Functions and Graphs

Constant Function

$y = a$ or $f(x) = a$

Graph is a horizontal line passing through the point $(0, a)$

Line/Linear Function

$y = mx + b$ or $f(x) = mx + b$

Graph is a line with point $(0, b)$ and slope m

Slope

Slope of the line containing the two points (x_1, y_1) and (x_2, y_2) is

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{rise}}{\text{run}}$$

Slope – intercept form

The equation of the line with slope m and y-intercept $(0, b)$ is

$$y = mx + b$$

Point – Slope form

The equation of the line with slope m and passing through the point (x_1, y_1) is

$$y = y_1 + m(x - x_1)$$

Parabola/Quadratic Function

$y = a(x-h)^2 + k$ $f(x) = a(x-h)^2 + k$

The graph is a parabola that opens up if $a > 0$ or down if $a < 0$ and has a vertex at (h, k)

Parabola/Quadratic Function

$y = ax^2 + bx + c$ $f(x) = ax^2 + bx + c$

The graph is a parabola that opens up if $a > 0$ or down if $a < 0$ and has a vertex at $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$

Parabola/Quadratic Function

$x = ay^2 + by + c$ $g(y) = ay^2 + by + c$

The graph is a parabola that opens right if $a > 0$ or left if $a < 0$ and has a vertex at $\left(g\left(-\frac{b}{2a}\right), -\frac{b}{2a}\right)$

Circle

$(x-h)^2 + (y-k)^2 = r^2$

Graph is a circle with radius r and center (h, k)

Ellipse

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

Graph is an ellipse with center (h, k) with vertices a units right/left from the center and vertices b units up/down from the center.

Hyperbola

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

Graph is a hyperbola that opens left and right, has a center at (h, k), vertices a units left/right of center and asymptotes that pass through center with slope $\pm\frac{b}{a}$.

Hyperbola

$$\frac{(y-k)^2}{b^2} - \frac{(x-h)^2}{a^2} = 1$$

Graph is a hyperbola that opens up and down, has a center at (h, k), vertices b units up/down from the center and asymptotes that pass through center with slope $\pm\frac{b}{a}$.

Common Algebraic Errors

Error	Reason/Correct/Justification/Example
$\frac{2}{0} \neq 0$ and $\frac{2}{0} \neq 2$	Division by zero is undefined!
$-3^2 \neq 9$	$-3^2 = -9$, $(-3)^2 = 9$ Watch parenthesis!
$(x^2)^3 \neq x^5$	$(x^2)^3 = x^2x^2x^2 = x^6$
$\frac{a}{b+c} \neq \frac{a}{b} + \frac{a}{c}$	$\frac{1}{2} = \frac{1}{1+1} \neq \frac{1}{1} + \frac{1}{1} = 2$
$\frac{1}{x^2 + x^3} \neq x^{-2} + x^{-3}$	A more complex version of the previous error.
$\frac{\cancel{a} + bx}{\cancel{a}} \neq 1 + bx$	$\frac{a+bx}{a} = \frac{a}{a} + \frac{bx}{a} = 1 + \frac{bx}{a}$ Beware of incorrect canceling!
$-a(x-1) \neq -ax - a$	$-a(x-1) = -ax + a$ Make sure you distribute the "-"!
$(x+a)^2 \neq x^2 + a^2$	$(x+a)^2 = (x+a)(x+a) = x^2 + 2ax + a^2$
$\sqrt{x^2 + a^2} \neq x + a$	$5 = \sqrt{25} = \sqrt{3^2 + 4^2} \neq \sqrt{3^2} + \sqrt{4^2} = 3 + 4 = 7$
$\sqrt{x+a} \neq \sqrt{x} + \sqrt{a}$	See previous error.
$(x+a)^n \neq x^n + a^n$ and $\sqrt[n]{x+a} \neq \sqrt[n]{x} + \sqrt[n]{a}$	More general versions of previous three errors
$2(x+1)^2 \neq (2x+2)^2$	$2(x+1)^2 = 2(x^2 + 2x + 1) = 2x^2 + 4x + 2$ $(2x+2)^2 = 4x^2 + 8x + 4$ Square first then distribute!
$(2x+2)^2 \neq 2(x+1)^2$	See the previous example. You can not factor out a constant if there is a power on the parethesis!
$\sqrt{-x^2 + a^2} \neq -\sqrt{x^2 + a^2}$	$\sqrt{-x^2 + a^2} = (-x^2 + a^2)^{\frac{1}{2}}$ Now see the previous error
$\frac{a}{\left(\frac{b}{c}\right)} \neq \frac{ab}{c}$	$\frac{a}{\left(\frac{b}{c}\right)} = \frac{\left(\frac{a}{1}\right)}{\left(\frac{b}{c}\right)} = \left(\frac{a}{1}\right)\left(\frac{c}{b}\right) = \frac{ac}{b}$
$\frac{\left(\frac{a}{b}\right)}{c} \neq \frac{ac}{b}$	$\frac{\left(\frac{a}{b}\right)}{c} = \frac{\left(\frac{a}{b}\right)}{\left(\frac{c}{1}\right)} = \left(\frac{a}{b}\right)\left(\frac{1}{c}\right) = \frac{a}{bc}$

