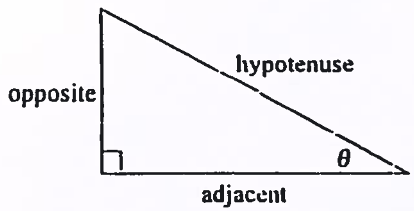
Pythagorean thm a+b=c

Definition of the Trig Functions

Right triangle definition

For this definition we assume that

$0 < \theta < \frac{\pi}{2}$ or $0° < \theta < 90°$.



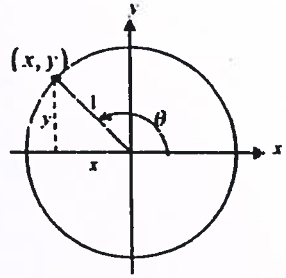
$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$ $\quad \csc\theta = \frac{\text{hypotenuse}}{\text{opposite}}$

$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$ $\quad \sec\theta = \frac{\text{hypotenuse}}{\text{adjacent}}$

$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$ $\quad \cot\theta = \frac{\text{adjacent}}{\text{opposite}}$

Unit circle definition

For this definition θ is any angle.



$\sin\theta = \frac{y}{1} = y$ $\quad \csc\theta = \frac{1}{y}$

$\cos\theta = \frac{x}{1} = x$ $\quad \sec\theta = \frac{1}{x}$

$\tan\theta = \frac{y}{x}$ $\quad \cot\theta = \frac{x}{y}$

Facts and Properties

Domain

The domain is all the values of θ that can be plugged into the function.

$\sin\theta$, θ can be any angle

$\cos\theta$, θ can be any angle

$\tan\theta$, $\theta \neq \left(n+\frac{1}{2}\right)\pi,\quad n = 0, \pm1, \pm2, \ldots$

$\csc\theta$, $\theta \neq n\pi,\quad n = 0, \pm1, \pm2, \ldots$

$\sec\theta$, $\theta \neq \left(n+\frac{1}{2}\right)\pi,\quad n = 0, \pm1, \pm2, \ldots$

$\cot\theta$, $\theta \neq n\pi,\quad n = 0, \pm1, \pm2, \ldots$

Range

The range is all possible values to get out of the function.

$-1 \le \sin\theta \le 1$ $\quad \csc\theta \ge 1$ and $\csc\theta \le -1$

$-1 \le \cos\theta \le 1$ $\quad \sec\theta \ge 1$ and $\sec\theta \le -1$

$-\infty < \tan\theta < \infty$ $\quad -\infty < \cot\theta < \infty$

Period

The period of a function is the number, T, such that $f(\theta+T) = f(\theta)$. So, if ω is a fixed number and θ is any angle we have the following periods.

$\sin(\omega\theta) \rightarrow T = \frac{2\pi}{\omega}$

$\cos(\omega\theta) \rightarrow T = \frac{2\pi}{\omega}$

$\tan(\omega\theta) \rightarrow T = \frac{\pi}{\omega}$

$\csc(\omega\theta) \rightarrow T = \frac{2\pi}{\omega}$

$\sec(\omega\theta) \rightarrow T = \frac{2\pi}{\omega}$

$\cot(\omega\theta) \rightarrow T = \frac{\pi}{\omega}$

Formulas and Identities

Tangent and Cotangent Identities

$\tan\theta = \frac{\sin\theta}{\cos\theta}$ $\quad \cot\theta = \frac{\cos\theta}{\sin\theta}$

Reciprocal Identities

$\csc\theta = \frac{1}{\sin\theta}$ $\quad \sin\theta = \frac{1}{\csc\theta}$

$\sec\theta = \frac{1}{\cos\theta}$ $\quad \cos\theta = \frac{1}{\sec\theta}$

$\cot\theta = \frac{1}{\tan\theta}$ $\quad \tan\theta = \frac{1}{\cot\theta}$

Pythagorean Identities

$\sin^2\theta + \cos^2\theta = 1$

$\tan^2\theta + 1 = \sec^2\theta$

$1 + \cot^2\theta = \csc^2\theta$

Even/Odd Formulas

$\sin(-\theta) = -\sin\theta$ $\quad \csc(-\theta) = -\csc\theta$

$\cos(-\theta) = \cos\theta$ $\quad \sec(-\theta) = \sec\theta$

$\tan(-\theta) = -\tan\theta$ $\quad \cot(-\theta) = -\cot\theta$

Periodic Formulas

If n is an integer.

