\ln(1+x)}\right]'$

$= \left(\frac{1}{x}\ln(1+x)\right)' \cdot e^{\frac{1}{x}\ln(1+x)}$

$= \left(-\frac{1}{x^2}\ln(1+x) + \frac{1}{x}\cdot\frac{1}{1+x}\right)(1+x)^{\frac{1}{x}}$

$= \left(\frac{-(1+x)\ln(1+x) + x}{x^2+x^3}\right)(1+x)^{\frac{1}{x}}$

check $\lim_{x\to 0} \left(\frac{-(1+x)\ln(1+x)+x}{x^2+x^3}\right)$

$\overset{(\frac{0}{0})}{\underset{L'}{=}} \lim_{x\to 0} \frac{-\ln(1+x) - 1 + 1}{2x+3x^2}$

$\overset{(\frac{0}{0})}{\underset{L'}{=}} \lim_{x\to 0} \frac{-\frac{1}{1+x}}{2+6x} = -\frac{1}{2}$.

So this limit exists and equals "$-\frac{1}{2}$"

Since both limits, $\lim_{x\to 0} \underbrace{(1+x)^{\frac{1}{x}}}_{=a}$ and $\lim_{x\to 0} \underbrace{\left(\frac{-(1+x)\ln(1+x)+x}{x^2+x^3}\right)}_{b}$, exist, so we can say $\boxed{\lim_{x\to 0} ab = \lim_{x\to 0} a \cdot \lim_{x\to 0} b}$.