Basic Properties & Facts

Arithmetic Operations

$ab+ac=a(b+c)$ $\quad a\left(\frac{b}{c}\right)=\frac{ab}{c}$

$\frac{\left(\frac{a}{b}\right)}{c}=\frac{a}{bc}$ $\quad \frac{a}{\left(\frac{b}{c}\right)}=\frac{ac}{b}$

$\frac{a}{b}+\frac{c}{d}=\frac{ad+bc}{bd}$ $\quad \frac{a}{b}-\frac{c}{d}=\frac{ad-bc}{bd}$

$\frac{a-b}{c-d}=\frac{b-a}{d-c}$ $\quad \frac{a+b}{c}=\frac{a}{c}+\frac{b}{c}$

$\frac{ab+ac}{a}=b+c,\ a\neq 0$ $\quad \frac{\left(\frac{a}{b}\right)}{\left(\frac{c}{d}\right)}=\frac{ad}{bc}$

Exponent Properties

$a^n a^m = a^{n+m}$ $\quad \frac{a^n}{a^m}=a^{n-m}=\frac{1}{a^{m-n}}$

$\left(a^n\right)^m=a^{nm}$ $\quad a^0=1,\ a\neq 0$

$(ab)^n=a^n b^n$ $\quad \left(\frac{a}{b}\right)^n=\frac{a^n}{b^n}$

$a^{-n}=\frac{1}{a^n}$ $\quad \frac{1}{a^{-n}}=a^n$

$\left(\frac{a}{b}\right)^{-n}=\left(\frac{b}{a}\right)^n=\frac{b^n}{a^n}$ $\quad a^{\frac{n}{m}}=\left(a^{\frac{1}{m}}\right)^n=\left(a^n\right)^{\frac{1}{m}}$

Properties of Radicals

$\sqrt[n]{a}=a^{\frac{1}{n}}$ $\quad \sqrt[n]{ab}=\sqrt[n]{a}\sqrt[n]{b}$

$\sqrt[m]{\sqrt[n]{a}}=\sqrt[nm]{a}$ $\quad \sqrt[n]{\frac{a}{b}}=\frac{\sqrt[n]{a}}{\sqrt[n]{b}}$

$\sqrt[n]{a^n}=a$, if n is odd

$\sqrt[n]{a^n}=|a|$, if n is even

Properties of Inequalities

If $a<b$ then $a+c<b+c$ and $a-c<b-c$

If $a<b$ and $c>0$ then $ac<bc$ and $\frac{a}{c}<\frac{b}{c}$

If $a<b$ and $c<0$ then $ac>bc$ and $\frac{a}{c}>\frac{b}{c}$

Properties of Absolute Value

$$|a|=\begin{cases} a & \text{if } a\geq 0 \\ -a & \text{if } a<0 \end{cases}$$

$|a|\geq 0$ $\quad |-a|=|a|$

$|ab|=|a||b|$ $\quad \left|\frac{a}{b}\right|=\frac{|a|}{|b|}$

$|a+b|\leq |a|+|b|$ Triangle Inequality

Distance Formula

If $P_1=(x_1,y_1)$ and $P_2=(x_2,y_2)$ are two points the distance between them is