$\sin(\theta + 2\pi n) = \sin\theta$ $\quad \csc(\theta + 2\pi n) = \csc\theta$

$\cos(\theta + 2\pi n) = \cos\theta$ $\quad \sec(\theta + 2\pi n) = \sec\theta$

$\tan(\theta + \pi n) = \tan\theta$ $\quad \cot(\theta + \pi n) = \cot\theta$

Double Angle Formulas

$\sin(2\theta) = 2\sin\theta\cos\theta$

$$\cos(2\theta) = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$$

$\tan(2\theta) = \frac{2\tan\theta}{1-\tan^2\theta}$

Degrees to Radians Formulas

If x is an angle in degrees and t is an angle in radians then

$\frac{\pi}{180} = \frac{t}{x} \quad \Rightarrow \quad t = \frac{\pi x}{180}$ and $x = \frac{180t}{\pi}$

Half Angle Formulas

$\sin^2\theta = \frac{1}{2}\left(1-\cos(2\theta)\right)$

$\cos^2\theta = \frac{1}{2}\left(1+\cos(2\theta)\right)$

$\tan^2\theta = \frac{1-\cos(2\theta)}{1+\cos(2\theta)}$

Sum and Difference Formulas

$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$

$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$

$\tan(\alpha \pm \beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}$

Product to Sum Formulas

$\sin\alpha\sin\beta = \frac{1}{2}\left[\cos(\alpha-\beta) - \cos(\alpha+\beta)\right]$

$\cos\alpha\cos\beta = \frac{1}{2}\left[\cos(\alpha-\beta) + \cos(\alpha+\beta)\right]$

$\sin\alpha\cos\beta = \frac{1}{2}\left[\sin(\alpha+\beta) + \sin(\alpha-\beta)\right]$

$\cos\alpha\sin\beta = \frac{1}{2}\left[\sin(\alpha+\beta) - \sin(\alpha-\beta)\right]$

Sum to Product Formulas

$\sin\alpha + \sin\beta = 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$

$\sin\alpha - \sin\beta = 2\cos\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$

$\cos\alpha + \cos\beta = 2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$

$\cos\alpha - \cos\beta = -2\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$

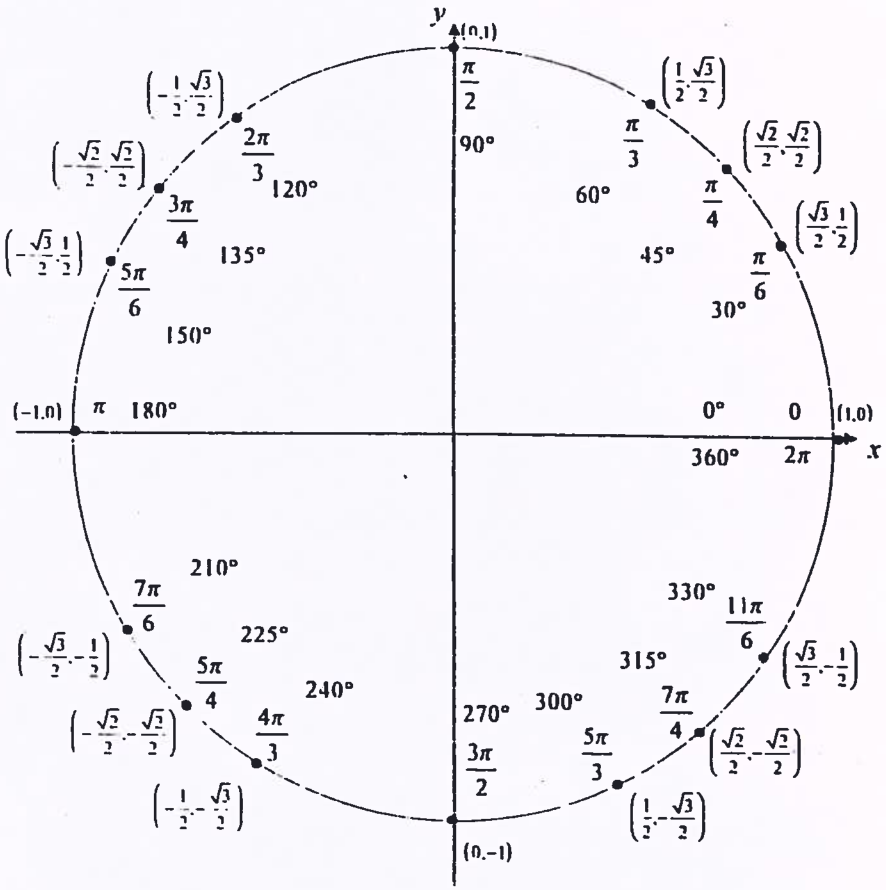
Cofunction Formulas

$\sin\left(\frac{\pi}{2}-\theta\right) = \cos\theta$ $\quad \cos\left(\frac{\pi}{2}-\theta\right) = \sin\theta$

$\csc\left(\frac{\pi}{2}-\theta\right) = \sec\theta$ $\quad \sec\left(\frac{\pi}{2}-\theta\right) = \csc\theta$

$\tan\left(\frac{\pi}{2}-\theta\right) = \cot\theta$ $\quad \cot\left(\frac{\pi}{2}-\theta\right) = \tan\theta$

Unit Circle



For any ordered pair on the unit circle (x,y) : $\cos\theta = x$ and $\sin\theta = y$

Inverse Trig Functions

Definition

$y=\sin^{-1}x$ is equivalent to $x=\sin y$

$y=\cos^{-1}x$ is equivalent to $x=\cos y$

$y=\tan^{-1}x$ is equivalent to $x=\tan y$

Inverse Properties

$\cos\left(\cos^{-1}(x)\right)=x$ $\quad \cos^{-1}\left(\cos(\theta)\right)=\theta$

$\sin\left(\sin^{-1}(x)\right)=x$ $\quad \sin^{-1}\left(\sin(\theta)\right)=\theta$

$\tan\left(\tan^{-1}(x)\right)=x$ $\quad \tan^{-1}\left(\tan(\theta)\right)=\theta$

Domain and Range

Function	Domain	Range
$y=\sin^{-1}x$	$-1\le x\le 1$	$-\frac{\pi}{2}\le y\le\frac{\pi}{2}$
$y=\cos^{-1}x$	$-1\le x\le 1$	$0\le y\le\pi$
$y=\tan^{-1}x$	$-\infty<x<\infty$	$-\frac{\pi}{2}<y<\frac{\pi}{2}$

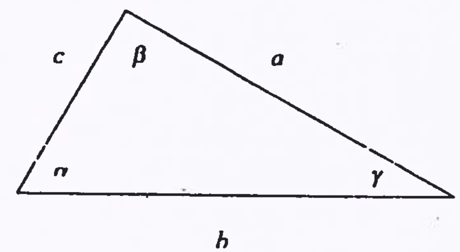
Alternate Notation

$\sin^{-1}x=\arcsin x$

$\cos^{-1}x=\arccos x$

$\tan^{-1}x=\arctan x$

Law of Sines, Cosines and Tangents



Law of Sines

$$\frac{\sin\alpha}{a}=\frac{\sin\beta}{b}=\frac{\sin\gamma}{c}$$

Law of Cosines

$$a^2=b^2+c^2-2bc\cos\alpha$$

$$b^2=a^2+c^2-2ac\cos\beta$$

$$c^2=a^2+b^2-2ab\cos\gamma$$

Mollweide's Formula

$$\frac{a+b}{c}=\frac{\cos\frac{1}{2}(\alpha-\beta)}{\sin\frac{1}{2}\gamma}$$

Law of Tangents

$$\frac{a-b}{a+b}=\frac{\tan\frac{1}{2}(\alpha-\beta)}{\tan\frac{1}{2}(\alpha+\beta)}$$

$$\frac{b-c}{b+c}=\frac{\tan\frac{1}{2}(\beta-\gamma)}{\tan\frac{1}{2}(\beta+\gamma)}$$

$$\frac{a-c}{a+c}=\frac{\tan\frac{1}{2}(\alpha-\gamma)}{\tan\frac{1}{2}(\alpha+\gamma)}$$