$$d(P_1,P_2)=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

Complex Numbers

$i=\sqrt{-1}$ $\quad i^2=-1$ $\quad \sqrt{-a}=i\sqrt{a},\ a\geq 0$

$(a+bi)+(c+di)=a+c+(b+d)i$

$(a+bi)-(c+di)=a-c+(b-d)i$

$(a+bi)(c+di)=ac-bd+(ad+bc)i$

$(a+bi)(a-bi)=a^2+b^2$

$|a+bi|=\sqrt{a^2+b^2}$ Complex Modulus

$\overline{(a+bi)}=a-bi$ Complex Conjugate

$\overline{(a+bi)}(a+bi)=|a+bi|^2$

Logarithms and Log Properties

Definition

$y=\log_b x$ is equivalent to $x=b^y$

Example

$\log_5 125=3$ because $5^3=125$

Special Logarithms

$\ln x=\log_e x$ natural log

$\log x=\log_{10} x$ common log

where $e=2.718281828\ldots$

Logarithm Properties

$\log_b b=1$ $\quad \log_b 1=0$

$\log_b b^x=x$ $\quad b^{\log_b x}=x$

$\log_b\left(x^r\right)=r\log_b x$

$\log_b(xy)=\log_b x+\log_b y$

$\log_b\left(\frac{x}{y}\right)=\log_b x-\log_b y$

The domain of $\log_b x$ is $x>0$

Factoring and Solving

Factoring Formulas

$x^2-a^2=(x+a)(x-a)$

$x^2+2ax+a^2=(x+a)^2$

$x^2-2ax+a^2=(x-a)^2$

$x^2+(a+b)x+ab=(x+a)(x+b)$

$x^3+3ax^2+3a^2x+a^3=(x+a)^3$

$x^3-3ax^2+3a^2x-a^3=(x-a)^3$

$x^3+a^3=(x+a)\left(x^2-ax+a^2\right)$

$x^3-a^3=(x-a)\left(x^2+ax+a^2\right)$

$x^{2n}-a^{2n}=\left(x^n-a^n\right)\left(x^n+a^n\right)$

If n is odd then,

$x^n-a^n=(x-a)\left(x^{n-1}+ax^{n-2}+\cdots+a^{n-1}\right)$

x^n+a^n

$=(x+a)\left(x^{n-1}-ax^{n-2}+a^2x^{n-3}-\cdots+a^{n-1}\right)$

Quadratic Formula

Solve $ax^2+bx+c=0$, $a\neq 0$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

If $b^2-4ac>0$ - Two real unequal solns.

If $b^2-4ac=0$ - Repeated real solution.

If $b^2-4ac<0$ - Two complex solutions.

Square Root Property

If $x^2=p$ then $x=\pm\sqrt{p}$

Absolute Value Equations/Inequalities

If b is a positive number

$|p|=b \Rightarrow p=-b$ or $p=b$

$|p|<b \Rightarrow -b<p<b$

$|p|>b \Rightarrow p<-b$ or $p>b$

Completing the Square

Solve $2x^2-6x-10=0$

(1) Divide by the coefficient of the x^2

$$x^2-3x-5=0$$

(2) Move the constant to the other side.

$$x^2-3x=5$$

(3) Take half the coefficient of x, square it and add it to both sides

$$x^2-3x+\left(-\frac{3}{2}\right)^2=5+\left(-\frac{3}{2}\right)^2=5+\frac{9}{4}=\frac{29}{4}$$

(4) Factor the left side

$$\left(x-\frac{3}{2}\right)^2=\frac{29}{4}$$

(5) Use Square Root Property

$$x-\frac{3}{2}=\pm\sqrt{\frac{29}{4}}=\pm\frac{\sqrt{29}}{2}$$

(6) Solve for x

$$x=\frac{3}{2}\pm\frac{\sqrt{29}}{2}$$